International Journal of Modern Physics A
Vol. 29, No. 32 (2014) 1430076 (10 pages)

DOI: 10.1142/S0217751X14300762

World Scientific
www.worldscientific.com

From symmetries to quarks and beyond*

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Received 11 September 2014
Accepted 15 September 2014
Published 30 December 2014

Attempts to understand the plethora of meson baryon and meson resonances by the introduction of symmetries, which led to the invention of quarks and the quark model, and finally to the formulation of QCD, are described.

1. Introduction

In this paper[a] I would like to look at the sequence of events that led to the quark model, how it evolved, and some of its consequences. As always, these events did not follow a simple linear path. This journey went on for about 25 years, from the late 1950s to the mid-1970s. During this exciting period, there was a happy confluence of lots of data to be explained and some imaginative theoretical constructs. Avoiding some dead ends, elementary particle physics progressed from the Sakata model,[1] to the symmetry era culminating in the Eightfold Way of Gell-Mann[2] and Ne'eman,[3] to quarks and the simple quark model, to the study of SU(6), the introduction of color, and eventually to Quantum Chromodynamics (QCD). The interplay between experiment and theory was crucial to the progression of our understanding of each of these topics.

2. Personal Perspective

I was fortunate to be at the Weizmann Institute in the fall of 1961. Carl Levinson and I had been using the group SU(3) for dynamical nuclear physics calculations, first at Princeton University in 1960 and then at Weizmann during the winter and spring of 1961. SU(3) is the group of unitary 3 × 3 matrices with determinant 1.

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[*] This article will also appear in "50 Years of Quarks," edited by H. Fritzsch and M. Gell-Mann (World Scientific, 2015).
[a] LIGO document number: P1400170.

We had returned from giving talks about our nuclear structure work at conferences in Europe in the fall of 1961, Carl at the Varenna summer school, and I, at a conference in Manchester. We arrived just in time to hear a seminar by Yuval Ne'eman, in which he described his PhD thesis. Working at Imperial College with Abdus Salam as his advisor, Yuval had produced work that paralleled that which Murray Gell-Mann had produced at Caltech. At no point in the seminar, did Ne'eman use the term SU(3), although it was evident that he had made great use of it in the transformations presented in his talk. When the seminar was over, Carl and I rushed up to Yuval, and asked him, "Isn't this just SU(3)?" Yuval agreed that it was. We said, "We know all about SU(3), but we do not know any particle physics. Please give us something to read. Yuval did, and we were on our merry way to learning about a whole new field. That very first night we were able to write down the mass breaking formula, but we did not know that the breaker had to transform like an $I = 0$, $Y = 0$ member of an octet. It took Murray to point out this crucial fact at the CERN conference in the summer of 1962.[4] Okubo,[5] independently, wrote down the mass formula

$$M = m_0\{1 + aY + b[I(I+1) - Y^2/4]\}\,. \tag{1}$$

I is the isospin and Y is the hypercharge.

3. G2 versus SU(3)

During the period 1960–1961, a big question was whether the proper classification group for the ever-growing list of mesons and baryons was G2 or SU(3). Ralph Behrends[6] was a proponent of G2, whereas Gell-Mann and Ne'eman advocated SU(3). The deciding factor for which extension of SU(2) isospin symmetry to use was the prediction for the number of pseudoscalar mesons. At the time, there existed seven pseudoscalars, namely three pions and four kaons. G2 predicted that there should be seven pseudoscalars, whereas SU(3) predicted that there should be an additional $I = 0$, $Y = 0$ meson. The issue was settled with the discovery of the pseudoscalar $\eta(548)$ meson, so G2 was ruled out and SU(3) was deemed the correct choice.

4. Sakata Model and its Demise

An early model (1956) to describe the baryons and mesons was the Sakata model. It was based on a fundamental triplet, composed of the physical proton, neutron and $\Lambda(1115)$ particles, called B. In this model, mesons were formed as $B\bar{B}$ ($3 \times \bar{3}$) composites, giving the now familiar octets and singlet. The pseudoscalar mesons could be accommodated in an octet of SU(3) as well. However, it was not clear in which representations the eight spin-$\frac{1}{2}$ baryons should be in the Sakata Model, whereas in the Eightfold Way they could be accommodated in a single octet. Fortunately, we (C. A. Levinson, H. J. Lipkin, S. Meshkov, A. Salam and R. Munir),[7] were able to eliminate the Sakata model by looking at the prediction for proton anti-proton annihilation going into $K_L K_S$ compared to $K_L K_L$. The Sakata model forbids annihilation into $K_L K_S$ pairs, whereas it is allowed in the Eightfold Way. Experimentally, these decays are produced at a macroscopic rate, so the Sakata model was ruled out.

An aspect of the Sakata model that turned out to be very useful was that to describe meson decays to two other mesons, the couplings for $BB\bar{B}\bar{B}$ were needed. Fortunately, Ikeda, Ogawa and Ohnuki,[8] and Sawada and Yonezawa[9] had produced tables of these. From their work, I was able to abstract a complete set of 8×8 Clebsch–Gordan coefficients for SU(3), which Levinson, I and Harry Lipkin, who joined us a few weeks after we heard Ne'eman's seminar, exploited in our subsequent work.

5. Breaking SU(3)

Once it was established that SU(3) was the correct symmetry model of the strong interaction, there were some obvious problems. Just looking at the wide range of masses of the mesons and baryons, in their respective multiplets, it was clear that there was a large symmetry breaking going on. This was explained by Gell-Mann[2] and Okubo,[3] in 1962. They assumed that the symmetry breaker transformed like an $I = 0$, $Y = 0$ member of an octet. This neatly explained and correlated the observed splittings.

From the fall of 1961 through 1963 there was a lot of activity on the symmetry front. Using what we now call flavor SU(3) was not the favored area in which to work for most theorists. Many were busy in the complex plane and did not take kindly to the idea of using symmetries. However, there gradually developed a number of physicists in the United States, Israel and Europe who were interested in exploring the use and properties of SU(3) symmetry.

I enjoyed working on SU(3) with Carl Levinson and Harry Lipkin at the Weizmann Institute, and with Gaurang Yodh and George Snow at the University of Maryland. In our early work Levinson, Lipkin and I made copious use of Weyl reflections and applied them to decay widths, scattering amplitudes in hadronic processes, photoproduction and other electromagnetic processes.[10–12] E. C. G. Sudarshan and his group at Syracuse did analogous work.[13–16]

Later, we invented the U-spin and V-spin subgroups of SU(3)[17] and observed that the photon is a U-spin scalar,[12] useful in dealing with electromagnetic processes. The classification of the decuplet, baryon octet, and the pseudoscalar and vector meson octets according to I-spin and U-spin assignments are illustrated in the figure above. The upper part of Fig. 1 shows the usual I-spin display. The lower part shows the U-spin display of these particles. Just as I-spin transformations are perpendicular to axis 1, so U-spin transformations are perpendicular to axis 2. U-spin multiplets all have the same electric charge. V-spin transformations are perpendicular to axis 3.

Yodh, Snow and I made the first successful comparison of SU(3) predictions with experiment for scattering processes.[18] In the modern era, Jonathan Rosner

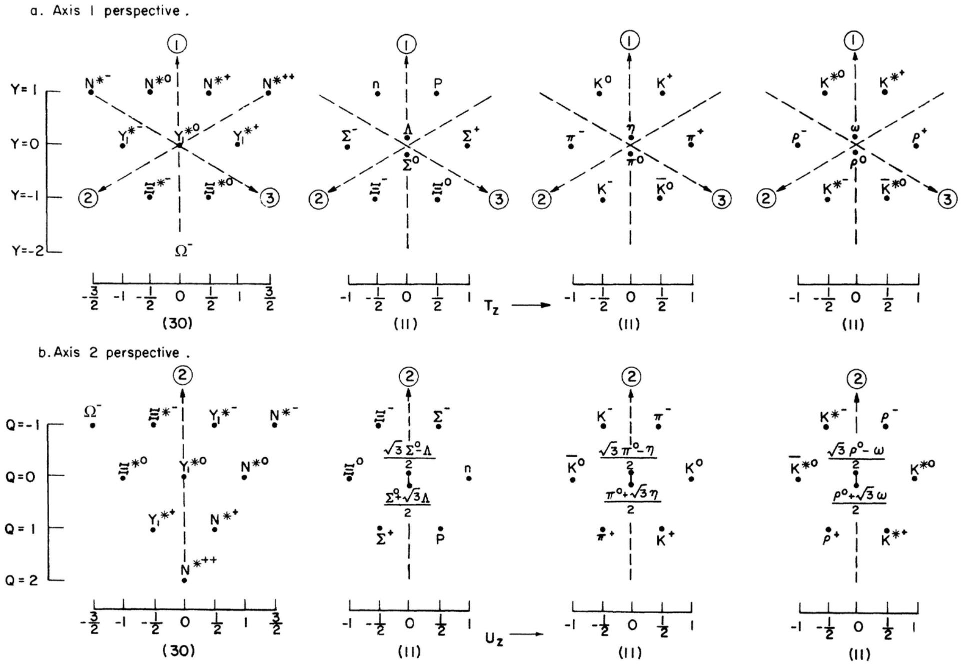

Fig. 1. Representation diagrams of the group SU_3. The (30) baryon-resonance representation diagram and the (11) baryon, meson and vector-meson representation diagrams are displayed.

and Michael Gronau continue to make extensive use of U-spin in studying CP asymmetry in the weak decays of B mesons.[19]

As mentioned earlier, there was lots of data — some good, some not — that asked for explanation. Both meson and baryon resonances were being produced at a great rate. Knowing which new resonance was real was difficult, especially for a theorist. My favorite way of determining which result to believe was to consult Nick Samios at Brookhaven National Laboratory. He was never wrong!

6. SU(3) Wins

In SU(3) the product:

$$8 \times 8 = 27 + 10 + \overline{10} + 8 + 8 + 1\,. \tag{2}$$

For baryons and baryon resonances, only 10, 8 and 1 multiplets existed. Why there were no 27 or $\overline{10}$ representations, no one knew. The spin-$\frac{3}{2}$ positive parity decuplet, 10, was supposed to have a linear mass spacing, according to the Gell-Mann Okubo mass formula. The mass splitting formula, Eq. (1), simplifies for the decuplet to:

$$M = M_0(1 + a'Y)\,, \tag{3}$$

the familiar linear mass relation. This ordering is illustrated in the figure above. The non-strange N^* (1236) was the lowest state. The strangeness -1 Σ (1385) was next. The strangeness -2 state of the decuplet, the Ξ^*, was predicted to be at 1533 MeV. Once it was found at its predicted mass, the general belief that SU(3) was a good symmetry grew. To confirm this belief, it was necessary to find the last member of the decuplet, the strangeness -3 Ω^-. In 1963, Samios and his group at Brookhaven,[20] found the Ω^-, exactly where it was supposed to be, with a mass of 1672 MeV, once again in accordance with the SU(3) prediction. Joy reigned.

The mesons were even more restricted in representation content than the baryons, with only 8*s* and 1*s* occurring. The question at the time was how to explain the lack of 27 and $\overline{10}$ representations for both baryons and mesons as well as the lack of 10s for mesons.

7. Quarks and Aces

In one of the cleverest and simplest inventions in physics history, this mystery was solved. In 1964, Murray Gell-Mann[21] and, independently, George Zweig,[22] proposed a simple mechanism to get only 10, 8 and 1 for baryons, and 8 and 1 for mesons. They introduced a fundamental triplet of fractionally charged objects, (u, d, s) which Gell-Mann called quarks and which Zweig called aces. The u quark has charge 2/3 while the d and s quarks each had charge $-1/3$.

The simple SU(3) multiplications

$$3 \times 3 \times 3 = 10 + 8 + 8 + 1 \quad \text{for baryons, and} \tag{4}$$

$$3 \times \bar{3} = 8 + 1 \quad \text{for mesons} \tag{5}$$

produced the representations that occurred in nature, for baryons and mesons of various spins and parities. The quarks in the triplets, with fractional charge, were peculiar objects. Whether they could be physically detected was a matter of hot debate at the time. Were they an index symmetry or real physical particles? Many exotic experiments were carried out, including looking for them in oyster shells, with no success. Whatever they were, their introduction solved a longstanding mystery. For several years, the group SU(3) had been successful in many aspects, but there had been no real understanding of what the 3 was. Now we knew — quarks! Later, the fact that no free quarks were observed was embedded into the modern view of confinement, described by the theory of asymptotic freedom and QCD.

A proposal that lent additional credence to the validity of the quark model was the Zweig rule,[22] proposed also by Okubo,[23] that the decay of the ϕ meson into K^+K^- and not into $\rho\pi$ was due to the fact that the ϕ meson is an $s\bar{s}$ composite. This accounts for the narrowness of the decay width.

Once we had the quark model it was easy to understand the structure of the baryon octet and decuplet. The neutron, proton and N^* (1236) were made of u and d quarks, the Λ, Σ and Y_1^* were made of u, d and one s quark. The Ξ^* was made of one non-strange quark and two strange quarks, and the Ω^- contained three strange quarks.

The quark model gave a simple interpretation of Cabibbo's observation[24] in the prequark SU(3) era that the weak current mixed non-strange and strange weak decay amplitudes. It could now be interpreted as a mixture of d and s currents. He found that strangeness non-changing decays dominated, with a coupling $G\cos\theta_c$, compared to the weaker strangeness changing decay strength $G\sin\theta_c$. G is a universal weak coupling strength and θ_c is the Cabibbo angle, 13 degrees.

8. Gluons and Nambu

After fractionally charged quarks and aces were proposed, Han and Nambu[25] introduced an alternative scheme which included three triplets of integrally charged fundamental particles, held together by the exchange of vector gauge bosons, that we now call gluons. Although the integrally charged particles are no longer viable candidates, the concept of the gluon introduced by Fritzsch and Gell-Mann, eventually was experimentally verified, flourished and is now part of our field theory of strong interactions, QCD.

9. SU(6) and Color

Almost immediately after quarks were proposed they were given spin by Beg, Lee, Pais,[26] Pais,[27] Radicati and Gursey[28] and slightly later by Sakita and Wali.[29] When quarks are given a spin, however, there is a spin statistics problem. The SU(6) multiplets come from combining three quarks, each with spin-$\frac{1}{2}$, written as 6.

Baryons are the composites:

$$6 \times 6 \times 6 = 56 + 70 + 70 + 20\,, \tag{6}$$

where $56 = 10^{3/2} + 8^{1/2}$. (SU(6) may be decomposed as SU(3) flavor × SU(2) spin). The problem that arose was that 56 is a symmetric combination, and the proton must be an antisymmetric state under the exchange of the constituents. What to do?

O. W. Greenberg,[30] on leave from the University of Maryland at the Institute for Advanced Study (IAS), solved that problem. He invoked parastatistics, now called color, and explicitly wrote down all of the baryon states in an $\mathrm{SU}(6) \times \mathrm{O}(3)$ model, though he did not call it that. In modern parlance, the symmetry problem for the 56 was solved by combining it with an antisymmetric color singlet giving a totally antisymmetric state.

Only color singlets are allowed in this scheme. (Periodic attempts have been made, at times, to invoke colored states, but to no avail.) Note that Sudarshan and Mahanthappa,[31] in a paper entitled, "SU(6)×O(3) Structure of Strongly Interacting Particles," also examined this problem, as did Richard Dalitz.[32]

In addition to solving the symmetry problem, the success of the approach described above, also cleared up a then extant problem of exactly how to describe the large catalog of resonances. Were baryon resonances described as composites of 4 quarks and an antiquark or as qqq in an L wave, and were mesons to be regarded, analogously, as $qq\bar{q}\bar{q}$ or $q\bar{q}$ in an L wave? The simplicity of the $\mathrm{SU}(6) \times \mathrm{O}(3)$ model, which included color, was almost universally accepted, and answered this question. A remarkable transition had taken place over a relatively short time span. Originally, we developed a symmetry description, based on the flavor group SU(3), which, while yielding many useful results, did not have a theoretical underpinning. The quark model of Gell-Mann and Zweig provided the basis for these successes. We have several earlier examples in physics and chemistry, where much was accomplished by developing and exploiting ad hoc models that "worked" and eventually led to the formulation of the true underlying principles. One example is the success in building the periodic chart long before we understood atomic structure, that the nucleus of the atom contained neutrons and protons, and that quantum mechanics was the fundamental theory that led to our true understanding of atomic physics. Another example, a bit later, was the development of the Bohr atom.

10. Combining Internal and Space-Time Symmetries

It was clearly interesting to try to combine internal and space-time symmetries. This effort took place all over the world through 1964 and 1965. My memory — a bit hazy since it was 50 years ago — was of going to the second Coral Gables Conference in January 1965 and hearing presentations by Salam, and several other groups claiming to have solved the problem by invoking the ill-fated symmetry $U(\widetilde{12})$. Shortly thereafter, I went to visit at Weizmann Institute and Harry Lipkin and I began to look at various subgroups of the symmetry. We found that with a subgroup decomposition into a particular SU(6) × SU(2) we could, within the SU(6), combine internal symmetries with a restricted version of the Lorentz Transformation. We were able to do this for collinear processes such as two body decays (3-point functions), but not for scattering amplitudes (4 point functions). We named the relevant SU(2) space-time symmetry W-spin and, combining it with flavor SU(3), called the combined symmetry $\mathrm{SU}(6)_W$.[33,34] W stood for Weizmann Institute. We did this with constituent quarks and learned that Dashen and Gell-Mann[35] had done similar work but with current quarks. In fact, at the then annual Washington APS meeting in the spring of 1965, Murray rushed up to me and said, excitedly, "Don't worry. Your work is OK." By that time, it had been accepted that $U(\widetilde{12})$ was not a good symmetry, but Murray was pointing out that our $\mathrm{SU}(6)_W$ subgroup symmetry was fine. Barnes, Carruthers and Von Hippel[36] also did analogous work.

The W-spin operators are invariant under Lorentz Transformations in the z direction, so it is a collinear symmetry. The W-spin classification for a particle with arbitrary momentum in the z direction is the same as the classification at rest. The generators of $\mathrm{SU}(2)_W$ are:

$$W_z = \sigma_z/2\,, \tag{7}$$

$$W_x = \beta\sigma_x/2\,, \tag{8}$$

$$W_y = \beta\sigma_y/2\,, \tag{9}$$

where β is the intrinsic parity of spin-$\frac{1}{2}$ particles in the rest frame.

A virtue of this symmetry is that it correctly describes decays that are forbidden in the standard SU(6) approach. For example the decay, $\rho \to \pi\pi$, was forbidden in the usual SU(6) but was allowed in SU$(6)_W$. Later, in 1973, together with Fred Gilman and Moshe Kugler,[37,38] and making use of the Melosh transformation[39] between constituent quarks and current quarks, we successfully analyzed the decay amplitudes of a myriad of meson and baryon resonances that had been produced in a SLAC partial wave analysis of $\pi N \to \pi\pi N$ and $\gamma N \to \pi\pi N$ experiments.[40]

11. Higher Mass Quarks

There had been predictions for the existence of a fourth heavier quark by Bjorken and Glashow[41] in 1964, shortly after quarks were invented and somewhat later in 1970, by Glashow, Illiopoulos and Maiani.[42] These predictions were fulfilled in 1974 by the discovery of the J/Ψ resonance at 3.10 GeV by a BNL group headed by Sam Ting,[43] and a SLAC group headed by Burton Richter.[44] The J/Ψ is a very narrow $c\bar{c}$ resonance. The charmed quark c with a charge of 2/3 has a mass of 1.275 GeV and charge +2/3. Shortly thereafter, the b quark with mass 4.18 GeV and a charge of −1/3 was discovered. The last quark to be found was the top quark, t, with a huge mass of 173 GeV and charge of +2/3. Searches for higher mass quarks have not yielded evidence for any new quarks. With these three heavy quarks, the Cabibbo model for the light quark transitions has been expanded to give the Cabibbo, Kobayashi, Maskawa 3×3 transition matrix.[45] The spectroscopy related to the c and b quarks is vast. In fact, it is much more extensive than that of the original u, d, s system. A prescient paper by Appelquist and Politzer,[46] written before the discovery of the J/Ψ resonance, is a guide to the study of this fertile heavy quark spectroscopy.

12. The Path to Quantum Chromodynamics (QCD)

Just as the path from symmetries led to the invention of the concept of quarks, so the sparkling success of the quark model in so many areas culminated in leading Harald Fritzsch and Murray Gell-Mann to the formulation of Quantum Chromodynamics.[47–49] Quantum Chromodynamics is a non-Abelian quantum field theory of strong interactions. In the QCD Lagrangian, which is of a Yang–Mills type, quarks, which come in three colors, are coupled to an octet of colored gluons. All physically observable systems are SU(3) color singlets. This quark–gluon field theory incorporates confinement of all colored states such as quarks and gluons. It is a major component of the Standard Model of Elementary Particle Physics.

13. Final Comments

Writing this historical review has reminded me of the travails and joys that accompanied our progress over the quarter century involved. We proceeded for a long time without a fundamental theory, piecing together an array of disparate experimental clues, interspersed with occasional clever theoretical constructs. The process marked the importance of invoking new mathematical techniques, in this case, group theory. The gradual acceptance of the role of unitary groups, in particular, marked a big change in the attitudes of physicists, many of whom preferred more analytic approaches. Fortunately, Harald Fritzsch and Murray Gell-Mann were clever enough to produce a grand synthesis of the earlier endeavors that culminated in the formulation of Quantum Chromodynamics.

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