CALT-68-2433
HUTP-03/A020
hep-th/0303063

Gravity Induced C-Deformation

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We study F-terms describing coupling of the supergravity to $\mathcal{N} = 1$ supersymmetric gauge theories which admit large N expansions. We show that these F-terms are given by summing over genus one non-planar diagrams of the large N expansion of the associated matrix model (or more generally bosonic gauge theory). The key ingredient in this derivation is the observation that the chiral ring of the gluino fields is deformed by the supergravity fields, generalizing the C-deformation which was recently introduced. The gravity induced part of the C-deformation can be derived from the Bianchi identities of the supergravity, but understanding gravitational corrections to the F-terms requires a non-traditional interpretation of these identities.

1. Introduction

The connection between supersymmetric gauge theories and matrix models (or more generally bosonic gauge theories) has led to exact non-perturbative computation of F-terms starting from perturbative computations in the gauge theory [1]. In the context of gauge theory on flat space, only the planar diagrams are relevant for the computation of F-terms. However if one goes beyond flat space or consider certain deformations, it is expected that the non-planar diagrams become relevant for computing F-terms. In particular in a recent paper [2], we introduced the notion of the C-deformation of $\mathcal{N}=1$ gauge theories. Without the deformation, the gluino fields $\mathcal{W}_\alpha$ in these theories satisfy the chiral ring relation,

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 0, \tag{1.1}$$

as pointed out in [3]. This relation plays an important role in classifying chiral primary fields in these theories. In [2], we showed that a self-dual two-form $F_{\alpha\beta}$ can be used to deform this relation as

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = F_{\alpha\beta}. \tag{1.2}$$

In string theory, $F_{\alpha\beta}$ has the interpretation as the graviphoton field strength of the $\mathcal{N}=2$ supergravity coupled to the branes. We can view this as the *defining property* of the gluino fields, modifying the condition that they be Grassmannian variables. We called this the C-deformation and showed that the non-planar diagrams of matrix models captures the $F_{\alpha\beta}F^{\alpha\beta}$ dependence of the glueball superpotential.

Another place where non-planar diagrams should enter involves gravitational corrections. In particular it was conjectured in [1] that certain R^2 type terms can be computed exactly by studying the non-planar perturbative gauge theory amplitudes with a single handle. They are expressed in terms of the glueball fields and evaluated at the extremum of the superpotential computed by the planar diagrams. This conjecture was motivated by the meaning of topological string amplitudes in the context of low energy effective theories of superstring compactifications [4,5,6,7] together with the large N duality conjectures [8], proven in [9] and embedded in superstrings in [7]. This prediction has already been tested in a number of cases: for the gravitational correction for $\mathcal{N}=4$ Yang-Mills in the third paper in [1], and for certain $\mathcal{N}=2$ supersymmetric gauge systems in [10,11]. Our aim in this paper is to prove this conjecture.

In [2], we showed that the effective superpotential of the C-deformed gauge theory (1.2) is computed by the full matrix model partition function including non-planar diagrams. More explicitly, if we define the glueball superfield S_i by

$$S_i = \frac{1}{32\pi^2}\epsilon^{\alpha\beta}\mathrm{Tr}_i \mathcal{W}_\alpha \mathcal{W}_\beta, \tag{1.3}$$

where Tr_i is over the i-th gauge group of rank N_i, their effective superpotential is given by

$$\Gamma_1 = \sum_{g=0}^{\infty} \int d^4x d^2\theta \ (F_{\alpha\beta}F^{\alpha\beta})^g \ N_i \frac{\partial F_g}{\partial S_i}(S), \tag{1.4}$$

where F_g is given by the matrix model partition function computed by a sum over genus g diagrams with S_i playing the role of the 't Hooft loop counting parameters. There is another series of gravitational corrections predicted in [4,7], which takes the form,

$$\Gamma_2 = \sum_{g=1}^{\infty} \ g \int d^4x d^2\theta \ \mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}(F_{\rho\sigma}F^{\rho\sigma})^{g-1}F_g(S), \tag{1.5}$$

where $\mathcal{W}_{\alpha\beta\gamma}$ denotes the $\mathcal{N}=1$ gravitino superfield. In this paper, we will show that (1.4) continues to hold and (1.5) computes the mixed gravitational/glueball superpotential of the $\mathcal{N}=1$ gauge theory if we postulate that the gluino fields obey the relation

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma + F_{\alpha\beta}. \tag{1.6}$$

If we set the Lorentz violating parameter $F_{\alpha\beta}=0$, only planar contribution in (1.4) and genus one contribution in (1.5) survive. From the string theory point of view, the relation arises as follows. The supersymmetry variation of the open string worldsheet with the gravitino background $\mathcal{W}_{\alpha\beta\gamma}$ gives rise to boundary terms. We can cancel these boundary terms and restore the supersymmetry if we assume this relation (1.6). This is essentially the same as the way we derived the C-deformation (1.2) for the graviphoton background.

It turns out that, when $F_{\alpha\beta}=0$, the relation (1.6) can also be understood in the conventional framework of supergravity theory — it follows from the supergravity tensor calculus. This is in contrast to the deformation by $F_{\alpha\beta}$, which does not have such a conventional interpretation via $\mathcal{N}=1$ supersymmetry. However, we will point out that a proper interpretation of the gravitational corrections (1.5) requires a non-traditional interpretation of this standard relation. In particular in the case of $U(1)$ gauge theories, the traditional interpretation of (1.6) would be that the left-hand side and the right-hand side of the equations vanish separately (the left-hand side being zero is due to the Grassmannian property of $\mathcal{W}_a$, and this forces the right-hand side to be equal to zero also). However we shall find that preservation of supersymmetry in the presence of constant $\mathcal{W}_{\alpha\beta\gamma}$ gravitational background requires only the weaker relation where we postulate (1.6) but do not impose the standard Grassmannian properties on $\mathcal{W}_\alpha$. Despite this non-traditional interpretation, this seems to be the natural choice since supersymmetry only requires the weaker relation and it is the one that leads to large N dualities in superstring theory. In particular without this non-traditional interpretation of the relation (1.5), we shall see that the large N superstring duality proposed in [7] would not hold.

It turns out that there are also planar contributions to superpotential terms of the form $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}\ S^n$. However as will be shown in [12], once one substitutes the expectation value for the glueball field which extremizes the superpotential, this contribution becomes trivial. This is also consistent with the large N superstring duality [7] since there is no R^2 correction coming from genus 0 on the closed string dual.

This paper is organized as follows. In section 2, we derive the relation (1.6) from the point of view of string theory. In section 3, we derive the same relation from the supergravity tensor calculus. In section 4, we show that this deformation leads to the gravitational corrections (1.5) including the more general situations not necessarily embedded in string theory. We also discuss certain mixed gravitational/gauge interactions which violate Lorentz invariance and which could serve as an experimental signature for the C-deformation.

2. Deformation of the chiral ring I: string theory perspective

In this section, we consider gravitational corrections to the $\mathcal{N}=1$ gauge theory in four dimensions which is defined as the low energy limit of Type II superstring with D$(N+3)$ branes wrapping on n cycles on a Calabi-Yau three-fold and extending in four flat dimensions. We will concentrate on the universal spacetime part of this computation. Even though the string context may appear to be restrictive (in that one is limited to field theories arising from string theory), the more general field theory setup discussed in [13] can be effectively related to the spacetime part of the string computation, as we have demonstrated in our previous paper [2]. In the string context the perturbative computation is better organized since one worldsheet topology corresponds to many Feynman diagrams.

Once we understand what is going on in string theory, we can directly translate each step into the more general field theory context. This is the reason why we start our discussion from the string theory perspective.

The F-terms of the low energy effective theory are given by (1.4) and (1.5), where

$$F_g(S) = \sum_{h=0}^{\infty} F_{g,h} S^h, \tag{2.1}$$

and $F_{g,h}$ is the topological string partition function for genus g worldsheet with h boundaries ending on D branes wrapping on these cycles.[1] According to [14], these topological string partition functions can be computed using the Chern-Simons theory (or its dimensional reduction). In particular, for a specific class of D5 branes wrapping on 2-cycles, the dimensional reduction of the Chern-Simons theory turns out to be a matrix model [1].

In the previous paper [2], we explained how the gravitational corrections of the type (1.4) arises from the string theory computation and showed that it can also be obtained from purely gauge theoretical Feynman diagram computation if we deform the chiral ring as (1.2). In this paper, we study the second series of gravitational corrections (1.5). As in the previous paper, we start our discussion on the string worldsheet, which we describe using the covariant quantization of superstring developed in [15]. As demonstrated in [16], this is the most economical way to establish the relation between topological string amplitudes and the F-terms in Type II superstring compactified on a Calabi-Yau three-fold, which was originally derived in the NSR formalism in [4,5]. In the formalism of [15], the four-dimensional part of the worldsheet Lagrangian density that is relevant for our discussion is simply given by

$$\mathcal{L} = \frac{1}{2}\partial X^\mu \bar{\partial} X_\mu + p_\alpha \bar{\partial}\theta^\alpha + p_{\dot{\alpha}} \bar{\partial}\theta^{\dot{\alpha}} + \bar{p}_\alpha \partial\bar{\theta}^\alpha + \bar{p}_{\dot{\alpha}} \partial\bar{\theta}^{\dot{\alpha}}, \tag{2.2}$$

where p's are $(1,0)$-forms, $\bar{p}$'s are $(0,1)$-forms, and $\theta, \bar{\theta}$'s are 0-forms. The remainder of the Lagrangian density consists of the topologically twisted $\mathcal{N}=2$ supersymmetric sigma-model on the Calabi-Yau three-fold and a chiral boson which is needed to construct the R current. We work in the chiral representation of supersymmetry, in which spacetime supercharges are given by

$$\begin{aligned} Q_\alpha &= \oint p_\alpha \\ Q_{\dot{\alpha}} &= \oint p_{\dot{\alpha}} - 2i\theta^\alpha \partial X_{\alpha\dot{\alpha}} + \cdots, \end{aligned} \tag{2.3}$$

[1] For simplicity, we consider the case with a single cycle. Correspondingly, there is only one boundary-counting parameter S. Generalization to cases with more cycles is straightforward.

where $X_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} X_{\mu}$, and $\cdots$ in the second line represents terms containing $\theta^{\dot{\alpha}}$ and $\theta^2 = \epsilon_{\alpha\beta}\theta^{\alpha}\theta^{\beta}$. The second set of supercharges $\bar{Q}_{\alpha}, \bar{Q}_{\dot{\alpha}}$ are defined by replacing p, θ by $\bar{p}, \bar{\theta}$. These generate the $\mathcal{N} = 2$ supersymmetry in the bulk. When the worldsheet is ending on D branes and extending in four dimensions, the boundary conditions for the worldsheet variables are given by

$$\begin{aligned} &(\partial - \bar{\partial})X^{\mu} = 0, \\ &\theta^{\alpha} = \bar{\theta}^{\alpha}, \quad p_{\alpha} = \bar{p}_{\alpha}. \end{aligned} \tag{2.4}$$

Here we assume that the boundary is located at Im $z = 0$. These boundary conditions preserve one half of the supersymmetry generated by $Q + \bar{Q}$.

In these conventions, the vertex operators for the graviphoton $F_{\alpha\beta}$ and the gravitino $\mathcal{W}_{\alpha\beta\gamma}$ are given by

$$\int F^{\alpha\beta} p_{\alpha} \bar{p}_{\beta}, \tag{2.5}$$

and

$$\int \mathcal{W}^{\alpha\beta\gamma}\left(p_{\alpha} X_{\beta\dot{\beta}} \bar{\partial} X_{\gamma\dot{\gamma}} + \bar{p}_{\alpha} X_{\beta\dot{\beta}} \partial X_{\gamma\dot{\gamma}}\right)\epsilon^{\dot{\beta}\dot{\gamma}} + \int \mathcal{W}^{\alpha\beta\gamma} p_{\alpha} \bar{p}_{\beta} (\theta_{\gamma} - \bar{\theta}_{\gamma}), \tag{2.6}$$

respectively. The gluino $\mathcal{W}_{\alpha}$ couples to the boundary γ_i of the worldsheet $(i = 1, \cdots, h)$ as

$$\oint_{\gamma_i} \mathcal{W}^{\alpha} p_{\alpha}. \tag{2.7}$$

We can make a simple counting of fermion zero modes to determine topology of worldsheets that contribute to a particular F-term. On a genus g surface with h boundaries, there are $(2g + h - 1)$ zero modes for each p_{α} $(\alpha = 1, 2)$. One possible ways to absorb these zero modes, as was done in [2], is to insert $2g$ graviphotons and $2h - 2$ gluinos. In order for these insertions to actually absorb the zero modes, we need two gluinos for each boundary except for one. We cannot insert gluinos on all boundaries since the sum $\sum_{i=1}^{h} \gamma_i$ is homologically trivial and

$$\sum_{i=1}^{h} \oint_{\gamma_i} p_{\alpha} = 0. \tag{2.8}$$

Therefore the topological string computation on genus g worldsheet with h boundaries gives the combination $NhS^{h-1}(F^2)^g$, where the factor N comes from the gauge group trace on the boundary where the gluino is not inserted, h comes from the choice of such a boundary, each boundary with gluino insertion is counted with the factor $S = \mathrm{Tr}\, \mathcal{W}^2$, and we have $2g$ graviphoton insertions. As we pointed out in [2], there is more to the story — in order to correctly reproduce the F-term computation, we need to take into account the effect due to the deformation of the chiral ring (1.2) — but the counting of the zero modes is correct as it is. Taking into account the C-deformation, we found in the previous paper that the F-term contribution from genus g worldsheet with h boundaries is $F_{g,h}$, and it can be expressed as a sum over the matrix model 't Hooft diagrams of the corresponding topology. This gives rise to the first series of gravitational corrections (1.4).

To understand the second series (1.5), we need to consider two insertions of the gravitino vertex operator (2.6). For simplicity of discussion, let us first turn off $F_{\alpha\beta} = 0$. There are two possible terms for gravitino vertex operator. Either one uses the first part of the gravitino vertex operator (2.6) which involves only one p or the second term which involves two p's. We cannot use mixed types, because that will not lead to absorption of all p zero modes. Note that both types of terms have one net p charge. Thus we can absorb two net p zero modes from the two gravitino insertions $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}$. To absorb the rest, we will use the gluino fields on the boundary. If we choose n boundaries and put two gluinos $\mathcal{W}_\alpha\mathcal{W}^\alpha$ on each, we have for the condition of the absorption of the p zero modes that

$$2n + 2 = 2(2g + h - 1) \quad \rightarrow \quad n = 2g + h - 2.$$

Since $n \leq h$, we have either $g = 0$ and $n = h - 2$ or $g = 1$ and $n = h$. Namely possible F-terms are $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}\ S^{h-2}$ from $g = 0$ and $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}\ S^h$ from $g = 1$.

If we use the first term of the gravitino vertex (2.6), we do not have an option of $g = 1$ and $n = h$ since the gravitino vertex anti-commutes with $\oint_{\gamma_i} p_\alpha$ and therefore $\sum_{i=1}^h \oint_{\gamma_i} p_\alpha = 0$. Namely $\oint_{\gamma_i} p_\alpha$ are not linearly independent and we cannot insert the gluino vertex operators on all boundaries. Thus it only contributes to $g = 0$ and $n = h-2$, namely to *planar diagrams*. These planar contributions will be discussed in [12], where it will be shown to be non-vanishing. However, it will also be shown there that their contributions to the F-terms become trivial when we substitute the extremum value of S, thus the planar contributions effectively drop out, consistently with the superstring duality in [7].

If we use the second part of the gravitino vertex operator instead, the $g = 1$ contribution does not vanish. This is because the second term contains $(\theta^\gamma - \bar{\theta}^\gamma)$, and it has nontrivial correlation with $\oint p_\alpha$ on the boundary. The sum $\sum_i \oint_{\gamma_i} p_\alpha$ does not have to vanish, and we can insert gluino vertex operators on all boundaries. In fact, a simple application of the Cauchy integral formula gives

$$\sum_{i=1}^{h} \oint_{\gamma_i} \mathcal{W}^\alpha p_\alpha \cdot \int \mathcal{W}^{\alpha\beta\gamma} p_\alpha \bar{p}_\beta (\theta_\gamma - \bar{\theta}_\gamma) \sim \int \mathcal{W}^{\alpha\beta\gamma}\mathcal{W}_\gamma\ p_\alpha \bar{p}_\beta. \tag{2.9}$$

This can lead to non-zero result for $g = 1$ and $n = h$, giving rise to the gravitational correction of the form $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}\ S^h$ in (1.5). As in our previous paper [2], there is more to the story. The presence of the gravitino background modifies the chiral ring of the gluino field as

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma. \tag{2.10}$$

Taking this into account, we can reproduce the topological string amplitude $g\ F_{g,h}$ that multiplies to $\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}\ S^h$ in (1.5). On the other hand, this effect does not give contributions to planar diagrams. This is evident from the presence of the factor g in $g\ F_{g,h}$.

Let us explain how the deformation (2.10) arises from the string theory perspective. We follow the approach of [2] and look at the variation of the gravitino vertex operator under $\epsilon^{\dot\alpha}(Q+\bar Q)_{\dot\alpha}$. We find

$$\delta\left[\int \mathcal{W}^{\alpha\beta\gamma} p_\alpha \bar p_\beta(\theta_\gamma - \bar\theta_\gamma)\right] = 2i\epsilon^{\dot\alpha}\mathcal{W}^{\alpha\beta\gamma}\int d\Big(Y_{\alpha\dot\alpha}(p_\beta + \bar p_\beta)(\theta_\gamma - \bar\theta_\gamma)\Big), \tag{2.11}$$

where

$$Y_{\alpha\dot\alpha} = X_{\alpha\dot\alpha} + i\theta_\alpha\theta_{\dot\alpha} + i\bar\theta_\alpha\bar\theta_{\dot\alpha}. \tag{2.12}$$

Since the integrand of the right-hand side of (2.11) is total derivative and $\theta_\gamma = \bar\theta_\gamma$ on the boundaries, it would vanish if there are no other operators inserted on the boundaries. The only non-zero contribution comes from the operator product singularity of (2.11) with the gluino vertex operator as

$$\delta\left[\int \mathcal{W}^{\alpha\beta\gamma} p_\alpha \bar p_\beta(\theta_\gamma - \bar\theta_\gamma)\right]\cdot \oint \mathcal{W}^\alpha p_\alpha = 4\epsilon^{\dot\alpha}\oint \mathcal{W}^{\alpha\beta\gamma}\mathcal{W}_\gamma\ Y_{\alpha\dot\alpha} p_\beta. \tag{2.13}$$

Comparing with our previous paper (see eq. (2.21) of [2] and the subsequent discussion), we find that the boundary terms can be cancelled by imposing the relation (2.10). It is evident from [2] that, if the graviphoton $F_{\alpha\beta}$ is turned on, this is further deformed as

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma + F_{\alpha\beta} \mod \bar D. \tag{2.14}$$

Note that the identity is modulo $D_{\dot\alpha}$ since that is all we need to cancel the boundary terms.

In the flat supergravity background, the definition of the gluino superfield

$$\mathcal{W}_\alpha = \frac{1}{4i}[D^{\dot\alpha}, D_{\alpha\dot\alpha}] \tag{2.15}$$

and the fact that this superfield is chiral $D_{\dot\alpha}\mathcal{W}_\beta = 0$ imply [3],

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 0 \mod \bar{D}. \tag{2.16}$$

As shown in [13] and [17] using direct field theory analysis, the effective superpotential in this case receives contributions only from planar diagrams, consistently with the topological string computation discussed in the above. In section 4, we will show that the superpotential for the gluino obeying the deformed relation (2.14) is computed by the full partition function of the matrix model including non-planar diagrams and reproduce the gravitational corrections (1.5) as well as (1.4) predicted by the topological string computation [4] and the large N duality [7].

3. Deformation of the chiral ring II: supergravity perspective

It turns out that the gravitino part of the deformed chiral ring relation (2.14)

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma, \tag{3.1}$$

can also be understood from the standard supergravity tensor calculus [18]. In fact, the Bianchi identity implies[2]

$$[D^{\dot\alpha}, D_{\alpha\dot\alpha}]_{\beta\gamma} = 4i\mathcal{W}_\alpha\epsilon_{\beta\gamma} - 8i\mathcal{W}_{\alpha\beta\gamma}, \tag{3.2}$$

where we are considering these operators acting on chiral spinor superfields (which is why we have spinor indices $\beta\gamma$ in the above). The second term above arises from the Lorentz action on the spinor field. Let us repeat the derivation of (2.16) in the supergravity background using this relation. We use the fact that $\mathcal{W}_{\dot\alpha}$ is chiral to show

$$\left\{[D^{\dot\alpha}, D_{\alpha\dot\alpha}], \mathcal{W}_\beta\right\} = \left\{D^{\dot\alpha}, [D_{\alpha\dot\alpha}, \mathcal{W}_\beta]\right\} = 0 \mod \bar{D}.$$

Substituting (3.2) to the left-hand side of this equation, we find

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} - 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma = 0. \tag{3.3}$$

again modulo $\bar{D}$. This is what we wanted to show. We have found that the gravitino part of the deformation (2.14) is due to the standard supergravity tensor calculus. However a proper understanding of the F-terms (1.5) requires a non-traditional interpretation of this relation, as we shall see below.

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[2] We are ignoring the other chiral superfield R which appears as the torsion in $[D_\alpha, D_{\beta\dot\beta}]$ since it vanishes on-shell.

4. Non-planar diagrams in the field theory limit

The field theory limit of the above string theory computation is straightforward, and is very similar to our discussion in the previous paper [2]. We will only point out some salient features. In [2], the graviphoton vertex operator $\int F^{\alpha\beta} p_\alpha \bar{p}_\beta$ disappears in the field theory limit, where $p = \bar{p}$. Its effect, however, survives if we include the C-deformation on the gluino fields. Similarly here, the relevant part of the gravitino vertex operator, the second term in (2.6), vanishes in the field theory limit. Effects of the gravitino background survives in the field theory if we include the C-deformation for the gluino field, which as we discussed before would be needed if we wish to preserve supersymmetry and in fact follows from the supergravity tensor calculus. Note that here we still have a choice on algebraic properties of the fields, and it is not dictated just from the tensor calculus leading to (3.1). For example consider the case where we consider the $\mathcal{W}_1$ component of a $U(1)$ gauge field and suppose $\mathcal{W}_{111}$ background is non-zero. Then the chiral relation (3.1) gives

$$(\mathcal{W}_1)^2 - \mathcal{W}_{111}\mathcal{W}_2 = 0 \mod \bar{D}$$

So far, this is perfectly standard supergravity tensor calculus as discussed in the last section. However, what does one take $(\mathcal{W}_1)^2$ to be? Usually we set it to zero by the Grassmannian property of the gluino field $\mathcal{W}_\alpha$, which would then mean that $\mathcal{W}_{111}\mathcal{W}_2 = 0$ modulo $\bar{D}$. It would just mean that this term is not going to appear in any F-term, as it is trivial as a chiral superfield. Thus we would have found no corrections involving mixed gravitational/glueball fields for non-planar diagrams in contradiction with the large N duality [7]. This is what one would obtain in the standard, non C-deformed treatment of Feynman diagrams. However the C-deformation we consider is the weaker statement which requires (3.1) but does not postulate the additional condition that the gluino fields are Grassmannian variables. In particular we *do not* require $(\mathcal{W}_1)^2 = 0$. This is how we end up getting a non-trivial result from Feynman diagrams, following the discussion in [2].

Given the relation (2.14), it is straightforward to reproduce the two series of gravitational corrections (1.4) and (1.5) from purely field theory Feynman diagram computations. In our previous paper [2], we have shown how this is done for the first series (1.4) when we have the C-deformation,

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = F_{\alpha\beta}. \tag{4.1}$$

By simply replacing $F_{\alpha\beta}$ by $F_{\alpha\beta} + 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma$ and noting,

$$\begin{aligned} &\mathrm{Tr}\Big[(F_{\alpha\beta} + 2\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^\gamma)(F^{\alpha\beta} + 2\mathcal{W}^{\alpha\beta}{}_\gamma \mathcal{W}^\gamma)\Big]^g \\ &= N(F_{\alpha\beta}F^{\alpha\beta})^g + 4g(F_{\alpha\beta}F^{\alpha\beta})^{g-1}\, (\mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma})\ \mathrm{Tr}\ \mathcal{W}_\alpha\mathcal{W}^\alpha, \end{aligned} \tag{4.2}$$

we see that the deformation (2.14) generates both types of the F-terms simultaneously.

4.1. Other corrections

Note that the full corrections expected from string theory (including the $U(1)$ fields $\mathcal{W}_\alpha$) to the F-terms can be summarized by the term [7],

$$\Gamma = \int d^4x d^2\theta \int d^2\hat{\theta} \Big[(F_{\alpha\beta} + \hat{\theta}^\gamma \mathcal{W}_{\alpha\beta\gamma})(F^{\alpha\beta} + \hat{\theta}_\delta \mathcal{W}^{\alpha\beta\delta}) \Big]^g F_g(S + \hat{\theta}^\alpha \mathcal{W}_\alpha + \hat{\theta}^2 N).$$

This includes, in addition to the terms already discussed in this paper and the previous paper [2], some mixed terms involving the $U(1)$ superfield $\mathcal{W}_\alpha$ and the gravitino superfield of the form,

$$2g \int d^4x d^2\theta F^{\alpha\beta} \mathcal{W}_{\alpha\beta\gamma} \mathcal{W}^\gamma \cdot (F^2)^{g-1} \frac{\partial F_g}{\partial S}.$$

These terms can also derived easily along the lines we have discussed here and in the previous paper. There is one very interesting aspect of these terms, however, that we wish to point out. In the background of non-zero graviphoton field strength, these terms generate "photon/graviton interactions" which violate Lorentz invariance. As noted in [2], the non-gravitational F-terms have the property that they screen violation of Lorentz invariance in the graviphoton background. The terms we are finding here after integration over the $d^2\theta$ will involve terms like $F^{U(1)}R$ where $F^{U(1)}$ is the field strength in the gluino multiplet and the indices are contracted appropriately with the Lorentz violating parameter $F_{\alpha\beta}$. If $F_{\alpha\beta} \neq 0$, this generates non-Lorentz invariant mixing of the photon and the graviton. They could give interesting signatures of the C-deformation, and it would be amusing to see if it is realized in Nature.

Acknowledgments

We thank R. Dijkgraaf, M. Grisaru, and D. Zanon, whose insights have contributed significantly to this paper. We are also grateful to N. Berkovits for valuable discussion.

C.V. thanks the hospitality of the theory group at Caltech, where he is a Gordon Moore Distinguished Scholar.

The research of H.O. was supported in part by DOE grant DE-FG03-92-ER40701. The research of C.V. was supported in part by NSF grants PHY-9802709 and DMS-0074329.

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