The Effect of Oriented Cracks on Seismic Velocities

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We have considered the problem of elastic wave velocities in a matrix containing aligned ellipsoidal fluid-filled cracks. This problem is relevant to a variety of geophysical applications, including crustal and mantle seismology and the behavior of stressed and dilatant rock. When the cracks are ellipsoids of revolution, the composite is transversely isotropic and is describable with five elastic constants. For aligned oblate spheroids the major reduction in velocity occurs along the axis of symmetry. The opening of new cracks, the widening of old cracks, or the reduction of pore pressure accompanying crustal dilatancy can be expected to cause a large decrease in compressional velocity and considerable compressional wave anisotropy.

Laboratory experiments indicate that crack porosity significantly depresses the seismic velocities even in low-porosity igneous rocks [*Birch,* 1960]. *Anderson and Spetzler* [1970], using the Eshelby-Walsh theory, showed that flat cracks were much more effective in reducing the elastic constants than spherical pores were. Thus they were able to show that the properties of the low-velocity zone could be explained with a small amount of partial melting. Likewise, the low velocities in crystalline rocks at low pressures can be understood in terms of narrow cracks [*Nur and Simmons,* 1969*a, b*] but not in terms of spherical pores. The properties of sea ice can also be explained in terms of thin films of intercrystalline melt [*Spetzler and Anderson,* 1968]. The role of dilatancy in crustal properties, particularly with regard to premonitory effects of earthquakes [*Nur,* 1972; *Whitcomb et al.,* 1973; *Anderson and Whitcomb,* 1973; *Scholz,* 1974], has stimulated interest in the effects of cracks and intergranular pore fluids on seismic wave velocities. The large changes in seismic velocities, particularly the V_P/V_S ratio, prior to earthquakes have been attributed to the opening of new dry cracks or to the reduction of pore pressure by the expansion of old cracks [*Nur,* 1972; *Anderson and Whitcomb,* 1973].

Most of the work to date on the properties of composites or solids containing cracks has assumed isotropy, i.e., random orientation of grains and cracks. Geophysical aggregates can be expected to be anisotropic even if the matrix is isotropic, due to preferred orientation of cracks. The crack fabric of rock can be caused by tectonic stress fields or temperature gradients. Layering, recrystallization, cooling, and tectonic deformation can all be expected to result in cracks with a preferred orientation, both on microscopic (intergranular cracks) and on macroscopic (preferred orientation of dikes, joints) scales. *Nur* [1971] has discussed the problem of stress-induced crack orientation and gives a general treatment for velocities in a matrix containing oriented dry cracks.

In the case of crustal deformation the orientation of cracks is controlled by the orientation of the principal stresses. In particular, the occurrence of dilatancy should be associated with cracks aligned in a statistical sense. We shall investigate here the extreme case in which the cracks are parallel.

The overall elastic symmetry of a material containing parallel penny-shaped cracks is axial or transversely isotropic. Only five elastic constants are then independent. The extreme case of a laminated medium has been examined by *Postma* [1955] and *Rytov* [1956]. *Rudski* [1911] gives ordering relations between the elastic constants for such material.

In this paper we investigate the effect of aligned fluid-filled cracks on the elastic properties of an otherwise homogeneous isotropic material. The theory is valid for ellipsoidal cracks of arbitrary shape, but calculations are presented only for the case of oblate spheroids. The composite material has hexagonal elastic symmetry. The five independent elastic constants are then combined to give the wave velocities as a function of propagation direction.

Shear wave birefringence occurs for all directions of propagation except along the unique axis (for obvious reasons of symmetry). This effect is seen in the experiments of *Nur and Simmons* [1969]. The compressional velocity is most affected in the direction of the unique axis, and is, as expected, dependent on the bulk modulus of the fluid phase. Maximum and minimum velocities bracket those for a medium with an isotropic crack distribution.

THEORY

We use the results of *Eshelby* [1957] to calculate the overall elastic constants of an isotropic matrix containing oriented ellipsoidal inhomogeneities. This theory has been used by *Walsh* [1969] to calculate the elastic constants of a solid containing penny-shaped fluid-filled cracks with random orientation. Such a material is isotropic in the large and is described by two elastic constants.

We consider an isotropic matrix containing ellipsoidal fluid-filled zones where the axial ratios are $a/b = a/c = \alpha < 1$. The Eshelby theory neglects interactions between inhomogeneities, and so we require a dilute solution of cracks; i.e., the porosity must be numerically much less than α. In calculating the velocities we assume low-frequency disturbances in the sense defined by *Walsh* [1969].

Let κ_1 and μ_1 be the bulk modulus and rigidity of the homogeneous isotropic matrix and κ_2 and μ_2 those of the inhomogeneities. For a liquid inhomogeneity we let $\mu_2 \to 0$; for an empty cavity $\kappa_2 \to 0$ as well. We will denote by C_{ij} the elastic coefficients of the composite. The Eshelby equations are not strictly valid in the limit $\mu_2 = 0$ because of the change in boundary conditions.

Eshelby [1957] has calculated the interaction energy of an ellipsoidal inhomogeneity with a strain field that would in absence of the inhomogeneity be a uniform strain in an infinite medium. Consider an infinite medium of matrix material with an ellipsoidal inhomogeneity of volume V oriented parallel to

TABLE 1. Velocities and Anisotropies in a Composite Containing Oriented Fluid-filled Cracks as a Function of Porosity ϕ and Fluid Bulk Modulus κ_f

ϕ	κ_f, kbar	$\bar{V}_P$, km/s	V_{P1}, km/s	V_{P2}, km/s	$\bar{V}_S$, km/s	V_{S1}, km/s	V_{S2}, km/s	A_P, %	A_S, %
0.01	100	6.43	6.54	6.31	3.70	3.78	3.56	3.4	5.7
	10	6.28	6.47	5.76	3.68	3.78	3.56	10.9	5.7
	1	6.23	6.45	5.57	3.67	3.78	3.56	13.7	5.7
	0.1	6.22	6.45	5.54	3.67	3.78	3.56	16.9	5.7
0.02	100	6.30	6.49	6.03	3.60	3.76	3.31	7.1	12.0
	10	6.02	6.36	4.80	3.57	3.76	3.31	24.6	12.0
	1	5.94	6.32	4.32	3.56	3.76	3.31	31.6	12.0
	0.1	5.93	6.32	4.26	3.56	3.76	3.31	32.6	12.0

Aspect ratio α is 0.05; $\bar{V}_P$ and $\bar{V}_S$ are isotropic velocities for random crack orientation.

the coordinate axes. If we introduce a strain e_i^A, $i = 1, \cdots, 6$, uniform at infinity, the elastic energy in the medium differs from that of a homogeneous medium loaded in the same manner at infinity by an amount $E_{\text{int}}(e_k^A)$.

Eshelby [1957, equation 4.10] gives E_{int} as

$$E_{\text{int}} = -\frac{1}{2} V\left[\lambda_1\left(\sum_{i=1}^{3} e_i^T\right)\left(\sum_{i=1}^{3} e_i^A\right) + 2\mu_1\left(\sum_{i=1}^{3} e_i^T e_i^A + \frac{1}{2}\sum_{i=4}^{6} e_i^T e_i^A\right)\right] \qquad (1)$$

where $e_i^T = A_{ij} e_j^A$ is the stress-free strain of the equivalent transformed inclusion of the inhomogeneity as discussed by Eshelby. The A_{ij} is a function of the shape of the ellipsoid and the elastic moduli of the inhomogeneity and matrix.

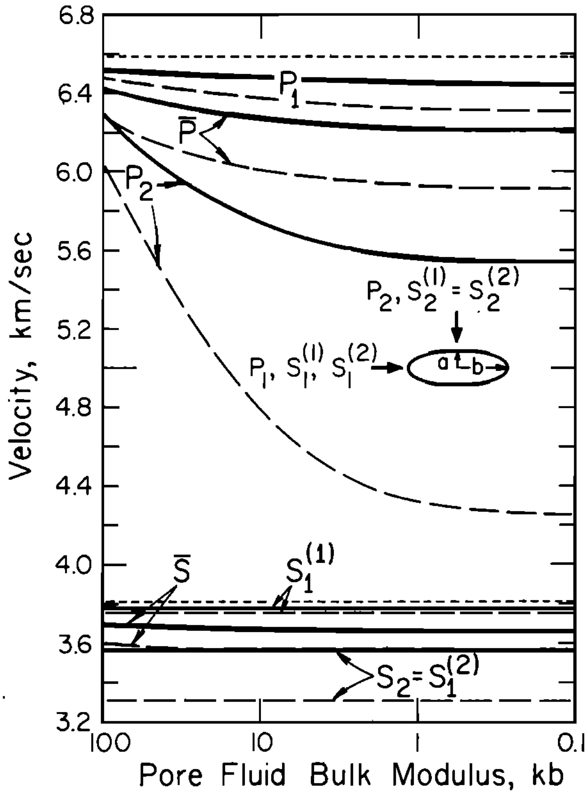

Fig. 1. Vertical and horizontal velocities as a function of pore fluid bulk modulus for a granite matrix containing aligned horizontal cracks with aspect ratio equal to 0.05. Solid curves are for porosity of 0.01, and long-dashed curves are for porosity of 0.02. Subscripts 1 and 2 are for horizontal and vertical propagation, respectively. The P and S are average velocities for a matrix containing randomly oriented cracks. The two shear velocities are equal in the vertical direction.

If we require that the largest dimension of the inhomogeneity be small in comparison with unity, E_{int} is approximately the interaction energy of the ellipsoid in a unit volume of matrix material subjected to surface tractions producing strains e_i^A at the surface. A medium made of unit volumes identical to this will have a gross strain e_i^A, and the elastic energy per unit volume is

$$E(e_k^A) = \frac{1}{2}\left\{\lambda_1\left(\sum_{i=1}^{3} e_i^A\right)\left[\sum_{i=1}^{3}(e_i^A - Ve_i^T)\right] + 2\mu_1\left[\sum_{i=1}^{3} e_i^A(e_i^A - Ve_i^T) + \frac{1}{2}\sum_{i=4}^{6} e_i^A(e_i^A - Ve_i^T)\right]\right\}$$

where V is now the volume fraction of inhomogeneities.

However, we have for the composite material the relation

$$E(e_k^A) = \tfrac{1}{2} C_{ij} e_i^A e_j^A$$

where the left-hand side can be evaluated for any applied strain. This equation is used to evaluate the constants C_{ij}. For this purpose we choose the applied strain components to be zero except for the mth and nth ones, which are set to unity. We can thus define the 21 quantities:

$$E_{mn} = E(\delta_{km} + \delta_{kn})$$

where δ_{pq} is the Kronecker delta.

Then we have for diagonal E_{mn} elements

$$C_{ii} = E_{ii}$$

(no sum), and for off-diagonal elements we have

$$E_{ij} = 2C_{ij} + C_{ii} + C_{jj}$$

giving

$$C_{ij} = \tfrac{1}{2}(E_{ij} - E_{ii} - E_{jj})$$

Thus if we can evaluate E_{int}, we can obtain the elastic constants of the composite. This evaluation can be done numerically by using (1).

The principal limitation of this procedure is the requirement that the inhomogeneities be a dilute solution. For oblate spheroids with axes a and b and $a/b = \alpha < 1$ we require that $b \ll 1$, so that the condition on the porosity is

$$V = \frac{4\pi}{3} b^3 \alpha \ll \alpha$$

For oriented spheroidal inclusions, only five elastic constants are independent: C_{11}, C_{33}, C_{13}, C_{44}, and $(C_{11} - C_{12})/2$ [e.g., *Love,* 1934; *Anderson,* 1961], where the X_3 axis is the axis of rotational symmetry. In analogy to the problem of a laminated solid we have the constraints $C_{11} > C_{33}$ and $(C_{11} - C_{12})/2 > C_{44}$ [*Postma,* 1955]. For horizontal cracks, $V_{P(\text{horizontal})} > V_{P(\text{vertical})}$, and $V_{SH(\text{horizontal})} > V_{SV(\text{horizontal})} = V_{SH(\text{vertical})} = V_{SV(\text{vertical})}$. In other directions the waves are not purely transverse or purely longitudinal, nor are the wave fronts normal to the rays [e.g., *Love,* 1934].

The velocity equation in any plane containing the unique axis is

$$\left(H - \frac{m^2C}{2}\right)[(H - M^2a)(H - n^2h) - m^2n^2d^2] = 0$$

where

$$a = C_{11} - C_{44} \qquad C = C_{11} - C_{12} - 2C_{44}$$

$$d = C_{13} + C_{44} \qquad h = C_{33} - C_{44} \qquad H = \rho v^2 - C_{44}$$

Here, V is the velocity, and m and n are direction cosines denoting the direction of propagation; $n = 1$ denotes propagation in the direction of the unique axis.

The wave associated with the root $H = m^2C/2$ is purely transverse, and the slowness surface intersects the plane along an ellipse. The other roots are purely transverse or purely longitudinal only in the directions $n = 0$ or $n = 1$. In a direction parallel to the crack plane there are three velocities of propagation, $(C_{11}/\rho)^{1/2}$, $(C_{44}/\rho)^{1/2}$, and $[(C_{11} - C_{12})/2\rho]^{1/2}$, corresponding to a longitudinal wave and two transverse waves. In the direction of the unique axis the compressional velocity is $(C_{33}/\rho)^{1/2}$, and the two shear waves have the same velocity: $(C_{44}/\rho)^{1/2}$.

NUMERICAL RESULTS

We have computed the elastic constants and directional wave properties in a composite consisting of a matrix with the properties of granite containing oriented spheroidal cracks. We have investigated a range of porosities, spheroid aspect ratios, and fluid properties. The matrix is assumed to be isotropic with $\lambda = \mu = 390$ kbar, and $\rho = 2.7$ g/cm³, values appropriate for crack-free granite. We consider low-porosity aggregates and have ignored the effect of porosity on the density of the composite. The error is negligible for flat cracks, but for $\alpha \sim 1$ the effect of porosity on the elastic moduli and density is of the same order.

Table 1 and Figure 1 give the direction velocities for a composite containing oriented spheroids having aspect ratios $\alpha = 0.05$. The V_{P_1} and V_{S_1} are, respectively, the compressional and *SH* velocities for propagation parallel to the cracks; the fast direction V_{P_2} is the compressional velocity perpendicular to the plane of the cracks. The two shear velocities in this direction are equal. Also tabulated are the velocities $\bar{V}_p$ and $\bar{V}_S$ for an isotropic aggregate consisting of randomly oriented cracks, computed with the expressions of *Walsh*

TABLE 2. Vertical Velocities V_{P2} and V_{S2}, Horizontal Velocities V_{P1}, V_{S1}, and V_{S2}, and Velocity Ratios $\zeta = (V_P/V_S - 1)$ as a Function of Aspect Ratio α, Porosity ϕ, and Bulk Modulus of the Fluid Phase κ_f

α	ϕ	κ_f, kbar	V_{P1}, km/s	V_{P2}, km/s	V_{S1}, km/s	V_{S2}, km/s	ζ_1(1)	ζ_1(2)	ζ_2
1.00	0.01	100	6.532	6.532	3.764	3.764	0.74	0.74	0.74
		0.1	6.526	6.526	3.764	3.764	0.73	0.73	0.73
0.80	0.001	100	6.577	6.576	3.797	3.797	0.73	0.73	0.73
		0.1	6.576	6.575	3.797	3.797	0.73	0.73	0.73
0.80	0.005	100	6.555	6.550	3.783	3.781	0.73	0.73	0.73
		0.1	6.550	6.543	3.783	3.781	0.73	0.73	0.73
0.80	0.01	100	6.528	6.517	3.766	3.761	0.75	0.75	0.73
		0.1	6.517	6.503	3.766	3.761	0.73	0.73	0.73
0.50	0.001	100	6.578	6.574	3.797	3.796	0.73	0.73	0.73
		0.1	6.577	6.572	3.798	3.796	0.73	0.73	0.73
0.50	0.005	100	6.559	6.539	3.786	3.778	0.73	0.74	0.73
		0.1	6.553	6.527	3.786	3.778	0.73	0.73	0.73
0.50	0.01	100	6.534	6.495	3.771	3.755	0.73	0.74	0.73
		0.1	6.524	6.470	3.771	3.755	0.73	0.74	0.72
0.10	0.001	100	6.575	6.561	3.798	3.787	0.73	0.74	0.73
		10	6.575	6.539	3.798	3.787	0.73	0.74	0.73
		1	6.574	6.535	3.798	3.787	0.73	0.74	0.73
		0.1	6.574	6.534	3.798	3.787	0.73	0.74	0.73
0.10	0.005	100	6.561	6.476	3.790	3.737	0.73	0.76	0.73
		10	6.546	6.362	3.790	3.737	0.73	0.75	0.70
		1	6.542	6.338	3.790	3.737	0.73	0.75	0.70
		0.1	6.542	6.336	3.790	3.737	0.73	0.75	0.70
0.1	0.01	100	6.538	6.367	3.779	3.673	0.73	0.78	0.73
		10	6.508	6.134	3.779	3.673	0.72	0.77	0.67
		1	6.503	6.084	3.779	3.673	0.72	0.77	0.66
		0.1	6.502	6.079	3.779	3.673	0.72	0.77	0.66
0.05	0.01	100	6.536	6.312	3.780	3.564	0.73	0.83	0.77
		10	6.474	5.760	3.780	3.564	0.71	0.81	0.62
		1	6.454	5.569	3.780	3.564	0.71	0.81	0.56
		0.1	6.452	5.545	3.780	3.564	0.71	0.81	0.56
0.05	0.02	100	6.488	6.030	3.760	3.310	0.72	0.96	0.82
		10	6.364	4.799	3.760	3.310	0.69	0.92	0.45
		1	6.323	4.323	3.760	3.310	0.68	0.91	0.31
		0.1	6.318	4.261	3.760	3.310	0.68	0.91	0.29
0.01	0.001	100	6.577	6.550	3.798	3.693	0.73	0.78	0.77
		10	6.557	6.367	3.798	3.693	0.73	0.78	0.72
		1	6.533	6.145	3.798	3.693	0.72	0.77	0.66
		0.1	6.528	6.095	3.798	3.693	0.72	0.77	0.65

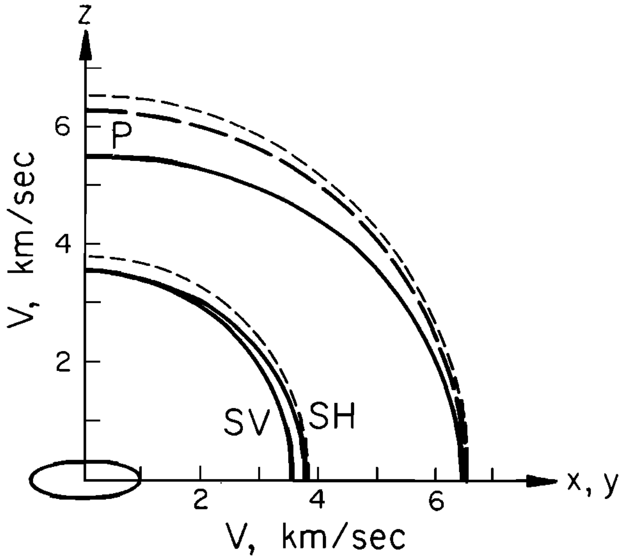

Fig. 2. Velocities as a function of angle and fluid properties in granite containing aligned ellipsoidal cracks, with $\phi = 0.01$, $\alpha = 0.05$, and $\kappa_f = 0.1$ and 100 kbar. The short-dashed curves give the velocity surface for the isotropic uncracked solid, the long-dashed curves the surface for liquid-filled cracks ($\kappa_f = 100$ kbar), and the solid curves the surface for gas-filled cracks ($\kappa_f = 0.1$ kbar).

[1969]. The A_P and A_S are the compressional wave and shear wave anisotropies ($\Delta V/V$). Note that the shear wave anisotropies ($\Delta V/V$). Note that the shear wave anisotropy is greater than the compressional wave anisotropy for liquid-filled cracks $\kappa = \kappa_2 > 10$ kbar, and the reverse is true for gas-filled cracks. Anisotropy reaches 32.6% for compressional waves in a solid containing 29% by volume of cracks containing a gas with $\kappa_f = 0.1$ kbar. For $\kappa_f < 0.1$ kbar there is little additional change in velocities as $\kappa_f \to 0$.

The ratio of compressional velocity to shear velocity is strongly dependent on direction and the nature of the fluid phase. In general, the V_P/V_S ratio is normal (1.73) or greater than normal when it is measured along the plane of the cracks. In the direction perpendicular to the cracks the V_P/V_S ratio is nearly normal for liquid-filled cracks but decreases rapidly as the bulk modulus of the fluid phase approaches that of a gas.

Table 2 summarizes the results of the computations for aspect ratios of 0.01 to 1.00 and porosities from 0.001 to 0.02 and fluid bulk moduli from 0.1 (gaslike) to 100 kbar (liquidlike). Also shown is the parameter $\zeta = V_P/V_S - 1$, which is important in the dilatancy model of the earthquake mechanism. For horizontal cracks

$$\zeta_1^{(1)} = V_{PH}/V_{SH} - 1 \qquad \zeta_1^{(2)} = V_{PH}/V_{SV} - 1$$

and

$$\zeta_2 = V_{PV}/V_{SH,PV} - 1$$

i.e., the subscript 1 refers to horizontally propagating waves, and the subscript 2 refers to vertically propagating waves. Note that the velocity ratios do not deviate markedly from the normal value of $\sim$1.73 until the aspect ratio of the cracks becomes less than about 0.1.

For small aspect ratios the V_P/V_S ratio decreases rapidly with decreasing pore fluid modulus for waves traveling in the direction normal to the cracks. For waves traveling parallel to the cracks this ratio and the ratio of the two shear velocities are almost independent of the nature of the pore fluid. However, there is a distinct shear wave anisotropy in this direction. Note the extremely small V_P/V_S ratios that are possible in the direction normal to the cracks.

Figure 2 gives the intersection of the velocity surface with a plane containing the unique axis. The short-dashed curves are the velocity surfaces, spheres, in the crack-free matrix. The long-dashed curves are for a solid containing 1% by volume of aligned spheroids with $\alpha = 0.5$ and a pore fluid bulk modulus of 100 kbar. The solid curves are for the same parameters as above but for a relatively compressible fluid in the pores with $\kappa_f = 0.1$ kbar. The two shear velocity surfaces do not depend on the pore fluid bulk modulus. Note the large compressional wave anisotropy for the solid containing the more compressible fluid. In a dilatant material we can expect large changes in compressional wave velocity in some directions and only slight changes in other directions.

SUMMARY

The seismic velocities in a solid containing flat oriented cracks depend on the elastic properties of the matrix, porosity, aspect ratio of the cracks, the bulk modulus of the pore fluid, and the direction of propagation. The V_P/V_S ratio is anisotropic; it increases slightly in some directions and decreases markedly in others during dilatancy. Substantial velocity reductions, compared with those of the uncracked solid, occur in the direction normal to the plane of the cracks. Shear wave birefringence occurs in rocks with oriented cracks.

Acknowledgments. This research was supported by USGS grant 14-08-0001-G52, Am. 1 and the Chevron Research Company. J. Walsh provided the authors with an extensive review of the manuscript. Contribution 2390, Division of Geological and Planetary Sciences, California Institute of Technology.

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(Received August 13, 1973;
revised May 2, 1974.)