Codes for Differential Signaling with Many Antennas

Babak Hassibi, Bertrand Hochwald,
Amin Shokrollahi and Wim Sweldens
Mathematical Sciences Center
Lucent Technologies
600 Mountain Avenue
Murray Hill, NJ 07974
e-mail:
{hassibi,hochwald,mshokrollahi,wim}@lucent.com

Abstract — **We construct signal constellations for differential transmission with multiple basestation antennas. The signals are derived using the theory of fixed-point-free groups and are especially suitable for mobile cellular applications because they do not require the handset to have more than one antenna or to know the time-varying propagation environment. Yet we achieve full transmitter diversity and excellent performance gains over a single-antenna system.**

I. Introduction

Differential phase-shift keying (DPSK) is a well-known technique for transmitting digital information across an unknown time-varying channel. Let the data consist of a sequence of integers $z_1, z_2, \ldots \in \{0, \ldots, L-1\}$ where L is the size of our alphabet (often a power of two). DPSK with a single transmitter transmits complex baseband signals that obey the recursion

$$s_t = v_{z_t} s_{t-1} \qquad t = 1, 2, \ldots,$$

where $s_0 = 1$ and the $v_{z_t} \in \{1, e^{2\pi i/L}, \ldots, e^{2\pi i(L-1)/L}\}$ are L points around the complex unit circle.

We can extend this differential scheme to $M > 1$ transmit antennas by transmitting $M \times M$ matrices that obey the recursion

$$S_\tau = V_{z_\tau} S_{\tau-1} \qquad \tau = 1, 2, \ldots, \tag{1}$$

where S_0 is the $M \times M$ identity matrix and V_{z_τ} are complex unitary data matrices [1]. (See also [3] for a similar differential scheme and [2] for a differential scheme based on orthogonal designs.) Each row of the transmission matrix S_τ specifies what is transmitted on the M transmit antennas; hence each S_τ specifies what the M antennas do for M time samples. The index τ marks each block of M time samples.

To use this method effectively, we need to design $L = 2^{RM}$ unitary data matrices $\mathcal{V} = \{V_0, \ldots, V_{L-1}\}$, where R is the data rate. We also need a simple decoding algorithm at the receiver. It can be shown that across an unknown flat-fading channel, the received signals on N antennas for M units of time obey the recursion

$$X_\tau = V_{z_\tau} X_{\tau-1} + W_\tau, \tag{2}$$

where W_τ is a $M \times N$ matrix of additive independent complex-Gaussian noise. Maximum likelihood decoding simply chooses the V-matrix that minimizes the Frobenius norm of the difference between X_τ and $V \cdot X_{\tau-1}$.

Unlike with DPSK, the choice of effective unitary data matrices V_{z_t} is nontrivial. At high SNR, the performance of a constellation can be shown to depend on

$$\zeta_\mathcal{V} = \frac{1}{2} \min_{\ell' \neq \ell} |\det (V_{\ell'} - V_\ell)|^{1/M}.$$

	Group type	L	M	Comments
1.	$G_{m,r}$	mn	n	
2.	$D_{m,r,\ell}$	$2mn$	$2n$	Contains quaternions
3.	$E_{m,r}$	$8mn$	$2n$	
4.	$F_{m,r,\ell}$	$16mn$	$4n$	if $n > 1$ or $\ell \not\equiv 1 \bmod m/3$
	$F_{m,1,\ell}$	$16mn$	2	if $\ell \equiv 1 \bmod m/3$
5.	$J_{m,r}$	$120mn$	$2n$	
6.	$K_{m,r,\ell}$	$240mn$	$4n$	

Table 1: There are six types of groups with $\zeta > 0$: For each group G, L is the order of G (the size of the constellation) and M is the dimension of the representation of G (number of transmit antennas). n is the smallest integer such that $r^n \equiv 1 \pmod{m}$.

We wish to choose the constellation $\mathcal{V}$ to have $0 \leqslant \zeta_\mathcal{V} \leqslant 1$ as large as possible.

II. Group Constellations

Equation (1) is the fundamental differential transmission and we see that if the set $\mathcal{V}$ forms a group, the transmitted signal S_τ is always an element of $\mathcal{V}$. In fact, the multiplication in (1) is replaced by a group table-lookup. We therefore ask if there are groups of unitary matrices that have large ζ. Perhaps surprisingly, this question was examined in the early 20th century in the mathematical literature (in a different context!) and answered in large part in [5] using the theory of fixed-point-free groups.

Briefly, a group that has a "representation" in unitary matrices with nonzero ζ is called fixed-point-free. Zassenhaus in [5] classifies most of the fixed-point-free groups. We complete the classification in [6]. We defer a detailed discussion of these groups to [6] and refer instead to Table 1 where the classes of the groups are outlined. Note that there are only six classes and any group with $\zeta > 0$ must fall within one of them. We also list their orders (size of constellation L) and the dimension of their representations (number of antennas M).

We see from the table that only the class $G_{m,r}$ gives signal constellations for odd M. The class $G_{m,r}$ also contains the diagonal signals corresponding to the cyclic groups studied in [1]. The class $D_{m,r,\ell}$ contains the generalized quaternion (also called dicyclic)

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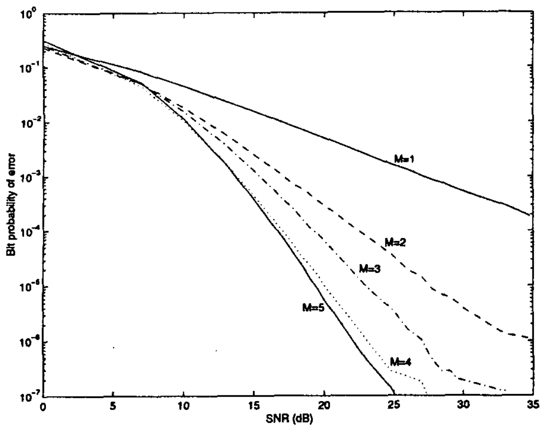

Figure 1: Performance of $M = 1$, 2, 3, 4, and 5 transmit antennas and $N = 1$ receiver antenna as a function of SNR ρ. The channel has unknown Rayleigh fading that is changing continuously according to Jakes' [4] model with parameter $f_D T_s = 0.0025$. The data rate is $R = 1$.

groups studied in [3]. Within the classes are many groups that perform very well when simulated on a wireless channel, as can be seen in the next section.

III. Performance Curves

Figure 1 shows the performance of using cyclic groups (corresponding in Table 1 to $G_{m,1}$) for $M = 1, \ldots, 5$ antennas transmitting across a continuously fading channel to one receive antenna with transmission rate $R = 1$. (With $M = 1$ antenna, standard differential BPSK is used.) The advantages of using more than one antenna are readily seen.

For higher rates, non-Abelian groups seem to provide better performance. As an example of a high-rate fixed-point-free group code, we plot in Figure 2 the performance of the group $F_{15,1,11}$, whose details defy a simple description and appear in [6]. This group has a representation as 240 complex 2×2 unitary matrices (rate $R = \log(240)/2 \approx 3.95$) suitable for differential transmission over a two-antenna fading channel. We also plot the performance of the best cyclic group with the same rate [1], a 2×2 orthogonal design [2] (which is not a group) and a generalized quaternion group code [3] with similar rates. All of these codes can also be used with a known channel with a performance gain of approximately 3 dB.

Another example, appearing in Figure 3, is the group $K_{1,1,-1}$ (again defying a simple description) with a representation in 240 complex 4×4 matrices (rate $R = \log(240)/4 \approx 1.98$). We also plot the performance of the best cyclic group with the same rate and see that the performance gains are appreciable.

References

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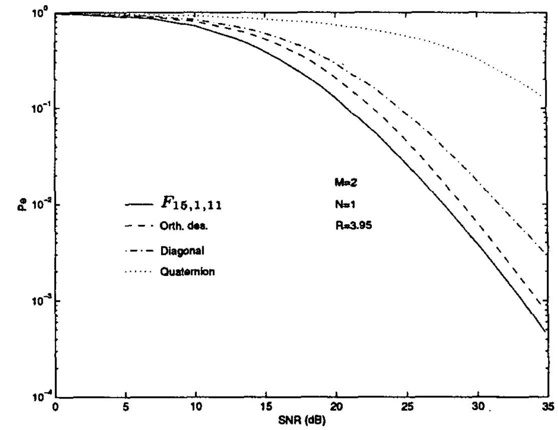

Figure 2: Block-error rate performance of the group $F_{15,1,11}$ for $M = 2$ transmitter antennas and $N = 1$ receiver antenna. The solid line is $F_{15,1,11}$, which has $L = 240$ unitary matrices ($R \approx 3.95$). The dashed line is an orthogonal design with 16th roots of unity ($R = 4$). The dash-dotted line is the best diagonal (Abelian group) construction ($R \approx 3.95$). The dotted line is the quaternion group ($D_{128,1,-1}$) with $L = 256$ matrices ($R = 4$).

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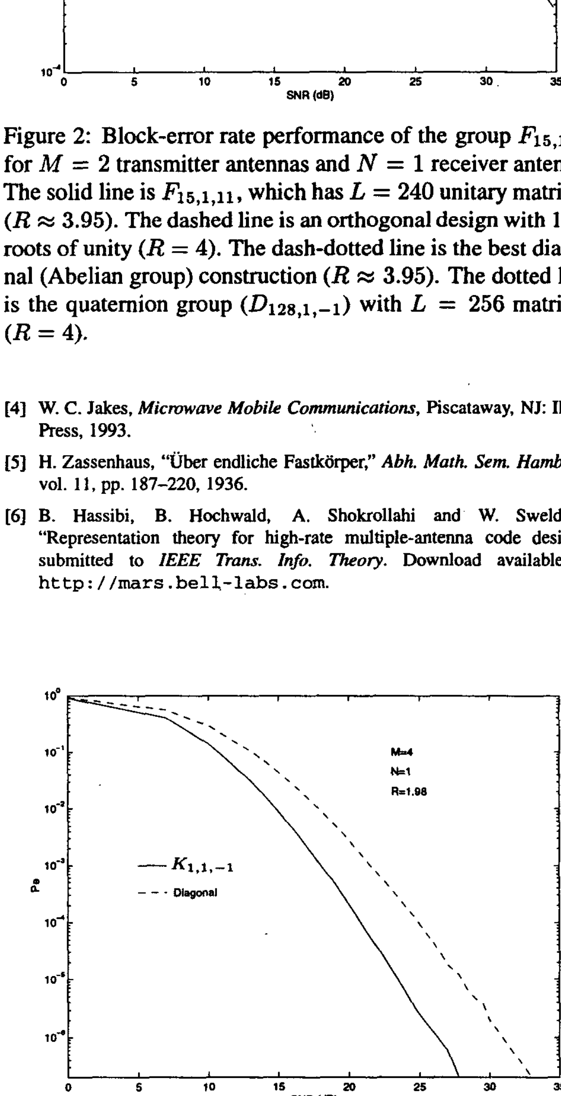

Figure 3: Block-error rate performance of the group $K_{1,1,-1}$ compared with the best diagonal code for $M = 4$ transmitter antennas and $N = 1$ receiver antenna. The solid line is $K_{1,1,-1}$ having $L = 240$ unitary matrices ($R \approx 1.98$). The dashed line is a diagonal construction with the same rate.