stadter *et al.*[3] gave a χ^2 of 134.7, which does not satisfy the criterion of a good fit to the *p-e* data alone. On minimizing the χ^2 with respect to the parameters, the minimum χ^2 was found to be 24.57 with parameters $A_1=-0.494$, $A_2=-0.482$, $A_3=-0.0157$, $A_4=-1.141$, $A_5=-17.05$, and $A_6=-15.66$.

It is to be noted that this may not be the best fit yet, since χ^2 is a very complicated function of the parameters and, hence, there are a large number of extremum values.

An attempt was then made to improve the fit by introducing an energy dependence of the Regge form in the form factors.

$$\bar{F}_1=F_1(q^2)\times(z_t)^{\alpha' t},$$
$$\bar{F}_2=F_2(q^2)\times(z_t)^{\alpha' t},$$

where the "Regge slope" α' is an unknown parameter, and

$$z_t=2[ME_0+q^2/2]/[(4M^2-q^2)(4m_e^2-q^2)]^{1/2}.$$

The minimum χ^2 for the same 53 pieces of data was 22.71 with the parameter values $A_1=-0.375$, $A_2=-0.455$, $A_3=0.056$, $A_4=-1.284$, $A_5=-20.26$, $A_6=-14.81$, and $\alpha'=0.0141$.

It is seen that the fit is not much improved by introducing the energy dependence in the form factors but that the form factors can withstand a considerable energy dependence corresponding to the value of α' given above.

The above analysis would be more meaningful in terms of the photon as a Regge pole for higher energy data which may soon be available. It is, however, understood that there is an energy dependence not only due to the possible Regge-pole character of the photon but also due to higher order exchanges as discussed by Frautschi[4] and Lévy.[5] In any case the slope α' is used here only as a phenomenological parameter. It gives a convenient measure of the energy dependence of form factors for high-energy scattering.

ACKNOWLEDGMENT

It is a pleasure to thank Professor David Wong for his constant guidance through the work.

[4] S. Frautschi (unpublished).
[5] Maurice Lévy, Phys. Rev. Letters 9, 235 (1962).

PHYSICAL REVIEW VOLUME 130, NUMBER 6 15 JUNE 1963

K-Leptonic Decay and Partially Conserved Currents†

GEORGE ZWEIG
California Institute of Technology, Pasadena, California
(Received 7 February 1963)

An operational definition for the partial conservation of the strangeness-changing vector current is given and applied to leptonic K^+ and K_2^0 decay. The K^* resonance is explicitly included in the calculation and quantitative agreement with experiment is obtained. A detailed comparison with the K^+ data of Brown *et al.* and Dobbs *et al.* is given. Because of rapid variations of a form factor, it is found that the data of these two groups are not in contradiction. From the K_2^0 experiment of Luers *et al.*, $I=\frac{1}{2}$ and $\frac{3}{2}$ currents are seen to exist. $\Lambda\beta$ decay is briefly considered. It is found that an explanation for the slowness of K leptonic decay and the vector part of $\Lambda\beta$ decay may be connected with the partial conservation of the strangeness-changing vector current.

I. DETERMINATION OF A THEORY FOR LEPTONIC K DECAY

ONE of the outstanding problems in the theory of weak interactions consists of finding a unifying principle for the strangeness changing and nonstrangeness changing decays. Attempts to use a universal Fermi interaction or to generalize the idea of a conserved nonstrangeness changing vector current have not been fruitful in the sense that an understanding of the experimental data has not been obtained.[1] Furthermore, the ideas developed in attempting to explain the striking success of the Goldberger-Treiman formula in $\pi-\mu\nu$ decay[2] have not been carried over successfully into the theory of K decays.[3] Many of the present difficulties may well stem from our inability to give operational definitions to such concepts as a "partially conserved current" and "universal interaction." In an attempt to sharpen our understanding of these terms, we have considered the leptonic decays of the K^+.

The assumption is made that the $K^+\to l^++\nu+\pi^0$ interaction is of the vector form, in which case we may

† This work was supported in part by the U. S. Atomic Energy Commission, and an IBM Fellowship.
[1] (a) J. Bernstein and S. Weinberg, Phys. Rev. Letters 5, 481 (1960); (b) H. Chew, *ibid.* 8, 297 (1962).
[2] J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).
[3] D. H. Sharp and W. G. Wagner, California Institute of Technology Synchrotron Laboratory Report CTSL-34, 1962 (unpublished).

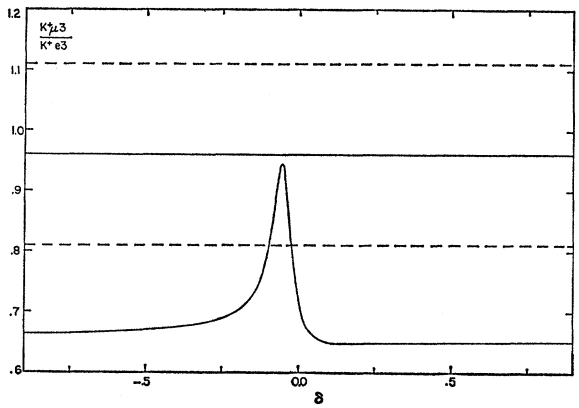

FIG. 1. The branching ratio $K_{\mu 3}^+/K_{e3}^+$ is plotted as a function of the parameter δ. The experimental value for the branching ratio of 0.96 ± 0.15 is represented by the horizontal solid and dashed lines. This indicates that the range of δ is limited to $-0.1\leq\delta\leq-0.025$. The sharp rise is due to the zero in the form factor $f_+(s)$ which suppresses the K_{e3}^+ rate more than the $K_{\mu 3}^+$ rate because K_{e3}^+ depends only on $f_+(s)$, while $K_{\mu 3}^+$ depends on both $f_+(s)$ and $f_-(s)$.

write for the decay amplitude,

$$\langle l^+\nu\pi^0|K^+\rangle = i\frac{G}{\sqrt{2}}\left(\frac{m_\nu}{E_\nu}\frac{m_l}{E_l}\right)^{1/2}\bar{\nu}\gamma_\alpha(1+\gamma_5)l^+\langle\pi^0|s_\alpha{}^V|K^+\rangle, \quad (1)$$

where $s_\alpha{}^V$ is the strangeness-changing vector current, and G is the weak interaction constant equal to 1.4×10^{-49} erg$\times$cm^3. By Lorentz invariance arguments, the matrix element $\langle\pi^0|s_\alpha{}^V(0)|K^+\rangle$ may be thrown into the form

$$1/\sqrt{2}\langle\pi^0|s_\alpha{}^V(0)|K^+\rangle = \tfrac{1}{2}(4E_KE_\pi)^{-1/2} \times[(p_K+p_\pi)_\alpha f_+(s)+(p_K-p_\pi)_\alpha f_-(s)], \quad (2)$$

where $s=-(p_K-p_\pi)^2$. The four-momenta of the K and π are p_K and p_π. Using causality arguments, one can show that $f_+(s)$ and $(m_K{}^2-m_\pi{}^2)f_+(s)+sf_-(s)$ satisfy subtracted dispersion relations.

It is not difficult to show that f_+ receives contributions (in the sense of dispersion theory) only from p-wave intermediate states. Also, since the matrix element $\langle\pi^0|\partial_\alpha s_\alpha(0)|K^+\rangle$ of the divergence of $s_\alpha{}^V$ is proportional to $(m_K{}^2-m_\pi{}^2)f_+(s)+sf_-(s)$, it is precisely this combination of form factors that receives contributions from s-wave intermediate states. We now explicitly take into account the K^* ($K\pi$ spin 1^- resonance at 884 MeV), the only known particle or resonance that contributes to our form factors.[4] Hence, we write

$$f_+(s)=\gamma\left\{\frac{1}{1-(s/M^2)}+v(s)\right\}, \quad (3)$$

$$\langle\pi^0|\partial_\alpha s_\alpha{}^V(0)|K^+\rangle \propto \Delta m^2 f_+(s)+sf_-(s)=\gamma\Delta m^2 d(s), \quad (4)$$

where $\Delta m^2=m_K{}^2-m_\pi{}^2$, M is the mass of the K^*, and γ is a coupling constant that measures the strength of the $K^*-K\pi$ interaction. Because we do not know of any zero mass particle that would give rise to poles in our form factors, we find

$$f_+(s)=\gamma\left\{d(0)+\frac{s}{M^2}\frac{1}{1-(s/M^2)}+v(s)-v(0)\right\}, \quad (5)$$

and

$$f_-(s)=-\gamma\frac{\Delta m^2}{M^2}\left\{\frac{1}{1-(s/M^2)}+\frac{M^2}{s}[v(s)-v(0)] -\frac{M^2}{s}[d(s)-d(0)]\right\}. \quad (6)$$

We now make the assumption that the current $s_\alpha{}^V$ is "partially conserved," by which we mean that $d(s)$ is slowly varying and $|d(s)|\ll1$, in the physical region for s.[5] This justifies neglecting the term $-(M^2/s)[d(s)-d(0)]$ is the expression for $f_-(s)$. Note that this definition for the partial conservation of $s_\alpha{}^V$ differs from the ones usually adopted. Previously, the partial conservation of $s_\alpha{}^V$ has been taken to mean $\partial_\alpha s_\alpha{}^V=0$ in the limit of some higher symmetry where baryon mass difference and meson mass difference vanish.[6] Alternative definitions have stipulated that $\langle\pi^0|\partial_\alpha s_\alpha{}^V|K^+\rangle\to0$ as $s\to\infty$.[7] Since neither one of these latter two conditions is directly measurable in any decay experiment, we have chosen to redefine the concept of a partially conserved current.

In order to obtain an expression for f_+ and f_- that may be easily compared with experiment, we make the rather crude approximation that $v(s)-v(0)$ is proportional to s. We may then write

$$f_+(s)=\lambda\left\{\frac{s}{M^2}\frac{1}{1-s/M^2}+\delta\right\}, \quad (7)$$

and

$$f_-(s)=-\lambda\frac{\Delta m^2}{M^2}\frac{1}{1-s/M^2}, \quad (8)$$

where

$$\delta=\frac{d(0)}{1+M^2[dv(s)/ds]|_{s=0}}\ll1$$

and

$$\lambda=\gamma\left(1+M^2\frac{dv(s)}{ds}\right)\Bigg|_{s=0}.$$

We now have a two-parameter theory. λ may be determined from the known K_{e3}^+ decay rate, while δ should follow from the observed $K_{\mu3}^+/K_{e3}^+$ branching ratio.

[4] Later in this paper we discuss the effects of other possible $K\pi$ resonances.

[5] A necessary condition for the validity of this assumption is that no particle exists with the quantum numbers of $\partial_\alpha s_\alpha{}^V$. Hence, according to our definition of a "partially conserved" current, it would not be correct to say that the axial vector nonstrangeness changing current $j_\alpha{}^A$ is "partially conserved" because $\partial_\alpha j_\alpha{}^A$ has the quantum numbers of the π meson. If we were to insist incorrectly on applying our above definition of partial conservation to $j_\alpha{}^A$, then we could no longer derive the Goldberger-Treiman relation.

[6] See, for example, S. Okubo, Nuovo Cimento **13**, 292 (1959).

[7] See, for example, reference 1(a).

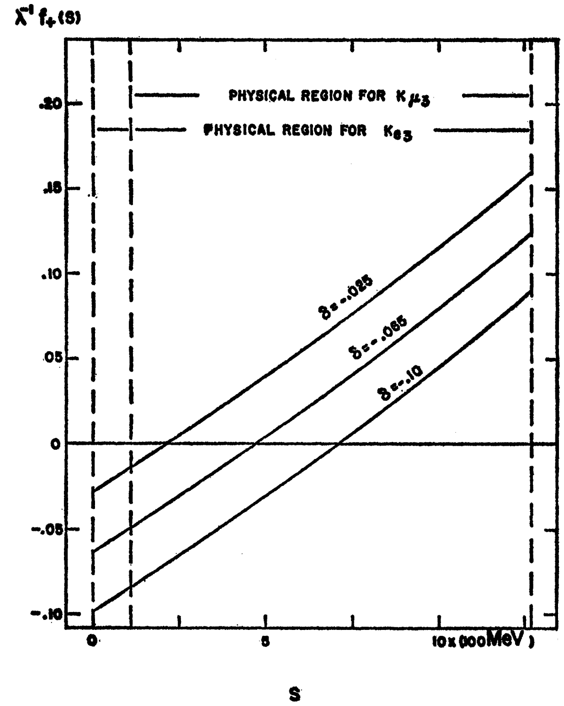

FIG. 2. The form factor $f_+(s)$ is given for three values of the parameter δ within the range determined by the branching ratio $K_{\mu3}^+/K_{e3}^+$. The coupling constant λ has been divided out of $f_+(s)$.

II. PREDICTIONS AND EXPERIMENTAL CONFIRMATIONS OF THE THEORY

In Fig. 1 we have plotted the branching ratio $K_{\mu3}/K_{e3}$ vs δ. The curve is flat except for a very sharp rise near $\delta=0$. The structure of this spike is a result of the combined hypotheses of a partially conserved current and "dominating" K^* pole. In the region of δ near the peak, not only the branching ratio, but also the spectra of all the particles, along with the longitudinal polarization of the μ, are extremely sensitive functions of δ. The size of δ should be compared with the pole term which has strength 1. A strictly conserved current would mean $\delta=0$, a theoretically impossible situation ($\delta=0$ also gives an incorrect branching ratio). Regardless of the value of δ, we may say in general that $K_{\mu3}^+/K_{e3}^+\leq 0.95$. The measured branching ratio is 0.96 ± 0.15. This gives $\delta=-0.05_{-0.05}^{+0.025}$. Figure 2 shows some typical f_+'s. Note that this form factor goes through zero in the physical region. Using the known rate for K_{e3}^+ decay, we may find λ^2 as a function of δ. The result is given in Fig. 3. Figure 4 shows the rate for K_{l3} as a function of δ, λ being held fixed.

If the theory is correct, it should be possible to fit both the π^0 and μ^+ energy spectra in $K_{\mu3}^+$ decay by picking some value of δ in the range $-0.025\leq\delta\leq -0.1$. Let us, therefore, look at Figs. 5 and 6 where the data from the experiment of Brown *et al.*[8] is displayed.

[8] J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe, and D. Sinclair, Phys. Rev. Letters **8**, 450 (1962).

We see that the constant form factors ($f_-/f_+\equiv\xi=-9$) used by Dobbs *et al.* and Boyarski *et al.*[9] in their experiments cannot possibly fit either the observed π^0 or μ^+ energy distributions as measured by Brown *et al.* The curve corresponding to $\delta=-0.065$ gives reasonable agreement with experiment. Note that Brown *et al.* use two parameters in their fit while we use one. We find that δ comes out small compared to one, as our theory predicts.

Using the δ obtained from the experiment of Brown *et al.*, we may compute what we would expect Dobbs *et al.* and Boyarski *et al.* to find in their experiments. The result is given in Fig. 7. Clearly, the form factors determined by Brown *et al.* do not fit the data of Dobbs *et al.* and Boyarski *et al.*, while $\delta=-0.065$ gives a result consistent with experiment.

Because Dobbs *et al.* and Boyarski *et al.* measure only the upper part of the μ spectrum, while Brown *et al.* measure the π^0 energy spectrum and the bottom part of the μ spectrum, it is possible that the data of these three groups are not in contradiction. A contradiction arises only if we assume that the form factors are essentially constant. Figures 8 and 9 give theoretical curves (without experimental biases) for the μ^+ and π^0 energy spectra.

Using the model with a fixed value for δ, the μ longitudinal polarization spectrum may be computed.

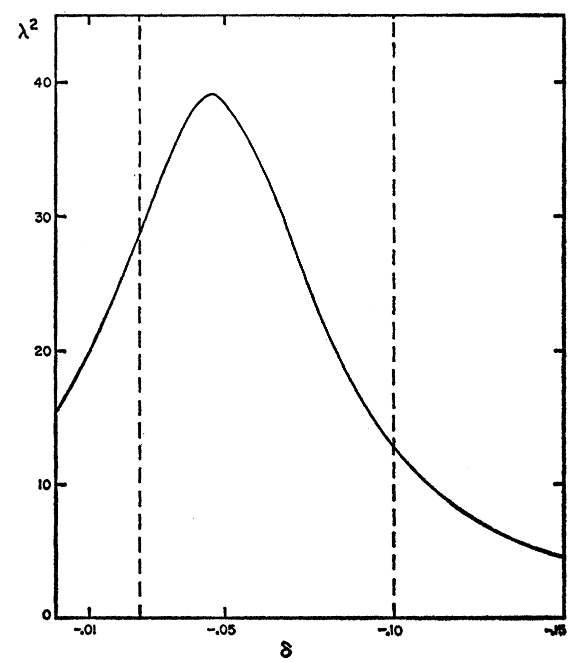

FIG. 3. The effective coupling constant squared λ^2 is plotted as a function of δ. The experimental rate of 4.0×10^6 sec^{-1}forK_{e3}^+ decay has been used. The dashed vertical lines indicate the restriction placed on δ by the known $K_{\mu3}^+/K_{e3}^+$ branching ratio.

[9] J. M. Dobbs, K. Lande, A. K. Mann, K. Reibel, F. J. Sciulli, H. Uto, D. H. White, and K. K. Young, Phys. Rev. Letters **8**, 295 (1962); A. M. Boyarski, E. C. Loh, L. Q. Niemela, D. M, Ritson, R. Weinstein, and S. Ozaki, Phys. Rev. **128**, 2398 (1962).

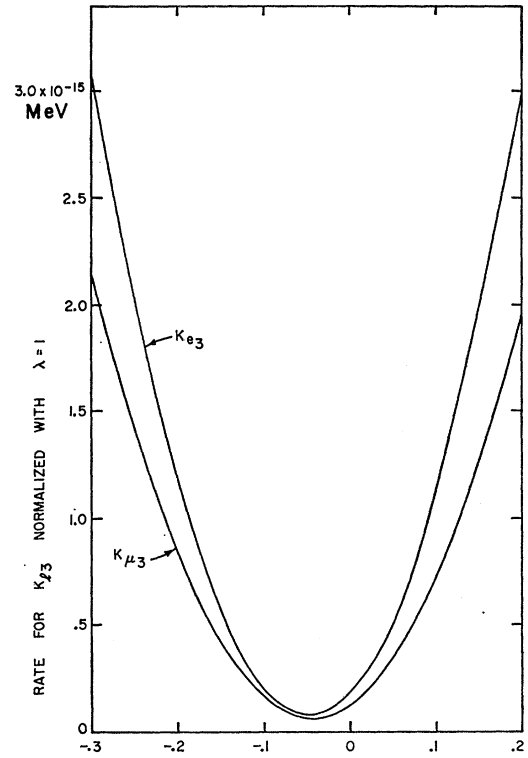

FIG. 4. The K leptonic decay rates are given as a function of δ with λ set equal to 1.

Figure 10 gives some typical polarization curves. For large μ kinetic energies ($T_\mu > 110$ MeV), the polarization comes out negative for all reasonable δ (all values of δ compatible with the $K_{\mu3}/K_{e3}$ branching ratio). For intermediate values of T_μ (35 MeV$<T_\mu<$75 MeV), the polarization is positive for all reasonable δ. For $T_\mu<75$

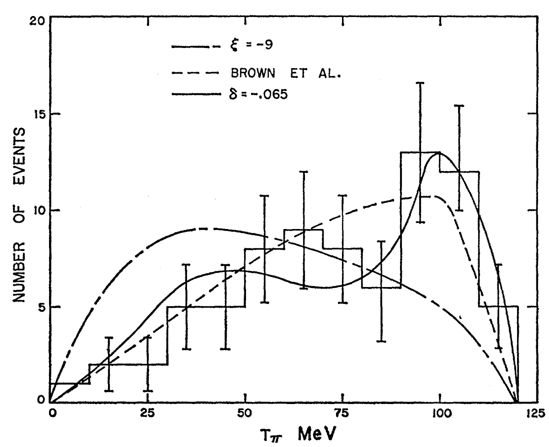

FIG. 5. The histogram gives the π energy spectrum in the $K_{\mu3}^+$ decay as measured by Brown *et al.* The kinetic energy of the π is T_π. The smooth theoretical curves have been corrected for experimental biases. Brown *et al.* use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ. The curve labeled $\xi=-9$ is the constant form factor theory implied by the experiments of Dobbs *et al.* and Boyarski *et al.*

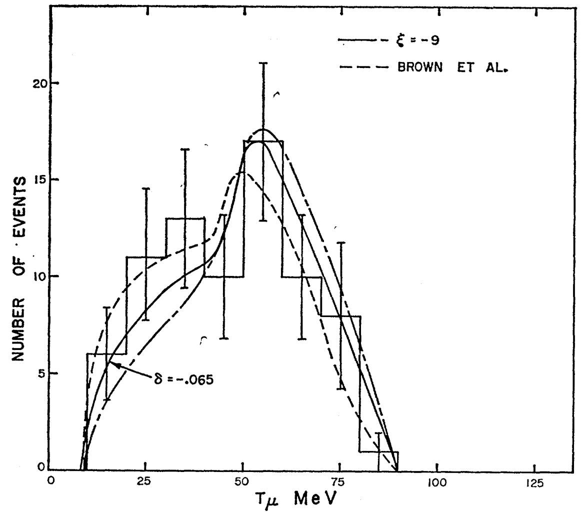

FIG. 6. The histogram gives the μ^+ energy spectrum in the $K_{\mu3}^+$ decay as measured by Brown *et al.* The kinetic energy of the μ is T_μ. The smooth theoretical curves have been corrected for experimental biases. Brown *et al.* use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ.

MeV, the polarization can be either positive or negative.

We would like to emphasize that certain very sensitive quantities, like the polarization of the μ in $K_{\mu3}^+$ decay or the π^0 energy spectrum in K_{e3}^+ decay, will not be very well determined within the framework of

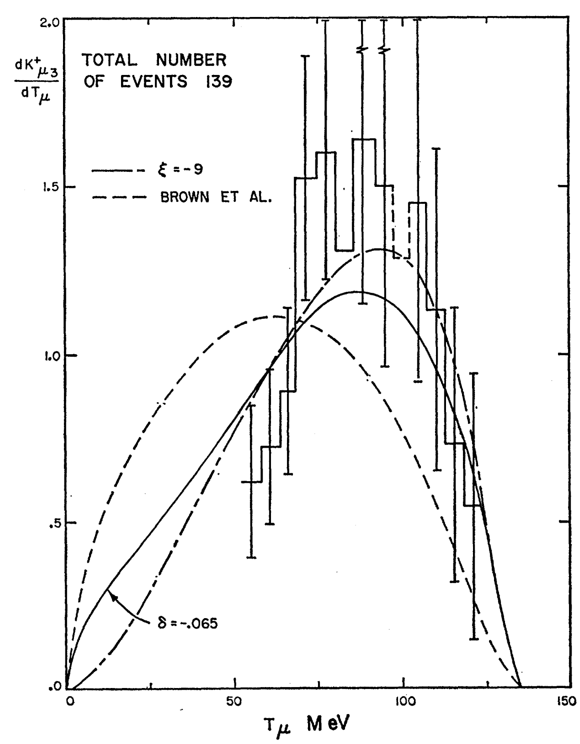

FIG. 7. The experimental μ^+ energy spectrum, as measured by Dobbs *et al.*, is represented by the histogram. The histogram has been corrected for experimental biases.

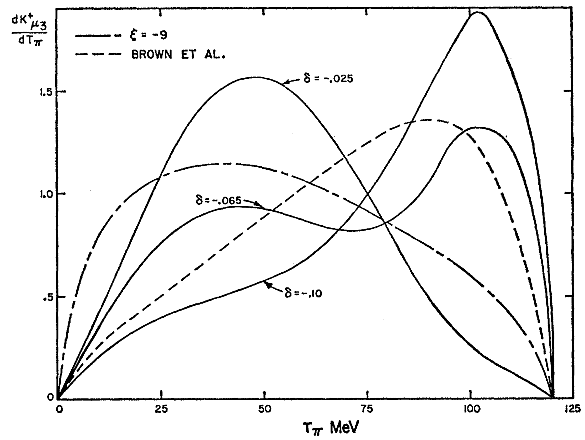

FIG. 8. The π energy spectrum predicted by various theories is given. Note the sensitivity of the spectrum to values of δ. The dip in the spectrum for the curve $\delta=-0.065$ is due to the zero of $f_+(s)$ in the physical region of s.

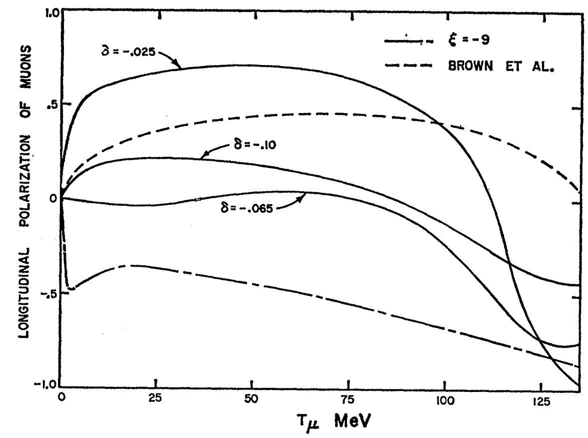

FIG. 10. The longitudinal polarization of the μ in $K_{\mu 3}{}^+$ decay is plotted as a function of μ kinetic energy. Although the polarization fluctuates wildly with small changes in δ, large μ energies always yield negative polarizations.

our approximations. Quadratic terms in s should also be included if we expect good agreement with experiment.

If we introduce a particle X to mediate the weak interactions, then we may summarize its effect by a change of form factors.

$$f_-\rightarrow f_- -\frac{\Delta m^2}{M_x{}^2}\frac{f_+}{1-(s/M_x{}^2)},\quad f_+\rightarrow\frac{1}{1-(s/M_x{}^2)}f_+,$$

where M_x is the mass of the X. Because we lack detailed knowledge of $v(s)$ and $d(s)$, the leptonic decays of the K meson seem to be a poor place for isolating the effects of the X. Figure 11 gives some indication of the size of X effects.

The concept of a universal Fermi interaction has never been very well defined. For example, to test for universality in $K_{l3}{}^+$ decay, it has been customary to consider $Gf_+(0)\equiv S_v$ as an effective coupling constant. Since it turns out that $S_v{}^2\ll G^2$, a universal form for the interaction is not apparent. However, if $f_+(s)$ is rapidly varying with s, then this test for universality may not be fair. Perhaps we should evaluate $Gf_+(s)$ at a different value of s when forming S_v and making our

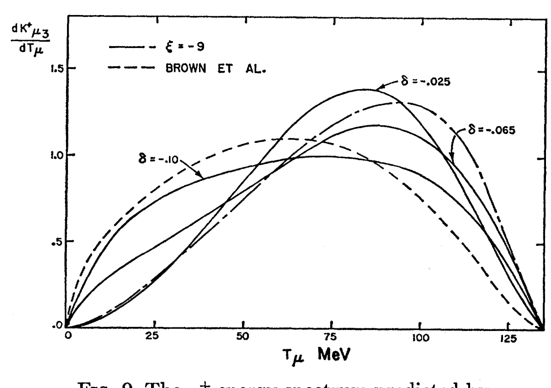

FIG. 9. The μ^+ energy spectrum predicted by various theories is given.

comparison with G. The slowness of the leptonic decay of the K^+ might then be explained on the basis of a partially conserved current. The rate is slow because the matrix element is of the order of $\langle\pi^0|\partial_\alpha s_\alpha{}^V|K^+\rangle$ which is a small quantity because $s_\alpha{}^V$ is partially conserved.

In concluding this section on K^+ decay, we would like to re-emphasize that the existence of a *partially* conserved current implies a profound deviation from what would be expected on the basis of phase-space arguments or almost constant form factors. If both the experiments of Brown *et al.* and Dobbs *et al.* prove to be correct, the hypothesis of almost constant form factors will no longer be tenable, while the assumption of a partially conserved current may finally attain some degree of experimental confirmation.

As a further application of our hypothesis of a "dominating" K^* and a partially-conserved current, we have computed the form factors for neutral K leptonic decay and have compared our results for $K_2{}^0\rightarrow e^\mp+\nu+\pi^\pm$ with the experiment of Luers *et al.*[10]

If we denote the corresponding form factors for $K_2{}^0$ leptonic decay by $h_+(s)$ and $h_-(s)$, we end up with the familiar form

$$h_+(s)=\lambda_2\left\{\frac{1}{1-(s/M^2)}-1+\delta_2\right\},\tag{9}$$

$$h_-(s)=-\lambda_2\frac{\Delta m^2}{M^2}\frac{1}{1-(s/M^2)}.\tag{10}$$

If there was only an $I=\frac{1}{2}$ current, then the spectra in K^+ and $K_2{}^0$ leptonic decay would be identical. In $K_{e3}{}^+$ decay we found that the π^0 energy spectrum had a zero when the kinetic energy of the π^0 was about 85 MeV (see Fig. 11). Since such a zero is not observed by Luers *et al.* in $K_{2e3}{}^0$ decay, we must have both $I=\frac{3}{2}$ and

[10] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1961).

$I=\frac{1}{2}$ currents. The present data do not allow a useful determination of λ_2 and δ_2.

It is interesting to note that if there exists a spin 1 $K\pi$ resonance other than the K^*,[11] then irrespective of the isotopic spin of this new particle, the form factors $f_+(s)$, $f_-(s)$, $h_+(s)$, and $h_-(s)$ still have the same effective representations (7), (8), (9), and (10) if we neglect quadratic terms in s. Only the physical interpretation of λ, δ, λ_2, and δ_2 changes. Hence, within the approximations made, our theory is not sensitive to the possible existence of other spin 1 $K\pi$ resonances.

Let us now briefly turn to the leptonic decay of the Λ. There, the strong interaction matrix elements of interest are

$$\langle p|s_\mu{}^V(0)|\Lambda\rangle=(m_pm_\Lambda/E_pE_\Lambda)^{1/2}\bar{u}_p\{i\gamma_\mu F_1(s)+i\tfrac{1}{2}[\gamma_\mu,\gamma_\nu s_\nu]F_2(s)+s_\mu F_3(s)\}u_\Lambda,$$

$$\langle p|s_\mu{}^A(0)|\Lambda\rangle=(m_pm_\Lambda/E_pE_\Lambda)^{1/2}\bar{u}_p\{i\gamma_\mu\gamma_5 G_1(s)+i\tfrac{1}{2}[\gamma_\mu,\gamma_\nu s_\nu]\gamma_5 G_2(s)+\gamma_5 s_\mu G_3(s)\}u_\Lambda,$$

where $s_\mu=(p_\Lambda-p_p)_\mu$ and $s=-s_\mu s_\mu$.

We consider the structure of $s_\mu{}^V$. Proceeding as before, we find that F_1 receives only p-wave contributions and that

$$\langle p|\partial_\alpha s_\alpha{}^V(0)|\Lambda\rangle\propto\Delta mF_1(s)+sF_3(s)=\omega\Delta mD(s)$$

receives only s-wave contributions. $\Delta m=m_\Lambda-m_p$. Because we do not know of any zero mass particle that would give rise to poles in our form factors, we find

$$F_1(s)=\omega\left\{D(0)+\frac{s}{M^2}\frac{1}{1-(s/M^2)}+V(s)-V(0)\right\},$$

$$F_3(s)=-\omega\frac{\Delta m^2}{M^2}\left\{\frac{1}{1-(s/M^2)}+\frac{M^2}{s}[V(s)-V(0)]-\frac{M^2}{s}[D(s)-D(0)]\right\},$$

where $V(s)$ represents all p-wave contributions to $F_1(s)$ other than those of the K^*. M is the mass of the K^*, and as in the case of K leptonic decay, we assume that $|D(s)|\ll 1$ in the physical region for s.

The point we wish to stress is that while $F_3(s)$ may be treated as being essentially constant, $F_1(s)$ is now a rapidly varying function of s and may even pass through zero in the physical region. Up to this time, it has been customary to take all form factors constant[12] and, because of the small momentum transfers involved $(m_\Lambda-m_p)^2\leq s\leq m_l^2$, the terms containing $F_2(s)$ and $F_3(s)$ have been neglected compared to the term containing $F_1(s)$. It is quite possible that this procedure is not justified.

Once again the concept of a universal Fermi interaction is ill defined because of the rapid variation of $F_1(s)$. As in leptonic K decay, an explanation for the slowness of the vector part of $\Lambda\beta$ decay may be connected with the partial conservation of $s_\alpha{}^V$. Because of the lack of experimental evidence and the wealth of unknown constants in the form factors, we are not able to say more about the problem at this time.

Note added in proof. A paper on K leptonic decay which arrives, from a different viewpoint, at a set of form factors essentially equivalent to those given in Eqs. (7) and (8) has been recently called to the attention of the author. It is as follows: N. Brene, L. Egardt, B. Qvist, and D. A. Geffen, Nucl. Phys. **30**, 399 (1962).

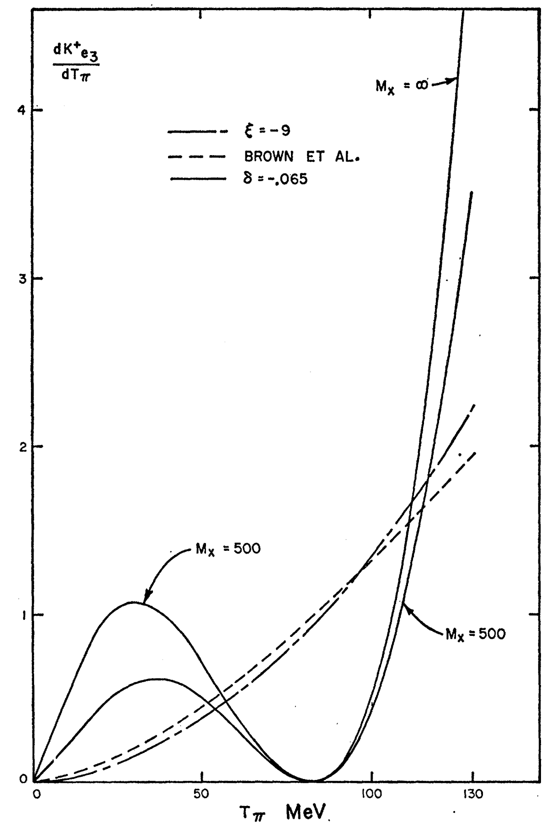

FIG. 11. The size of effects due to a vector boson X mediating the weak interactions is given for the electron energy spectrum in the $K_{e3}{}^+$ decay. Although strong interactions could give rise to similar variations in the electron spectrum, the zero in the spectrum is a definite peculiarity of our theory arising from the zero of $f_+(s)$ in the physical region of s. A direct measurement of this spectrum would be a crucial test for the hypothesis of a partially cnsoerved current.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Murray Gell-Mann for innumerable comments and criticisms. The author would also like to thank Professor Alvin Tollestrup for conversations regarding experimental aspects of the problem, along with Dr. George Trilling who has kindly supplied the data from the experiment of Brown *et al.*

[11] There is some experimental evidence for the existence of such a resonance with a mass of 730 MeV. See G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962); or in *Proceedings of the 1962 Annual International Conference on High-Energy Physics*, edited by J. Prantki (CERN, Geneva, 1962).

[12] See, for example, R. Norton, Phys. Rev. **126**, 1216 (1962).