

## Supplementary Videos

We present three movie clips showing tracer particle motion in oscillatory shear flows created using a Couette flow cell. The volume fraction  $\phi$  of the sample shown in the movie clips is 0.30; the  $x$  (flow) direction is horizontal and the  $z$  (axial or “vorticity”) direction is vertical. A small fraction ( $< 1\%$ ) of the particles are dyed black so that they are visible in the movie clips; the rest of the particles are rendered invisible by matching the refractive index of the suspending fluid to that of the particles. The diameter of the particles is  $d = 230 \pm 20 \mu\text{m}$ , which sets the length scale in the movie clips. The depth of focus spans the width of the gap (2.5 mm) of the Couette flow cell (in the  $y$  or velocity gradient direction) so that particles at all values of  $y$  are visible. The velocity field  $\mathbf{u}(x, y, z, t)$  inside the observation window in the flow cell is well-approximated by  $\mathbf{u}(x, y, z, t) = \dot{\gamma}(t) y \hat{\mathbf{e}}_x$ , where  $\hat{\mathbf{e}}_x$  is the unit vector in the flow direction and  $y$  is the spatial coordinate in the velocity gradient direction. Movie 1 shows the oscillatory shear flow; Movies 2 and 3 show periodically sampled particle positions at two different strain amplitudes, to demonstrate reversible and irreversible motion below and above the threshold measured in the paper.

### Supplementary Video 1: Continuous sampling

Continuously sampled movie clip of particle motion in an oscillatory shear flow. Particles at different positions in the shear flow (i.e. at different  $y$ -coordinates in the flow cell) move with different velocities. The period of the oscillatory flow in this movie clip is a few seconds, which is shorter than the period typically used in the experiments (25-100 s). The strain amplitude is  $\gamma_0 = 2.5$ . Although the particles do not return to their starting points after a cycle, this only becomes clear with the periodic sampling shown in Movie 3.

**Supplementary Video 2: Reversible flow**

Periodically sampled movie clip of particle motion in an oscillatory shear flow with a period of 25 s and a strain amplitude  $\gamma_0 = 1.0$ . The particle positions, sampled once per period, appear virtually stationary, indicating that each particle returns to its original position when the flow is reversed. This 20 s movie clip running at 30 frames/s spans 600 periods or 1200 flow reversals and constitutes a severe test of flow reversibility. The total experimental run time is approximately 4.2 hours (600 periods  $\times$  25 s/period). The very slow drift in the particle positions over the entire span of the measurement is due to imperfect synchronization between the oscillatory flow and the camera triggering.

**Supplementary Video 3: Irreversible flow**

Periodically sampled movie clip of particle motion in an oscillatory shear flow at a strain amplitude  $\gamma_0 = 2.5$ . The particle positions no longer appear stationary but randomly jump about from frame to frame, even though  $\gamma_0$  is only larger than that in Movie 2 by a factor of 2.5. Close observation reveals that the mean random motion in the  $x$  (horizontal or flow) direction is considerably larger than in the  $z$  (vertical, axial, or “vorticity”) direction.