to an exponent prime to l. For if p belongs to the exponent $f(\text{mod } l)$ and such that $ef = l - 1$ with e odd, and if r is a primitive root of l,

$$\mathfrak{p}(\zeta) = \mathfrak{p}(\zeta^{r^e})$$

which gives from (8a)

$$r^{e(l - 2a)} \equiv 1(\text{mod } l)$$

which is impossible since r is a primitive root of l and the exponent is odd.

[1] *Proc. Nat. Acad. Sci.*, **15**, 45 (1929).

[2] We say that an ideal $\mathfrak{p}$ belongs to an exponent n when $\mathfrak{p}^n$ is a principle ideal but $\mathfrak{p}^d$ is not a principal ideal for $0 < d < n$.

[3] *Trans. Amer. Math. Soc.*, **31**, 632 (1929).

[4] *Duke Math. Jour.*, **5**, 419–420 (1939).

[5] *Trans. Amer. Math. Soc.*, **31**, 632–633 (1929).

[6] Dickson, *Jour. für. Math.*, **135**, 134–141 (1909).

[7] *Proc. Nat. Acad. Sci.*, **16**, 303 (1930).

[8] Landau, *Math. Zeitschrift*, **2**, 52–154 (1918).

[9] Hasse, *Jahresbericht Deutschen Math.-Verein*, Ergänzungsbände VI, 61 (1930).

[10] Hasse, loc. cit., p. 112.

ON THE PHYSICAL CHARACTERISTICS OF THE HYDRA CLUSTER OF NEBULAE

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A. Observational Data.—In a previous paper[1] counts of the brighter nebulae in the Hydra cluster were communicated. The distance of the Hydra cluster was estimated as 7.3×10^6 parsecs and the average apparent velocity of recession to be expected was given as $v = 4100$ km./sec. No observations of the red shift for nebulae in the Hydra cluster were available at the time. Dr. Hubble subsequently obtained a spectrum plate for NGC 3309 which nebula is a member of the Hydra cluster. He kindly informs me that the apparent velocity of recession for NGC 3309 is of the order of 3950 km./sec., a value which is in good agreement with our original estimate.

B. The Radial Distribution of Nebulae in the Hydra Cluster.—As emphasized previously[1] the Hydra cluster exhibits spherical symmetry and is therefore suspected to have reached a statistically stationary state. In order to check this conclusion two additional tests are available. In the first place we may compare the observed radial distribution of nebulae in the Hydra cluster with the distribution derived by Emden[2] for the bounded isothermal gravitational gas sphere and secondly we can verify whether the ratio $\overline{w_r^2}/\rho_0$ of the square of the dispersion in radial velocities of the cluster nebulae to the central density ρ_0 of the cluster can be correctly determined from the observed *structural length or structural index* α of the cluster.[3]

If from the previously[1] given counts in the Hydra cluster of nebulae brighter than the apparent photographic magnitude $m_p = 16.2$ we subtract the nebulae which belong to the general background (1.8 nebulae per square degree), we obtain for the average number of nebulae N_r per square degree at various distances r from the center of the cluster the values given in table 1.

TABLE 1

RADIAL DISTRIBUTION OF NEBULAE IN THE HYDRA CLUSTER

r IN MINUTES OF ARC	N_r	$8.2\ N_r$	r_1	$1000\ D$	$1000\ D - 37$
0	...	....	0	3032	2995
10	241.8	1983	2.5	1815	1778
20	121.2	994	5	832	795
30	44.6	366	7.5	476	439
40	24.9	204	10	318	281
50	30.7	252	12.5	243	206
60	24.9	204	15	195	158
70	13.3	109	17.5	163	126
80	11.6	95	20	140	103
90	7.7	63	22.5	126	89
100	1.9	15.6	25	111	74
110	6.4	52.5	27.5	97	60
120	5.3	43.5	30	92	55
130	1.0	8.2	32.5	87	50
140	4.2	34.4	35	81	44
150	5.4	44.3	37.5	75	38
160	...	....	40	69.6	32.6
170	2.4	19.7	42.5	66.4	29.4
180	2.1	17.2	45	63.1	26.1
190	2.6	21.3	47.5	59.8	22.8
200	2.4	19.7	50	56.5	19.5
210	1.3	9.1	52.5	54.6	17.6
220	0.6	4.9	55	52.7	15.7
230	2.6	21.3	57.5	50.8	13.8
240	1.3	9.1	60	49	12
250	0.5	4.6	62.5	45	8
280	...	....	70	41.5	4.5
340	...	....	85	37	0

Column 4 gives the distances r_1 in the standard[3] Emden isothermal gas sphere to which the actual distances r from the center of the cluster are reduced through the relation $r = \alpha r_1$. The values of D represent the spatial densities in an infinite isothermal sphere while $D - 37/1000$ is the projected density in that bounded isothermal sphere which best

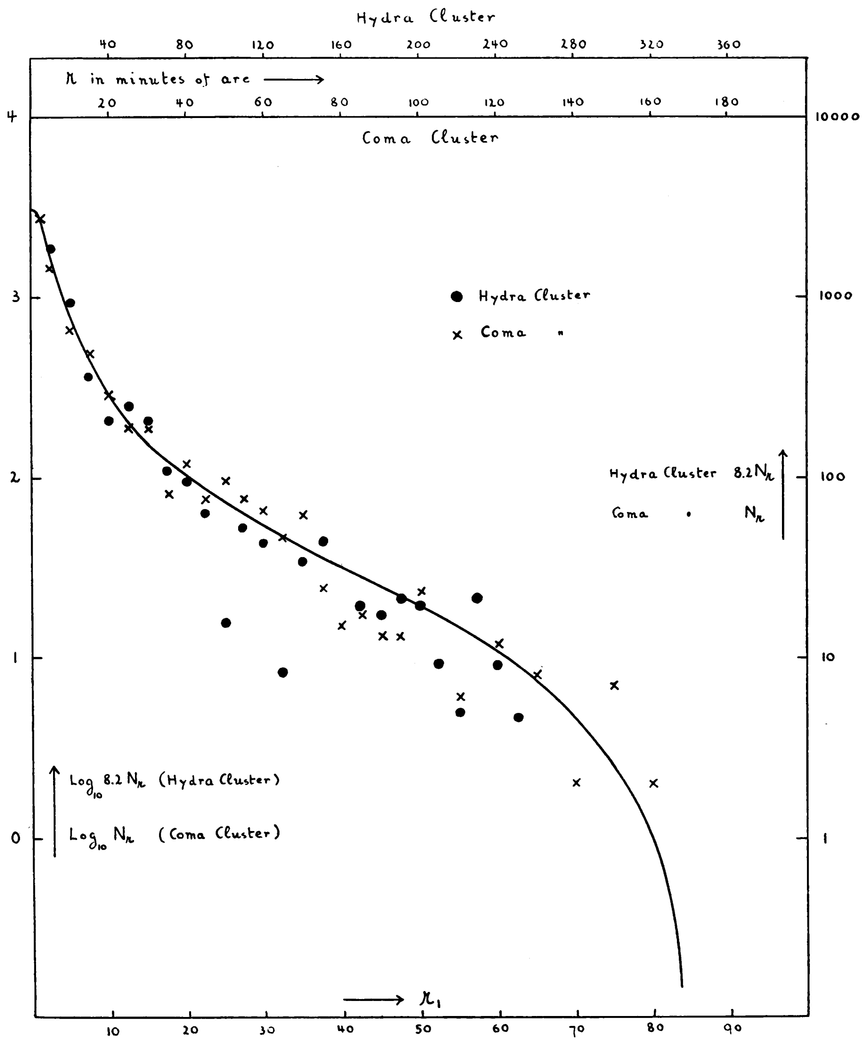

FIGURE 1

represents the radial distribution of the brighter nebulae in the Coma cluster with which we here compare the data on the Hydra cluster. From figure 1 we see that the properly reduced radial distributions of the Coma cluster and of the Hydra cluster agree among themselves, as well as with the theoretical curve, within the limits of statistical fluctuations to be expected for the relatively small number of nebulae involved in the counts.

There is some indication that the values of N_r for the Hydra cluster begin to fall off from the theoretical curve at slightly smaller values of r_1 than those for the Coma cluster. This checks the general expectation that, the smaller the population of a cluster (and the higher its velocity dispersion) the smaller should be the distance from the center of the cluster at which the Boltzmann distribution changes over into a Smoluchowski distribution.

The data for the Coma cluster plotted in figure 1 include the nebulae brighter than the apparent magnitude $m_p = 16.6$ or brighter than the absolute magnitude $M_p = -14.5$ while the data for the Hydra cluster include the nebulae brighter than the apparent magnitude $m_p = 16.2$ or the absolute magnitude $M_p = -13.1$.

C. Physical Characteristics of the Hydra Cluster.—From figure 1 we deduce that the structural length $\alpha = r/r_1$ for the Coma and Hydra clusters in angular measure are 2′ arc and 4′ arc, respectively. Since the distances of the two clusters are 13.8×10^6 and 7.3×10^6 parsecs, respectively, we obtain for the

Coma Cluster: $\alpha = 2.48 \times 10^{22}$ cm. (1)

Hydra Cluster: $\alpha = 2.56 \times 10^{22}$ cm. (2)

By definition[3] it is

$$\alpha = (\overline{w^2}/12\pi\Gamma\rho_0)^{1/2} \quad (3)$$

where $\overline{w^2}$ is the square of the velocity dispersion in a stationary cluster and ρ_0 is its central density. Since the velocity dispersion for the nebulae in the Hydra cluster is not known at the present we give in table 2 values for ρ_0 in dependence of a number of values for the velocity dispersion.

TABLE 2

$(\overline{w^2})^{1/2}$ in km./sec.	ρ_0 in g./cm.^{3}	$\overline{\mathfrak{M}}/\mathfrak{M}_\odot$
250	3.8×10^{-25}	5.8×10^{9}
500	1.5×10^{-24}	2.3×10^{10}
750	3.4×10^{-24}	5.2×10^{10}
1000	6.0×10^{-24}	9.2×10^{10}
1500	1.4×10^{-23}	2.1×10^{11}
2000	2.4×10^{-23}	3.7×10^{11}
2500	3.8×10^{-23}	5.8×10^{11}

In column 3 are given the ratios of the average mass $\overline{\mathfrak{M}}$ of the nebulae involved to the mass $\mathfrak{M}_\odot$ of the sun which correspond to the different values of the velocity dispersion given in column 1. The method of calculation of the ratios $\overline{\mathfrak{M}}/\mathfrak{M}_{\odot}$ is given in another place[3] where it is shown that for the same values of $(\overline{w^2})^{1/2}$ the ratios $\overline{\mathfrak{M}}/\mathfrak{M}_{\odot}$ for the Coma cluster are about half as large as those here obtained for the Hydra cluster. Now the nebulae included in the counts of the Coma cluster are brighter than the absolute magnitude $M_p = -14.5$ while those for the Hydra cluster are brighter than $M_p = -13.1$. Therefore if the same types of nebulae are included in the two clusters $\overline{\mathfrak{M}}$ (Coma) should be slightly larger than $\overline{\mathfrak{M}}$ (Hydra). This means that $2\overline{w^2}$ (Hydra) should be slightly smaller than $\overline{w^2}$ (Coma) or the velocity dispersion in the Hydra cluster should be smaller than that in the Coma cluster by a factor slightly smaller than 0.7. It will be interesting to check this conclusion by an investigation of the velocity dispersion in the Hydra cluster.

From our data we can also answer the question regarding the central density, that is the number of nebulae, say, per cubic mega-parsec (10^{18} cubic parsecs) in the two clusters. It was shown in another place[3] that

$$\rho_0 = \sigma_0/3.03\alpha \tag{4}$$

where ρ_0 is either the central spatial density or the number of nebulae per unit cube while σ_0 is the projected central density. From the extrapolation of our counts of nebulae to the center of the two clusters we consequently obtain the following results.

In the center of the Hydra cluster we obtain per cubic mega-parsec 8.7×10^5 nebulae which are brighter than the absolute magnitude $M = -13.1$ while the corresponding number for the Coma cluster is 2×10^6 nebulae per cubic mega-parsec which are brighter than the absolute magnitude $M = -14.5$. In comparison we mention that in the general field the average number of nebulae in the same range of absolute magnitudes is less than ten per cubic mega-parsec. The number of nebulae per unit volume in the center of the large clusters is therefore considerably higher than even experts in nebular counts might have expected. The high nebular density in clusters perhaps sheds some light on the remarkable fact that large symmetrical clusters of nebulae show all of the physical characteristics required for statistically stationary assemblies of objects whose interactions are governed by Newton's law. The peculiar possibility that these giant clusters whose elementary building stones are the nebulae themselves should provide the first quantitative test for Emden's results which were derived for gaseous spheres whose elementary building stones are atoms or molecules requires of course additional and thorough investigation.

In conclusion we mention that the clusters in Perseus, Cancer, Fornax and Pegasus as well as some of the groups in Pisces lend themselves to the same analysis as that applied here to the Hydra cluster and in another place to the Coma cluster.[3]

[1] Zwicky, F., these Proceedings, **27**, 264 (1941).

[2] Emden, R., *Gaskugeln*, Teubner, Leipzig, 1907.

[3] Zwicky, F., *Th. von Kármán Anniversary Volume*, May, 1941. See also a paper on the clustering of nebulae which is to appear shortly in the *Astrophys. Journal*.