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THE CAPTURE OF PROTONS BY Be^7

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Abstract: The capture of low-energy protons by Be^7 is calculated using a single-particle model for the ground state of B^8. All the model parameters are determined from the experimental data for the reactions $Li^7(n, n)Li^7$ and $Li^7(n, \gamma)Li^8$. The calculated results for $Be^7(p, \gamma)B^8$ are in agreement with the experimental data that are available and permit an extrapolation to energies corresponding to stellar temperatures.

1. Introduction

The capture of protons by Be^7 is of particular interest because this reaction occurs in the termination of the proton-proton chain in stellar interiors. The role of this reaction in stellar energy production has recently been reviewed in detail by Parker *et al.* [1]), and the relation of the reaction cross section to the feasibility of stellar neutrino detection has been discussed by Bahcall [2]). Since the B^8 decay is the major source of the high-energy neutrinos that will be detected by these experiments, its rate of production is of critical importance.

Experimental data for the $Be^7(p, \gamma)B^8$ reaction have been obtained by Kavanagh [3]) at proton energies of 0.80 and 1.40 MeV and have been compared with the direct capture calculations of Christy and Duck [4]) in order to obtain an extrapolation to stellar temperatures. These calculations take into account only extranuclear capture and are normalized to the experimental data in the region of 1 MeV.

The purpose of the present paper is to investigate an independent means for calculating the cross section for the $Be^7(p, \gamma)B^8$ reaction using the data available for the scattering and capture of neutrons by Li^7.

The capture of neutrons by Li^7 has been studied at neutron energies between 0.05 and 1.0 MeV by Imhof *et al.* [5]) and has been studied at thermal energies by several investigators [6]). Of special interest are measurements of the asymmetry in the beta decay that results when polarized thermal neutrons are captured by aligned Li^7 nuclei [7, 8]); these measurements indicate that over 80 % of the capture proceeds via the initial channel spin, $S = 2$. A preliminary measurement of the branching to the 2^+ ground state through the 1^+ first excited state has also been reported, but the

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branching ratio has not been measured accurately [9]. (The intensities of the gamma rays of energy 2.03, 1.06 and 0.96 MeV are in the ratio 0.8/0.2/0.3, respectively.)

The low-energy scattering of neutrons from Li^7 is well known, and the relative influence of the two channel spin states has been determined [10, 11]. The S wave scattering is dominated almost entirely by the $S = 2$ channel spin, which has a scattering length of $a_2 = -3.67 \pm 0.08$ fm. The S wave scattering for the $S = 1$ channel spin contributes only slightly (if at all) to the scattering and has $a_1 = 0.25 \pm 0.35$ fm.

2. Scattering and Bound State Calculations

A simple model for the bound and continuum states of $Li^8(B^8)$ is assumed that consists of a neutron (proton) plus a $Li^7(Be^7)$ core. The interaction of the nucleon and the core is taken to have the form of a Woods-Saxon potential plus the Coulomb potential of a uniformly charged sphere having the same radius as the nuclear potential. The shape parameters for the nuclear potential were obtained by interpolation in A from the optical model analyses of Johansson *et al.* [12] and have values of $R = 2.95$ fm and $a = 0.52$ fm, where

$$V_{\mathrm{WS}}(r) = \frac{-V_0}{1+\exp\left(\dfrac{r-R}{a}\right)}.$$

This model was used to calculate the values of V_0 which reproduced the positions of the low-lying P states of Li^8 and B^8. These values of V_0 are given in table 1 and show that the nuclear potential is nearly charge independent. Thus, one may infer that the potential that reproduces the S wave scattering of neutrons from Li^7 can also be used to predict the S wave scattering for protons on Be^7.

TABLE 1

The values of V_0 that reproduce the positions of the first three states of Li^8 and B^8

$J\pi$	Li^8 E_x (MeV)	Li^8 V_0 (MeV)	B^8 E_x (MeV)	B^8 V_0 (MeV)
2^+	0.000	32.89	0.000	32.62
1^+	0.978	30.54	0.79	30.27
3^+	2.26	26.42	2.17	26.30

The scattering length a_2 and the low energy cross section for $Li^7(n, n)Li^7$ are reproduced accurately by $V_0 = 3.56$ MeV. Thus, if one considers the trend of the results of table 1, a value of V_0 for $Be^7(p, p)Be^7$ would be between 3.29 and 3.56 MeV. (The S wave scattering for $S = 1$ is consistent with $V_0 = 0.0$ MeV.)

3. The Capture Calculations

The total cross section for E1 capture of neutrons (protons) to the ground state of Li^8(B^8) is given by

$$\sigma_t = \frac{5}{32} \frac{x^3 \pi e^2}{\hbar V_i k^2} \{\theta_2^2 \mathcal{J}_2^2 + \theta_1^2 \mathcal{J}_1^2\},$$

where x is the wave number of the gamma ray, k is the wave number of the incident nucleon, V_i is the velocity of the incident nucleon, and θ_s is the amplitude of channel spin s of the ground state. The $\mathcal{J}_s$ are the radial integrals

$$\mathcal{J}_s = \int_0^\infty U_f(r) R_{i,s}(r) r \, dr.$$

Here the radial part of the final state wave function is $U_f(r)/r$, and the initial state S wave scattering wave function for channel spin s is $R_{i,s}(r)/kr$.

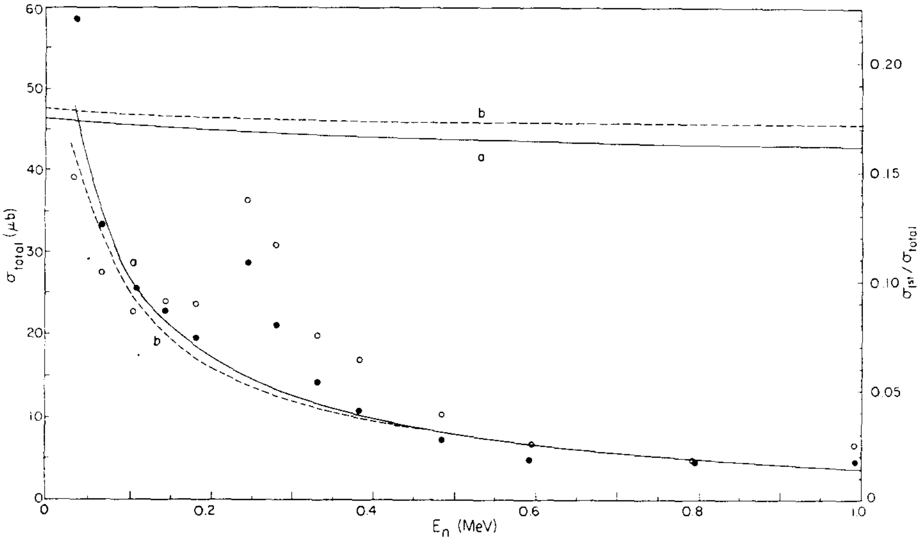

Fig. 1. The calculated and experimental total cross sections for Li^7(n, γ)Li^8 are shown for cases (a) (solid lines) and (b) (dashed lines). The approximately horizontal lines at the top of the figure give the ratio of the first excited state transition to the total transition rate and should be referred to the right-hand ordinate. The open and closed circles represent the normalization of the experimental data based on two separate reactions (see ref. [5]). (The anomaly seen near 0.25 MeV is the P wave 3$^+$ resonance mentioned in the text.)

The expression for the cross section for S wave capture to the 1^+ first excited state of Li^8 has the same functional form as that given above but must be multiplied by $\frac{3}{5}$.

Though most of the thermal capture ($\geq 80\,\%$) is known to proceed through the $S = 2$ channel spin, the $S = 1$ contribution at higher energies may not be negligible. Two possibilities were considered:

(a) All of the capture proceeds through $S = 2$. This gives a value of $\theta_2^2 = 0.55 \pm 0.11$ when the calculated results were normalized to the data of Imhof *et al.* [5]).

(b) The $S = 1$ channel spin contributes 20 % of the thermal capture (the maximum amount allowable). This gives $\theta_1^2/\theta_2^2 = 0.56$. When the calculations are normalized to the data, one finds $\theta_2^2 = 0.39 \pm 0.08$ and $\theta_1^2 = 0.22 \pm 0.04$.

The results are compared to the experimental data in fig. 1. A resonance in the neutron captuie is seen at $E_n = 0.258$ MeV, corresponding to an M1 capture of P wave neutrons. The assignment of this level as 3^+ was obtained from the scattering data [10]). Because the contribution to the cross section from P wave capture becomes negligible at low energies, the effect of this resonance has been ignored in the present treatment.

Also shown in this figure are the predicted branching ratios for cases (a) and (b) – these are seen to be in agreement with the branching ratio data that are available [9]). (For these calculations the amplitudes for $S = 1$ and $S = 2$ of the first excited state of Li^8 were assumed to be equal to those of the ground state. Such an assumption is not unreasonable because these states are identical in the L–S limit [13]). In any case, however, small differences in the θ values for these states would have no appreciable effect on the calculated results because of the small branching ratio.)

The size of the amplitudes θ_s represent in an approximate sense the degree to which the states can be approximated by a single-particle model. The 3^+ state allows a crude check on the values obtained; for this state, which can be formed only through $S = 2$ in this model, $\theta_2^2 \approx \Gamma_{\text{exp}}/\Gamma_{\text{model}} = 0.34 \pm 0.05$. Another confirming value comes from the DWBA analysis of the $Li^7(d, p)Li^8$(g.s.) reaction by Halton and Hodgson [14]); they obtain a value of $\theta^2 = 0.57 \pm 0.09$ for the ground state of Li^8.

The values of θ_s^2 obtained in cases (a) and (b) were used to calculate the capture of protons by Be^7. The results were found to be virtually identical for $V_0 = 3.29$ and 3.56 MeV, thus eliminating any ambiguity from the slight lack of charge symmetry introduced by this crude model. The $S = 1$ potential depth was taken to be $V_0 = 0.0$ MeV as in the case of Li^8. The cross section is shown in fig. 2; the lines indicate the extreme values of θ_2^2 that result when the precision of the $Li^7(n, \gamma)Li^8$ data are considered for cases (a) and (b). The calculated value agrees with the measured value at $E_p = 1.40$ MeV but is outside the assigned error for the 0.80 MeV point. (This disagreement may reflect the influence of resonant M1 capture at the 1^+ level ($E_p = 0.75$ MeV) of B^8.)

For the purpose of extrapolating the cross section to lower energies, the function S_0 is frequently used:

$$S_0 = E_{\text{c.m.}}\, \sigma_{\text{t, capture}} \exp(2\pi\eta),$$

where $\eta = Z_1 Z_2 e^2/\hbar V_i$. This function is shown in fig. 3 for $Be^7(p, \gamma)B^8$. Considering the uncertainties in the $Li^7(n, \gamma)Li^8$ data, the predicted value at zero energy is

$$S_0 = 0.012 \pm 0.003 \text{ keV} \cdot \text{b}.$$

If, however, we normalize these calculations directly to the $Be^7(p, \gamma)B^8$ data of Kavanagh [3]), we obtain at zero energy

$$S_0 = 0.021 \pm 0.008 \text{ keV} \cdot \text{b}.$$

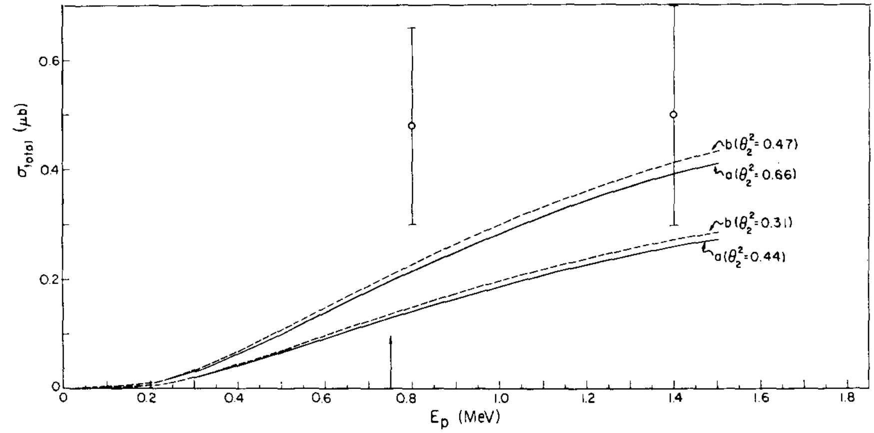

Fig. 2. The total cross section for $Be^7(p, \gamma)B^8$ for the two cases. The pair of lines for each case represent the extreme values of the amplitudes that result when the precision of the $Li^7(n, \gamma)Li^8$ measurements is considered. The arrow indicates the position of the P wave 1^+ state of B^8.

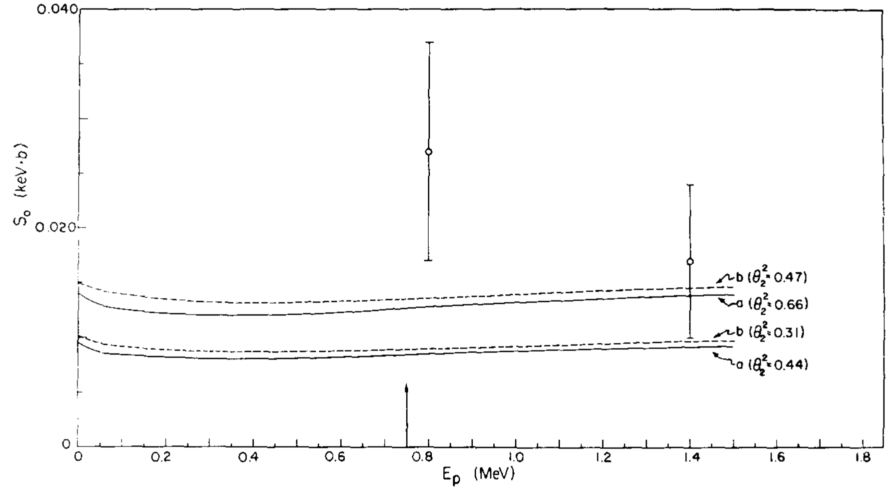

Fig. 3. The calculated and experimental values of S_0 are shown for the $Be^7(p, \gamma)B^8$ reaction. See fig. 2 for details.

These two values agree to within the combined errors, and if the two numbers are weighted inversely as their errors, we obtain a value of

$$S_0 = 0.0154 \pm 0.0035 \text{ keV} \cdot \text{b}.$$

The difference between the value of $S_0 = 0.021$ keV · b obtained here and that of 0.030 keV · b obtained by Christy and Duck is partly due to their assumption that only extranuclear capture contributes. They neglected contributions to the matrix element for radii less than 4.1 fm; this assumption is quite good at low energies but begins to break down before 1 MeV is reached. Since their calculations were normalized to the data at 1 MeV, the resultant value of S_0 would tend to be too large. Correcting their results for this contribution yields a value of $S_0 = 0.025$ keV · b. Considering the approximations used by Christy and Duck in evaluating the matrix element at low energies, the remaining difference between our results can probably be ignored.

Since the value of S_0 obtained here is somewhat smaller than that presently used, the calculated rate of production of B^8 in the sun is decreased, thereby reducing the importance of the B^8 decay as a source of stellar neutrinos.

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