TABLE 4

n	0	1	2
$\lambda^2 a_n(^1S)$	0.02522	0.6372	...
$\lambda^2 a_n(^3S)$	−0.4215	0.2720	1.77
$(\varphi_n^2/N_n)_{^1S}$	0.1845	0.2901	...
$(\varphi_n^2/N_n)_{^3S}$	0.01736	0.3791	0.3067

As one would expect, the convergence for $R = 4.732\ r_0$ is not so rapid as was the case for the potential hole where R could be taken to be r_0. Even for the latter case the first three levels should be taken to get an accurate result for the scattering amplitude.

* $\hbar$ denotes Planck's constant divided by 2π.

[1] Kapur, P. R., and Peierls, R., *Proc. Roy. Soc.*, **166**, 277 (1938).
[2] Wigner, E., *Zeit. Physik*, **83**, 253 (1933).
[3] Breit, G., Thaxton, H. M., and Eisenbud, L., *Phys. Rev.*, **55**, 1018 (1939).
[4] Share, S. S., Hoisington, L. E., and Breit, G., *Ibid.*, **55**, 1130 (1939).

ON THE FORMATION OF CLUSTERS OF NEBULAE AND THE COSMOLOGICAL TIME SCALE

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A. The Problem of the Short Time Scale in an Expanding Universe.—The red-shift in the spectra of nebulae is most easily interpreted as a Doppler effect caused by the mutual and approximately uniform recession of these nebulae from one another. This hypothesis of an expanding universe leads to an alarmingly short cosmological time scale with the consequence that some two billion years ago intergalactic space must have been practically non-existent and the nebulae were not clearly separable from one another. Nevertheless, the individual objects which constitute these nebulae such as the stars, and the double, triple and multiple systems of stars might well have existed in their present physical condition even in a very contracted universe. The argument that the earth and the stars have existed for periods longer than two billion years, and the fact that perhaps the present statistical distribution of stars could have been achieved only in a very much longer time than two billion years is therefore not sufficient to reject the hypothesis of the expanding universe. In order to show that the time scale demanded by this hypothesis is inacceptable we must endeavor to discuss objects which (*a*) in a contracted universe clearly could not have existed in their present form and (*b*) whose formation required intervals of time definitely larger than two billion years. If we succeed in finding any objects which meet these requirements (*a*) and (*b*) we shall have demonstrated the impossibility of a time scale which would allow us to interpret the nebular red-shift on the basis of an expanding universe.

B. The Clusters of Nebulae.—It is the contention of this investigation that the big and *regular* clusters of nebulae such as the Coma cluster represent cosmic objects which satisfy both of the requirements (*a*) and (*b*). Much work remains to be done to complete our knowledge of the clusters of nebulae, but some preliminary investigations suggest the probable general validity of the following relevant features.

(1) The large clusters of nebulae contain from 1000 to 10,000 nebulae. Their linear dimensions are presumably of the order of from one to ten megaparsecs.

(2) The most regular among the large clusters exhibit remarkable spherical symmetry, a fact which strongly indicates that they are stationary assemblies in the statistical mechanical sense. The Coma cluster is a good example of a highly symmetrical cluster.[1]

(3) The radial distribution of nebulae in the Coma cluster is remarkably similar to the distribution of luminous objects in individual globular nebulae.[1] The distribution curve can furthermore be very closely represented by the curve derived by Emden for the density distribution of matter in an isothermal gravitational gas sphere. This remarkable fact further strengthens our belief that the regular clusters represent statistically stationary configurations of objects whose interactions are governed by Newton's law of attraction.

(4) Clusters of nebulae delineate a segregation in types of nebulae inasmuch as the central parts of the clusters are rich in nebulae of the elliptical and globular types, whereas spirals are more abundant between the clusters.

(5) Investigations by Sinclair Smith[2] on the Virgo cluster indicate that the velocity distribution of the nebulae in the central parts of a cluster is the same as the velocity distribution at considerable distances from the center. Furthermore, according to Hubble[3] the dispersion among the peculiar motions of the so-called field nebulae in between the clusters seems to be rather smaller than the corresponding dispersion for the nebulae in the central regions of the great clusters.

In the following we shall try to analyze the preceding list of observational facts in the light of two hypotheses which would seem to exhaust the significant possibilities of explaining the formation of the great regular clusters in an expanding universe. The first hypothesis assumes that two billion years ago the nebulae were distributed at random. We shall show in section *C*, that in this case the nebulae at the present time still should be distributed at random, a conclusion which is in contradiction with the existence of the great regular clusters of nebulae. The second hypothesis assumes that these clusters, in a more condensed form, already were present in the contracted universe of two billion years ago. We shall show in section D that this hypothesis allows us to draw conclusions which are also in contradiction with the observations described in the preceding. We hold therefore that the universe cannot be expanding.

*C. Estimate of the Minimum Time Interval Which Is Necessary for the Formation of a Regular Cluster of Nebulae.**—For the statistical formation of a cluster of nebulae to take place, triple or multiple close encounters among nebulae are indispensable. Double impacts alone are not sufficient for the following reasons.

Suppose that a certain point C is destined to become the approximate center of a future large regular cluster of nebulae. In any random distribution a few nebulae only will be accidentally hovering around the point C. In order to enrich the neighborhood of C by a great number of hovering nebulae which might form a future cluster we must analyze the action of impacts between nebulae not belonging to the accidental original group. Consider two such nebulae of masses m_1 and m_2 which move with the vectorial velocities $\vec{v}_1$ and $\vec{v}_2$. The center of mass of this couple will continue to move with a velocity $\vec{v} = (m_1\vec{v}_1 + m_2\vec{v}_2)/(m_1 + m_2)$ relative to a system of coördinates Σ attached to C, even if an impact between the two nebulae takes place near C. Such impacts can therefore not effectively contribute to the number of nebulae hovering around C, except perhaps in the very rare case that a head-on impact takes place between two nebulae for which $\vec{v}$ is approximately equal to zero. The situation, however, is different for triple and multiple impacts. During such impacts one nebula may be effectively stopped down near C without any violation of the law of conservation of momentum because the remaining nebulae may carry away the surplus momentum.

Since among the multiple collisions which are effective in the production of clusters, triple collisions are by far the most numerous we shall now calculate their frequency in a given random distribution of nebulae.

Suppose that N nebulae are distributed at random in a unit volume of space and that the average velocity square is $\overline{v^2}$. We shall say that an n-fold collision is taking place if n nebulae are simultaneously enclosed by a sphere (impact volume) whose volume is $\sigma = 4\pi r^3/3$. The number Z of n-fold collisions per unit volume and unit time under these circumstances is[4]

$$Z \cong N^n \sigma^{n-1} (\overline{v^2})^{1/2}/n!\, r, \qquad (1)$$

where the unit of volume must be chosen so that $\sigma \ll 1$. We chose as the unit of volume a cube whose edge is L = 10 megaparsecs = 3×10^{25}

cm. In this "unit" volume there are, according to Hubble,[5] about $N = 5 \times 10^3$ nebulae. For an effective triple collision to take place, r and σ must certainly not be larger than about $r = 10^4$ parsecs $= 10^{-3} L$ and $\sigma = 4\pi r^3/3 \cong 4 \times 10^{-9} L^3$. Furthermore we have in order of magnitude

$$(\bar{v^2})^{1/2} \cong 10^7 \text{ cm./sec.} \cong 3.3 \times 10^{-19} L/\text{sec.} \qquad (2)$$

Substituting these values in (1) we obtain

$$Z = 10^{-22}/\text{sec.} = 3 \times 10^{-15}/\text{year.} \qquad (3)$$

In other words, it takes on the average about 3.3×10^{14} years for a single triple collision to occur in a volume L^3 which contains just about the number of nebulae ($N = 5 \times 10^3$) necessary for the formation of a large regular cluster. Before a cluster is really formed every one of these nebulae, on the average, will have to be involved more than once in a multiple impact, so that the time of formation τ of a large cluster out of a random distribution of nebulae is

$$\tau > 10^{18} \text{ years.} \qquad (4)$$

This period τ which is necessary for the formation of a regular cluster is so much larger than the time of only 2×10^9 years available in an expanding universe that the above considerations, if substantiated by further observations, would rule out any possibility of interpretation of the nebular red-shift on the basis of an expanding universe.

D. Clusters of Nebulae in an Expanding Universe.—It might be objected that in a contracted universe some billions of years ago clusters of nebulae have already existed and that the clusters observed at the present time are identical with the original clusters. The present greater dimensions of these clusters then would be due to the fact that the original member nebulae have moved to greater distances from the original center of a cluster, depending on their initial velocities. The observations discussed in the sections *B*4 and *B*5, however, make this possibility improbable. Indeed, in the case just discussed the nebulae on the outskirts of a cluster should show predominantly radial velocities directed away from the center of a cluster. These velocities should assume the largest values for the nebulae which have traveled to the greatest distances from the centers of the clusters to which they originally belonged. This conclusion is contradicted by the observations mentioned in section *B*5. Also it would seem impossible to explain the preponderance of spiral nebulae on the outskirts of clusters. In addition it would be difficult to explain the spatial distribution of nebulae in a cluster.

E. Other Observations Pointing towards a Stationary Universe rather than an Expanding Universe.—It was thought for a while that counts of nebulae to successively fainter magnitudes would lead to a decision about the expansion of the universe. It was, however, soon realized that this investigation which was partially carried through by Hubble,[5] would have to overcome two serious difficulties before conclusions drawn from it could be accepted with any degree of confidence. In the first place it would be necessary to secure thousands of long exposure plates of equally good quality taken with the large reflectors under good seeing conditions. Many years are required to accumulate such a set of plates, especially if duplicates of each field in the sky are desired. In the second place corrections for the effect of the redshift on the apparent magnitude of the distant nebulae must be applied. The calculation of these corrections and the interpretation of the resulting counts of nebulae are largely dependent on the real nature of the red-shift and on the theoretical model of the universe from which one starts. Counts of nebulae in themselves do not therefore enable us to reach directly a clear cut decision for or against the expansion of the universe. But even if the practical and theoretical difficulties just mentioned could be eliminated in their entirety, we now know that counts of nebulae cannot, in principle, lead to the decision desired because of the interference of gravitational lense effects[6] which, at the crucial distances of hundreds of millions of light years considered by Hubble, must become of ever increasing importance.

There is nevertheless a method, based on *specific* counts of nebulae in clusters, which is free from the three objections discussed in the preceding and which promises to supply us with additional decisive evidence concerning the nature of the red-shift from nebulae.[1] As already mentioned, in an expanding universe clusters at an earlier stage must have already existed in a very condensed state, since otherwise there would not have been time enough for them to be formed at all. This means that clusters now observed at very great distances must on the average be much more condensed than the nearby clusters. At the present time not enough clusters are known to make a test of this conclusion practical. With the completion of the 48-inch Schmidt telescope for the observatory on Palomar Mountain clusters at great distances will be quickly discovered in sufficient numbers to make possible the investigation just suggested. The 48-inch telescope also will enable us to accumulate a statistically complete set of data concerning the properties of clusters discussed in section *B*. If these additional data substantiate the preliminary investigations available at the present time the case against the hypothesis of an expanding universe will in our opinion be a strong one.

* For a discussion of the time of formation of double galaxies, see for instance E. Holmberg, *Ann. Observ. Lund*, No. 6, 103 (1937). The fact that the formation of double nebulae presumably involves a period of time longer than two billion years cannot, however, be used as a decisive argument against the theory of the expanding universe since the double nebulae do not satisfy the requirement (*a*) of Section *A*.

[1] F. Zwicky, *Astrophys. Jour.*, **86**, 217 (1937).
[2] S. Smith, *Ibid.*, **83**, 25 (1936).
[3] E. Hubble, *Ibid.*, **84**, 323 (1936).
[4] J. K. Syrkin, *Phys. Zeitschr.*, **24**, 236 (1923).
[5] E. Hubble, *The Realm of Nebulae*, Yale University Press (1936).
[6] F. Zwicky, *Phys. Rev.*, **51**, 290 (1937) and **51**, 679 (1937).

BODILY CONSTITUTION AND HUMAN LONGEVITY[1]

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I.—The problem with which this paper is concerned may be stated as follows: Consider two groups, *A* and *B*, of adult human beings, both drawn from the same universe, but in such a manner that the individuals in the *A* group are the *longest-lived* individuals in that universe, while the individuals in the *B* group are the *shortest-lived* to be found in the universe who also were, at the time they were observed and measured, of the same age in years, individual by individual, as the persons in the *A* group when they were observed. All the individuals in both the *A* group and the *B* group were in a state of sound health at the time of observation, as determined by medical examination. The question to be answered is: Were there significant differences between group *A* and group *B*, at the time of observation, in respect of bodily structure or functions that can be measured and expressed biometrically? The important part played by inheritance in the determination of human longevity has been abundantly demonstrated.[2] A necessary consequence of the fact that there is a genetic element involved in longevity is that each individual organism starts life with some degree of pre-determination as to the probable duration of its life. Environmental circumstances encountered along the way may modify the ultimate duration achieved by each individual, but apparently never to the complete elimination or obliteration of the effect of the genetic element, when the mean longevity of a group of substantial size is considered. Does this genetic influence on the distribution of longevity in the group *also* manifest itself in measurable morphological and physiological characteristics?

A first preliminary report of some of the results of our study of this problem was made in 1938 (Pearl, R.,[2(c)] pp. 473–476). The present paper is to be regarded as a further progress report of an investigation still being continued.