Nonprecessional spin-orbit effects on gravitational waves from inspiraling compact binaries to second post-Newtonian order

Benjamin J. Owen*
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125

Hideyuki Tagoshi†
National Astronomical Observatory, Mitaka, Tokyo 181, Japan

Akira Ohashi‡
Department of Physics, Kyoto University, Kyoto 606-01, Japan
(Received 30 October 1997; published 16 April 1998)

We derive all second post-Newtonian (2PN), non-precessional effects of spin-orbit coupling on the gravitational waveforms emitted by an inspiraling binary composed of spinning, compact bodies in a quasicircular orbit. Previous post-Newtonian calculations of spin-orbit effects (at 1.5PN order) relied on a fluid description of the spinning bodies. We simplify the calculations by introducing into post-Newtonian theory a δ-function description of the influence of the spins on the bodies' energy-momentum tensor. This description was recently used by Mino, Shibata, and Tanaka (MST) in Teukolsky-formalism analyses of particles orbiting massive black holes, and is based on prior work by Dixon. We compute the 2PN contributions to the waveforms by combining the MST energy-momentum tensor with the formalism of Blanchet, Damour, and Iyer for evaluating the binary's radiative multipoles, and with the well-known 1.5PN order equations of motion for the binary. Our results contribute at 2PN order only to the amplitudes of the waveforms. The secular evolution of the waveforms' phase—the quantity most accurately measurable by LIGO—is not affected by our results until 2.5PN order, at which point other spin-orbit effects also come into play. We plan to evaluate the entire 2.5PN spin-orbit contribution to the secular phase evolution in a future paper, using the techniques of this paper. [S0556-2821(98)05810-X]

PACS number(s): 04.25.Nx, 04.30.Db

I. INTRODUCTION

Inspiraling compact binaries are one of the main classes of gravitational wave source to be targeted by the coming generation of ground-based laser interferometers such as the Laser Interferometric Gravitational Wave Observatory (LIGO), VIRGO, GEO600, and TAMA [1]. There are two reasons for this. First, binary coalescences are expected to occur fairly often within the detection range of ''enhanced'' interferometers [2]. Astronomical lore estimates several neutron-star–neutron-star coalescences per year within 200 Mpc [3,4] and a similar rate of black-hole–black-hole coalescences within 200 Mpc to 1 Gpc [4–6]. Second, the signal from the final moments of inspiral is characterized by a complicated phase evolution containing detailed information about the physical parameters of the binary, such as the masses of the bodies and their spins about their own axes [1].

Because inspiral signals have such a complicated structure, and because they last many cycles within the frequency bands of ground-based interferometers, they are ideal candidates for the use of matched filtering [7]. Matched filtering, a signal-processing technique well-studied in the context of radar, can be used both to search for signals in noisy data and to estimate parameters once a signal is found. Matched filtering essentially entails cross-correlating noisy interferometer data with a set of theoretical template waveforms. If a template waveform is a good approximation to the signal waveform, the cross-correlation enhances the signal-to-noise ratio. In the context of matched filtering, a good approximation means (roughly speaking) one in which the phase evolution of the template matches that of the signal to within a half cycle out of the total spent in an interferometer's band. Because signals are expected to last up to tens of thousands of cycles in the bands of some interferometers, the templates must match any possible signal to a correspondingly high degree of precision.

Currently there is no exact solution to the generic two-body problem in general relativity. Thus, inspiral waveform templates are constructed using approximation schemes which must be carried out to high precision to be useful for matched filtering. These approximation schemes can be broadly grouped into two categories, the post-Newtonian approach and the black-hole perturbation approach.

The post-Newtonian approach is the longtime standard for gravitational wave generation. It involves expanding the Einstein equations and equations of motion in powers of the binary's orbital velocity v/c and gravitational potential $GM/rc^2 \sim (v/c)^2$, where the order in GM/rc^2 is referred to as the post-Newtonian (PN) order. Concurrently, the gravitational waveforms and luminosity are expanded in terms of time derivatives of symmetric, trace-free (STF) radiative multipoles, which are expressed as integrals of the matter source and gravitational fields. The radiative multipoles are

*Electronic address: owen@tapir.caltech.edu
†Electronic address: tagoshi@yso.mtk.nao.ac.jp
‡Electronic address: ohashi@tap.scphys.kyoto-u.ac.jp

combined with post-Newtonian equations of motion to yield explicit expressions for the waveforms, including the secular evolution of orbital phase and frequency due to radiation reaction. Recent summaries of the two main versions of the post-Newtonian approach are given by Blanchet, Damour, and Iyer [8] and by Will and Wiseman [9]. The post-Newtonian expansion of the waveforms of a binary currently has been carried out to 2.5PN order [10] [i.e., to $O(v/c)^5$ beyond the leading order radiation reaction and $O(v/c)^5$ beyond Newtonian gravity in all non-radiation reaction effects] in the case where the two bodies orbit in a quasicircular fashion and do not spin about their own axes. (By ''quasicircular'' we mean orbits that are circular aside from gradual inspiral due to gravitational radiation reaction.)

In the case where the bodies do spin, there are three types of spin effects to be considered. Effects of the first type, due to precession of the plane of the orbit, modulate the amplitude and frequency of the gravitational radiation in a complicated, non-monotonic fashion. Secular or dissipative effects, due to radiation reaction, contribute to the (monotonic) phase and frequency evolution of the orbit, and nondissipative effects contribute directly to the amplitudes of the various harmonics of orbital frequency in the waveforms, without affecting their phase evolution. All three types of effects can be further divided into spin-orbit contributions (i.e., terms involving one spin only) and spin-spin contributions (interactions between spins). Precessional effects were first extensively investigated by Apostolatos *et al.* [11] and by Kidder [12], and were found to complicate matters considerably. Therefore, like most other treatments of spin, ours will investigate the case where there is no precession—i.e., the spins are parallel or antiparallel to the orbital angular momentum—leaving precession for future studies.

Nonprecessional spin effects have been evaluated by Kidder, Will, and Wiseman [13,12] only to lowest order: 1.5PN for dissipative (and 1PN for non-dissipative) spin-orbit effects and 2PN for spin-spin effects.[1] The main reason for the discrepancy in progress between the spinning and nonspinning cases is the form of the matter source used in the Einstein equations. In the nonspinning case it is simple to write the energy-momentum tensor as a Dirac δ-function, which greatly simplifies the calculations. In order to derive spin effects, Refs. [13,12] treated the bodies as uniformly rotating balls of a perfect fluid. The perfect fluid energy-momentum tensor was integrated over a finite spatial volume, which made the multipole integrals much more cumbersome than in the δ-function case and introduced additional complications in the definition of the binary's center of mass. The net result was that spin calculations at a given post-Newtonian order seemed to require as much effort as spinless calculations at higher post-Newtonian order, and spin calculations were not pursued any further with this approach. (An additional difficulty is the lack of higher post-Newtonian order spin corrections to the equations of motion, which are extremely difficult to obtain for fluid balls.)

The more recent black-hole perturbation approach obtains high-order (in some cases exact) expressions for the influence of the radiation reaction on the orbital phase which are valid in the limit of extreme mass ratio. The basis of this approach is the perturbation of known, exact solutions of the Einstein equations (the Schwarzschild and Kerr spacetimes) with a test body using the Teukolsky equation [14] or an equivalent. During the last several years, analytical techniques for post-Newtonian expansion in the context of the black-hole perturbation approach have been developed to very high orders in v/c (for a recent review, see [15]). However, most black-hole perturbation papers treat the test body as a nonspinning point particle with a δ-function energy-momentum tensor, and thus do not give results for the case of two spinning bodies.

Recently, the black-hole perturbation approach has been extended to the case of two spinning bodies [16,17]. In [16], Mino, Shibata, and Tanaka calculated the gravitational waveforms and radiation reaction of a spinning particle falling into a Kerr black hole. In [17], Tanaka *et al.* obtained an expression for the non-precessional 2.5PN spin-orbit contribution to the secular phase evolution of a binary composed of a spinning test particle in quasicircular orbit around a Kerr black hole. These results were obtained using an energy-momentum tensor for the test body which mimics the effects of an extended, spinning object but can be expressed in terms of a δ-function for ease of calculation. This ''spinning-particle'' δ-function energy-momentum tensor is based on the work of Dixon [18]. We call it the MST tensor after Mino, Shibata, and Tanaka [16], who distilled it into the compact form we will use.

In this paper, we use the MST energy-momentum tensor for the first time in the curved-space, post-Newtonian approach to derive new gravitational-wave generation results. (Cho [19], in work parallel to our own, has recently used a similar approach to re-derive the waveforms of Kidder, Will, and Wiseman [13] in a slightly different form.) We reproduce (with a shorter calculation) the 1PN and 1.5PN spin-orbit corrections to the radiative multipoles derived in [12]. We also derive all of the (previously unknown) 2PN nonprecessional spin-orbit corrections to the waveforms, by calculating 2PN spin-orbit corrections to the radiative multipoles and combining them with the well-known 1.5PN equations of motion (in which there is no 2PN spin-orbit term). Because of the harmonics of the orbital frequency involved, there is no 2PN spin-orbit contribution to the radiation-reaction-induced secular phase evolution of the waveforms (the most accurately measurable effect).

In the future, we plan to use the methods of this paper to calculate all the non-precessional 2.5PN spin-orbit effects, including the nonvanishing radiation reaction and resulting secular evolution of the frequency and phase of the waveforms. That secular evolution is likely to be quite important for data analysis. Investigations by Tagoshi *et al.* [20], comparing post-Newtonian expansions to exact numerical results in the test-mass limit, indicate that spin effects are important for the extraction of information from observed waves at least up through 3PN order. To obtain the 2.5PN secular evolution requires the calculation not only of additional ra-

[1]Like non-spin effects, spin effects appear in the secular phase evolution of the waveforms at a certain order and at every order in v/c (0.5PN order) beyond it except for the first. Thus, dissipative spin-orbit effects appear at 1.5PN, 2.5PN, 3PN, . . . orders and spin-spin effects appear at 2PN, 3PN, 3.5PN, . . . orders.

diative multipoles, but also of the 2.5PN spin-orbit corrections to the equations of motion. The latter problem is qualitatively different (and more difficult); thus we will address it in a future paper.

This paper is organized as follows. In Sec. II we present the MST energy-momentum tensor [16] and review its properties. In Sec. III we review the post-Newtonian expansions of basic variables used in our calculations. Then in Sec. IV we calculate the STF radiative multipoles needed to obtain the 2PN spin-orbit terms in the waveforms. In Sec. V we evaluate all the 2PN (non-precessional) spin-orbit terms in the waveforms of a binary in quasicircular orbit with spins parallel or antiparallel to the orbital angular momentum, and in Sec. VI we briefly discuss their significance. In the Appendix we use our methods to derive the 1PN and 1.5PN STF radiative multipoles, and compare with the results of Refs. [13,12].

Throughout this paper, we use units such that Newton's gravitational constant and the speed of light equal unity. We also use the tensor notation conventions of [8,9]: curved brackets () on tensor indices to indicate symmetrization, square brackets [] to indicate antisymmetrization, and angled brackets $\langle\rangle$ or the superscript STF to indicate the symmetric trace-free part. A capitalized superscript L indicates a multi-index $i_1 \cdots i_l$; e.g., I^L represents I^{ijk} in the case $l=3$. We also write outer products of vectors in shorthand, e.g., $x^{ijk} = x^i x^j x^k$ and $x^L = x^{i_1} \cdots x^{il}$. Greek indices run from 0 to 3 and Latin indices from 1 to 3.

II. SPINNING PARTICLE ENERGY-MOMENTUM TENSOR

Our starting point is the spinning particle energy-momentum tensor given in terms of the Dirac δ-function [16],

$$T^{\alpha\beta}(x) = \int d\tau \left\{ p^{(\alpha}(x,\tau) u^{\beta)}(x,\tau) \frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}} - \nabla_\gamma \left[S^{\gamma(\alpha}(x,\tau) u^{\beta)}(x,\tau) \frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}} \right] \right\}. \tag{2.1}$$

Here $z^\mu(\tau)$ is the world line of the particle, $u^\mu(\tau) = dz^\mu/d\tau$, $p^\mu(\tau)$ is the particle's linear momentum, and $S^{\mu\nu}(\tau)$ is an antisymmetric tensor representing the particle's spin angular momentum. We focus only on spin-orbit interactions, i.e. discard all terms higher than first order in spin. In this case τ becomes the particle's proper time and u^μ becomes its four-velocity [see Eq. (2.4) of Ref. [16]].

The bitensors $p^\alpha(x,\tau)$, $u^\alpha(x,\tau)$, and $S^{\alpha\beta}(x,\tau)$ are space-time extensions of p^μ, u^μ, and $S^{\mu\nu}$ away from the particle's world line,[2] defined by

$$p^\alpha(x,\tau) = \overline{g}^\alpha{}_\mu(x,z(\tau)) p^\mu(\tau), \tag{2.2a}$$

$$u^\alpha(x,\tau) = \overline{g}^\alpha{}_\mu(x,z(\tau)) u^\mu(\tau), \tag{2.2b}$$

$$S^{\alpha\beta}(x,\tau) = \overline{g}^\alpha{}_\mu(x,z(\tau)) \overline{g}^\beta{}_\nu(x,z(\tau)) S^{\mu\nu}(\tau). \tag{2.2c}$$

Here $\overline{g}^\alpha{}_\mu(x,z)$ is a bitensor of parallel displacement with the properties

$$\lim_{x\to z} \overline{g}^\alpha{}_\mu(x,z(\tau)) = \delta^\alpha_\mu, \tag{2.3a}$$

$$\lim_{x\to z} \nabla_\beta \overline{g}^\alpha{}_\mu(x,z(\tau)) = 0. \tag{2.3b}$$

The definition of $S^{\mu\nu}$ is arbitrary up to the choice of a spin supplementary condition (the analogue of a gauge condition). We use

$$S^{\mu\nu} u_\mu = 0. \tag{2.4}$$

Note that in post-Newtonian theory at least three spin supplementary conditions are in common use. We choose Eq. (2.4) because it makes our radiative multipoles consistent with the standard post-Newtonian equations of motion, thus simplifying the calculations (cf. Ref. [12], Appendix A). We introduce a spin vector S^μ which is related to the spin tensor by

$$S^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} u_\rho S_\sigma, \tag{2.5a}$$

$$S^\mu u_\mu = 0, \tag{2.5b}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Cività tensor.[3] The spin supplementary condition is identically satisfied by Eq. (2.5a). On the other hand, we need to impose the condition (2.5b) on S^μ to fix the one remaining degree of freedom S^0.

For later convenience, we separate the MST energy-momentum tensor (2.1) into the usual point-particle piece (the first term) plus a spin-orbit piece,

$$T^{\alpha\beta}_{(\mathrm{SO})}(x) = -\int d\tau \nabla_\gamma \left[S^{\gamma(\alpha} u^{\beta)} \frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}} \right]. \tag{2.6}$$

When evaluating the radiative multipoles of a system of masses, we encounter integrals of the form

$$\int d^3x F^L(x) T^{\alpha\beta}(x). \tag{2.7}$$

The spin-orbit contribution to this expression can be evaluated by substituting Eq. (2.6), using Leibniz' rule to rewrite the integral, and discarding the spatial integral of a three-divergence. The result for a many-body system is

[2] We use indices $\alpha, \beta, \ldots$ to denote quantities associated with the field point x, and $\mu, \nu, \ldots$ to denote those associated with the worldline $z(\tau)$.

[3] We use the convention defined in Eq. (8.10) of Ref. [21].

$$\int d^3xF^L(x)T^{\alpha\beta}_{(\mathrm{SO})}(x)$$
$$=\sum_A\left[S_A^{\gamma(\alpha}v_A^{\beta)}\frac{\partial_\gamma F^L}{\sqrt{-g}}-\partial_0\left(S_A^{0(\alpha}v_A^{\beta)}\frac{F^L}{\sqrt{-g}}\right)\right.$$
$$\left.-(\Gamma^\gamma{}_{\gamma\delta}S_A^{\delta(\alpha}v_A^{\beta)}+S_A^{\gamma(\alpha}\Gamma^{\beta)}{}_{\gamma\delta}v_A^\delta)\frac{F^L}{\sqrt{-g}}\right], \tag{2.8}$$

where A labels the bodies, $v^\alpha=u^\alpha/u^0$, and ∂_γ is shorthand for $\partial/\partial x^\gamma$ evaluated at $x=x_A$.

III. POST-NEWTONIAN EXPANSIONS OF BASIC VARIABLES

We now switch from fully covariant expressions to post-Newtonian expansions in harmonic coordinates. Spatial indices on the right hand sides of the equations in this section can be raised and lowered freely with the Kronecker δ. We use the expansion parameter ϵ which is related to the orbital variables by $\epsilon\sim M/r\sim v^2$, where M is the total mass of the system, r is the orbital separation, and v the orbital velocity. We assume the bodies are compact; i.e. each body's spin has magnitude $|\boldsymbol{S}_A|\sim\chi m_A^2$, where χ is of order unity (see [12] for further discussion).

When evaluating the post-Newtonian expansions of basic variables in this section and the radiative multipoles in Sec. IV, we encounter divergent expressions—in our case, self-interaction terms. Such divergences are inevitable when using any δ-function source, and we follow previous authors in discarding them (see the discussion at the end of Ref. [8], Sec. II). We do not claim any rigorous justification for doing so; however, since it is asserted in the non-spinning case [8] that this procedure can be justified to $O(\epsilon^2)$, and since we consider corrections only up to $O(\epsilon)$ beyond lowest order spin effects, we expect that the formal use of the δ-function is justified to the same degree as in the non-spinning case. Informally, we note that the usual post-Newtonian equations of motion for spinning bodies can be obtained by taking the divergence of the MST energy-momentum tensor (2.1) and discarding self-interaction divergences [16].

The metric components in harmonic coordinates are well known [9] as

$$g_{00}=-[1-2U+O(\epsilon^2)], \tag{3.1a}$$

$$g_{i0}=O(\epsilon^{3/2}), \tag{3.1b}$$

$$g_{ij}=\delta_{ij}[1+2U+O(\epsilon^2)], \tag{3.1c}$$

$$\sqrt{-g}=1+2U+O(\epsilon^{3/2}), \tag{3.1d}$$

where only the lowest-order expression for the potential U is needed:

$$U(\boldsymbol{x})=\sum_A\frac{m_A}{|\boldsymbol{x}-\boldsymbol{x}_A|}+O(\epsilon^2). \tag{3.2}$$

By differentiating Eq. (3.1), we find the dominant Christoffel symbols

$$\Gamma^0{}_{i0}=\Gamma^i{}_{00}=-a^i, \tag{3.3a}$$

$$\Gamma^i{}_{jk}=\delta^{ij}a^k+\delta^{ik}a^j-\delta^{jk}a^i, \tag{3.3b}$$

where $a^i=\partial_i U$. All others are of higher post-Newtonian order, and can be neglected for the purposes of this paper. The metric components (3.1), together with the condition $u^\mu u_\mu=-1$, give us the expansion of the four-velocity

$$u^0=1+\left(\frac{v^2}{2}+U\right)+O(\epsilon^2), \tag{3.4a}$$

$$u^i=v^i\left[1+\left(\frac{v^2}{2}+U\right)+O(\epsilon^2)\right]. \tag{3.4b}$$

We express the components of the spin tensor in terms of the spatial components of the spin vector by combining Eqs. (2.5a), (2.5b), (3.1), and (3.4) to obtain

$$S^{i0}=(\boldsymbol{v}\times\boldsymbol{S})^i+O(\epsilon^{3/2}), \tag{3.5a}$$

$$S^{ij}=\epsilon^{ijk}\left[\left(1+\frac{1}{2}v^2-U\right)S^k-(\boldsymbol{v}\cdot\boldsymbol{S})v^k\right]+O(\epsilon^2), \tag{3.5b}$$

where ϵ^{ijk} is from here on used to indicate the antisymmetric symbol $[ijk]$. Substituting Eqs. (3.5) back into Eq. (2.6), we find that the post-Newtonian orders of the components of $T^{\alpha\beta}_{(\mathrm{SO})}$ are

$$T^{00}_{(\mathrm{SO})}\sim T^{ij}_{(\mathrm{SO})}\sim m\frac{mv}{r}\sim m\times O(\epsilon^{3/2}), \tag{3.6a}$$

$$T^{i0}_{(\mathrm{SO})}\sim m\frac{m}{r}\sim m\times O(\epsilon), \tag{3.6b}$$

where m is the mass of either body. This contrasts with the point-mass order counting,

$$T^{00}_{(\mathrm{PM})}\sim m, \tag{3.7a}$$

$$T^{i0}_{(\mathrm{PM})}\sim m\times O(\epsilon^{1/2}), \tag{3.7b}$$

$$T^{ij}_{(\mathrm{PM})}\sim m\times O(\epsilon). \tag{3.7c}$$

We also note that the second and third terms on the right hand side of Eq. (2.8) are $O(\epsilon)$ with respect to the first if F^L is an outer product of position vectors (as is the case when computing multipoles).

IV. STF RADIATIVE MULTIPOLES

In this section we calculate the symmetric, trace-free radiative multipoles necessary to obtain the 2PN spin-orbit contributions to the waveform. In Sec. IV C we specialize to the case of nonprecessing orbits.

The STF radiative multipoles are given to $O(\epsilon^{5/2})$ by Blanchet [Eq. (4.3) of Ref. [10]]. However, we only need the $O(\epsilon)$ expressions

$$I^L(t) = \int d^3x \hat{x}^L (T^{00} + T^{ii}) - \frac{4(2l+1)}{(l+1)(2l+3)} \frac{d}{dt} \times \int d^3x \hat{x}^{La} T^{0a} + \frac{1}{2(2l+3)} \frac{d^2}{dt^2} \times \int d^3x \hat{x}_L |\boldsymbol{x}|^2 (T^{00} + T^{ii}) + O(\epsilon^2), \tag{4.1}$$

$$J^L(t) = \epsilon^{ab\langle i} \left[\int d^3x \hat{x}^{L-1\rangle a} (1+4U) T^{0b} - \frac{2l+1}{(l+2)(2l+3)} \frac{d}{dt} \int d^3x \hat{x}^{L-1\rangle ac} T^{bc} + \frac{1}{2(2l+3)} \frac{d^2}{dt^2} \int d^3x \hat{x}^{L-1\rangle a} \left| \boldsymbol{x} \right|^2 T^{0b} \right] + O(\epsilon^2), \tag{4.2}$$

where $\hat{x}^L$ denotes the symmetric, trace-free part of x^L. We also define $I^L_{(\mathrm{SO})}$ and $J^L_{(\mathrm{SO})}$ by substituting $T^{\mu\nu}_{(\mathrm{SO})}$ for $T^{\mu\nu}$ in Eqs. (4.1) and (4.2). In Eqs. (4.1) and (4.2) we have discarded self-interaction terms, which are always divergent when using a δ-function source. We have also discarded terms involving only gravitational potentials (referred to as ''non-compact'' terms in [8]), whose spin-orbit contributions do not appear until higher post-Newtonian orders than considered in this paper.

Spin-orbit corrections to the multipoles and waveform follow an order-counting scheme different from the usual point-mass terms. Substituting Eqs. (3.6) into Eqs. (4.1) and (4.2), we find that the lowest order spin-orbit correction to a multipole appears in the current quadrupole $J^{ij}_{(\mathrm{SO})}$ [12]. This term contributes to the waveform amplitude at 1PN order, but because it is a subharmonic of the dominant (mass quadrupole) radiation, it does not contribute to the radiation reaction until 1.5PN order. The next-order effects appear in the current octupole $J^{ijk}_{(\mathrm{SO})}$ and the mass quadrupole $I^{ij}_{(\mathrm{SO})}$, and contribute to the waveforms and radiation reaction at 1.5PN order. (These are the terms given in [13]; we evaluate them with our methods in the Appendix.) Following this progression, the 2PN waveforms require evaluation of $J^{ijkl}_{(\mathrm{SO})}$ and $I^{ijk}_{(\mathrm{SO})}$ to lowest order and of $J^{ij}_{(\mathrm{SO})}$ to $O(\epsilon)$ beyond lowest order.

A. *N*-body case

We first evaluate the spin-orbit contributions to the multipole integrals (4.1) and (4.2) as sums over N bodies.

The expression for the current hexadecapole is the easiest to evaluate. It is needed only to lowest order and thus involves only the first term in Eq. (4.2),

$$J^{ijkl}_{(\mathrm{SO})} = \epsilon^{ab\langle i} \int d^3x \hat{x}^{jkl\rangle a} T^{0b}_{(\mathrm{SO})}, \tag{4.3}$$

which is straightforwardly obtained from the first term of Eq. (2.8) as

$$J^{ijkl}_{(\mathrm{SO})} = \frac{5}{2} \sum_A (S^i_A x^{jkl}_A)^{\mathrm{STF}}. \tag{4.4}$$

The mass octupole is also needed only to lowest order,

$$I^{ijk}_{(\mathrm{SO})} = \int d^3x \hat{x}^{ijk} (T^{00}_{(\mathrm{SO})} + T^{aa}_{(\mathrm{SO})}) - \frac{7}{9} \frac{d}{dt} \int d^3x \hat{x}^{ijka} T^{0a}_{(\mathrm{SO})}. \tag{4.5}$$

Again, using the first term of Eq. (2.8) it is straightforward to evaluate the integrals. When evaluating the time derivative in the second term we neglect time derivatives of the spins (i.e., precession). We could do this even if considering spin precession, because those derivatives appear $O(\epsilon)$ beyond the spins themselves [cf. Eqs. (F18),(F19) of Ref. [9], where due to a typographical error a factor of $(M/r)^3$ was omitted from in front of the last term in each equation].[4] We are left with

$$I^{ijk}_{(\mathrm{SO})} = \sum_A \left[\frac{9}{2} (\boldsymbol{v}_A \times \boldsymbol{S}_A)^i x^{jk}_A - 3 (\boldsymbol{x}_A \times \boldsymbol{S}_A)^i x^j_A v^k_A \right]^{\mathrm{STF}}. \tag{4.6}$$

Because the two-index current moment is needed to $O(\epsilon)$, we must keep all three terms in Eq. (4.2),

$$J^{ij}_{(\mathrm{SO})} = \epsilon^{ab\langle i} \left[\int d^3x \hat{x}^{j\rangle a} (1+4U) T^{0b}_{(\mathrm{SO})} - \frac{5}{28} \frac{d}{dt} \int d^3x \hat{x}^{j\rangle ac} T^{bc}_{(\mathrm{SO})} + \frac{1}{14} \frac{d^2}{dt^2} \int d^3x \hat{x}^{j\rangle a} T^{0b}_{(\mathrm{SO})} \right]. \tag{4.7}$$

Again, time derivatives of the spins appear only in higher-order terms and may be discarded at this order. Carefully evaluating the integrals according to Eq. (2.8), we obtain

$$J^{ij}_{(\mathrm{SO})} = \sum_A \left[\frac{3}{2} x^i_A S^j_A + \frac{1}{14} \boldsymbol{v}_A \cdot \boldsymbol{x}_A v^i_A S^j_A + \frac{2}{7} \boldsymbol{S}_A \cdot \boldsymbol{x}_A v^{ij}_A + \frac{11}{28} \boldsymbol{x}_A \cdot \boldsymbol{x}_A a^i_A S^j_A - \frac{17}{7} \boldsymbol{S}_A \cdot \boldsymbol{v}_A x^i_A v^j_A + \frac{1}{7} \boldsymbol{a}_A \cdot \boldsymbol{S}_A x^{ij}_A - \frac{17}{14} \boldsymbol{S}_A \cdot \boldsymbol{x}_A x^i_A a^j_A + \left(\frac{3}{2} U_A + \frac{43}{28} \boldsymbol{v}_A \cdot \boldsymbol{v}_A + \frac{11}{14} \boldsymbol{a}_A \cdot \boldsymbol{x}_A \right) x^i_A S^j_A \right]^{\mathrm{STF}}. \tag{4.8}$$

In addition to the spin-orbit multipoles, we need the lowest order contributions to I^{ijk} and J^{ij} from the usual point-mass energy-momentum tensor. These multipoles are given by

[4] Although we do not consider spin precession in this paper, the N-body and two-body multipoles we present are (instantaneously) valid even in the precessing case. One simply has to put in the spin vectors as (slowly varying) functions of time—which is easier said than done.

$$I^{ijk}_{(\mathrm{PM})}=\sum_A m_A x_A^{\langle ijk\rangle}, \tag{4.9}$$

$$J^{ij}_{(\mathrm{PM})}=\sum_A m_A(\boldsymbol{x}_A\times\boldsymbol{v}_A)^{\langle i}x_A^{j\rangle} \tag{4.10}$$

and contribute to the waveforms at 0.5PN order.

B. Two-body case

We now specialize to the case of two bodies of mass m_1 and m_2 in an arbitrary (possibly precessing) orbit, and express the multipoles in terms of the relative coordinate $\boldsymbol{x}$ (whose origin is at body 2).

It is convenient to use the mass parameters

$$M=m_1+m_2, \tag{4.11a}$$

$$\eta=m_1m_2/M^2, \tag{4.11b}$$

$$\Delta=(m_1-m_2)/M. \tag{4.11c}$$

It is also convenient to use the dimensionless, symmetrized spin parameters introduced by Will and Wiseman [9],

$$\boldsymbol{\chi}_a=\frac{1}{2}\left(\frac{\boldsymbol{S}_1}{m_1^2}-\frac{\boldsymbol{S}_2}{m_2^2}\right), \tag{4.12a}$$

$$\boldsymbol{\chi}_s=\frac{1}{2}\left(\frac{\boldsymbol{S}_1}{m_1^2}+\frac{\boldsymbol{S}_2}{m_2^2}\right). \tag{4.12b}$$

We eliminate the potentials and accelerations in Eq. (4.8) with the well-known Newtonian expressions

$$U_A=U(\boldsymbol{x}_A)=\frac{m_B}{r}, \tag{4.13}$$

$$\boldsymbol{a}_A=\frac{-M}{r^3}\boldsymbol{x}_A, \tag{4.14}$$

where $B\neq A$ and $r=|\boldsymbol{x}|$.

The spins of the bodies introduce a correction to the relation between $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and the relative coordinate $\boldsymbol{x}$,

$$\boldsymbol{x}_1=\boldsymbol{x}\left[\frac{m_2}{M}+\frac{1}{2}\eta\Delta\left(v^2-\frac{M}{r}\right)\right]-M\eta\boldsymbol{v}\times(\boldsymbol{\chi}_a+\Delta\boldsymbol{\chi}_s),$$
$$\boldsymbol{x}_2=\boldsymbol{x}\left[-\frac{m_1}{M}+\frac{1}{2}\eta\Delta\left(v^2-\frac{M}{r}\right)\right]-M\eta\boldsymbol{v}\times(\boldsymbol{\chi}_a+\Delta\boldsymbol{\chi}_s). \tag{4.15}$$

[Compare Eq. (3.13) of Ref. [12] and Eq. (F11) of Ref. [9], where the missing factor of η in the latter is a typographical error.] This correction is 1.5PN order; therefore it enters our 2PN calculation through contributions from the 0.5PN (point-mass) multipoles (4.9).

Applying the transformation (4.15) and including the contributions from the point-mass multipoles, we find the two-body forms of Eqs. (4.4), (4.6), and (4.8) to be

$$J^{ijkl}=\frac{5}{2}M^2\eta^2(\chi_a-\Delta\chi_s)^{\langle i}x^{jkl\rangle}, \tag{4.16}$$

$$I^{ijk}=-M\eta\Delta x^{\langle ijk\rangle}+M^2\eta^2\left\{\frac{3}{2}[\boldsymbol{v}\times(\boldsymbol{\chi}_a-5\Delta\boldsymbol{\chi}_s)]^i x^{jk}-3[\boldsymbol{x}\times(\boldsymbol{\chi}_a-\Delta\boldsymbol{\chi}_s)]^i x^{jk}\right\}^{\mathrm{STF}}, \tag{4.17}$$

$$\begin{aligned}J^{ij}={}&-\mu\Delta(\boldsymbol{x}\times\boldsymbol{v})^{\langle i}x^{j\rangle}+\frac{3}{2}M^2\eta(\chi_a+\Delta\chi_s)^{\langle i}x^{j\rangle}\\&+\frac{3}{4}M^2\eta\Delta\left(v^2-\frac{M}{r}\right)(\Delta\chi_a+\chi_s)^{\langle i}x^{j\rangle}\\&+\frac{1}{28}M^2\eta^2\left\{\left[\left(23\frac{M}{r}-13v^2\right)\chi_a+\Delta\left(47\frac{M}{r}-141v^2\right)\chi_s\right]^i x^j+2(\boldsymbol{x}\cdot\boldsymbol{v})(15\chi_a\right.\\&+13\Delta\chi_s)^i v^j+2\frac{M}{r^3}[\boldsymbol{x}\cdot(29\boldsymbol{\chi}_a-\Delta\boldsymbol{\chi}_s)]x^{ij}\\&-4[\boldsymbol{x}\cdot(5\boldsymbol{\chi}_a+9\Delta\boldsymbol{\chi}_s)]v^{ij}\\&\left.-4[\boldsymbol{v}\cdot(3\boldsymbol{\chi}_a-31\Delta\boldsymbol{\chi}_s)]x^iv^j\right\}^{\mathrm{STF}}.\end{aligned} \tag{4.18}$$

The first terms in Eqs. (4.17) and (4.18) are the lowest order non-spin terms. The second term in Eq. (4.18) is easily verified as identical to the lowest order (1PN) spin-orbit contribution obtained by Kidder [Eq. (3.20a) of Ref. [12]].

C. Quasicircular orbits

We now specialize to the case where the two bodies orbit each other in a circular trajectory, which adiabatically shrinks (inspirals) under a radiation reaction. For spinning bodies, this is only possible when both spin vectors are parallel or antiparallel to the orbital angular momentum, eliminating spin-orbit precession.

We express our results in terms of the orthonormal vectors $\boldsymbol{n}=\boldsymbol{x}/r$, $\boldsymbol{\lambda}=\boldsymbol{v}/v$, and $\boldsymbol{z}=\boldsymbol{n}\times\boldsymbol{\lambda}$ (parallel to all angular momenta). The majority of the terms in Eqs. (4.16)–(4.18) vanish, and we are left with the greatly simplified expressions

$$J^{ijkl}=\frac{5}{2}M^2\eta^2r^3(\chi_a-\Delta\chi_s)n^{\langle ijk}z^{l\rangle}, \tag{4.19}$$

$$I^{ijk}=-M\eta\Delta r^3n^{\langle ijk\rangle}+\frac{3}{2}M^2\eta^2vr^2[(\chi_a-5\Delta\chi_s)n^{ijk}+2(\chi_a-\Delta\chi_s)n^i\lambda^{jk}]^{\mathrm{STF}}, \tag{4.20}$$

$$J^{ij}=-M\eta\Delta r^2vn^{\langle i}z^{j\rangle}+\frac{3}{2}M^2\eta r(\chi_a+\Delta\chi_s)n^{\langle i}z^{j\rangle}+\frac{1}{14}M^3\eta^2(5\chi_a-47\Delta\chi_s)n^{\langle i}z^{j\rangle}. \tag{4.21}$$

V. WAVEFORM

Having evaluated the necessary radiative multipoles, we obtain the gravitational waveform

$$h^{ij}=\frac{1}{R}\sum_{l=2}^{\infty}\left[\frac{4}{l!}\overset{(l)}{I}{}^{ijL-2}N^{L-2}+\frac{8l}{(l+1)!}\epsilon^{pq(i}\overset{(l)}{J}{}^{j)pL-2}N^{qL-2}\right]^{\mathrm{TT}}, \tag{5.1}$$

where TT denotes the transverse traceless projection, (l) denotes the lth time derivative, and $\boldsymbol{N}$ is the unit vector pointing toward the observer [cf. Eq. (E5a) of Ref. [9]].

We evaluate the time derivatives using the equation of motion for circular orbits, $\boldsymbol{a}=-\omega^2\boldsymbol{x}$, where the standard form of the post-Newtonian expansion is given by

$$\omega^2=\frac{M}{r^3}\left\{1-(3-\eta)\frac{M}{r}-2[\Delta\chi_a+(1+\eta)\chi_s]\frac{Mv}{r}+O(\epsilon^2)\right\} \tag{5.2}$$

[cf. Eqs. (7.1) and (F20) of Ref. [9]]. The 1.5PN spin-orbit term in the equation of motion [the third term in Eq. (5.2)] contributes to the 2PN spin-orbit term in h^{ij} through time derivatives of the 0.5PN point-mass multipoles [cf. the first terms in Eqs. (4.20) and (4.21)]. If we were to consider spin-orbit precession, time derivatives of spin expressions appearing in the multipoles (4.16)–(4.18) would also factor into Eq. (5.1).

Evaluating the time derivatives in Eq. (5.1), using Eq. (5.2) and the identity $v=\omega r$ to write v and M/r in terms of $M\omega$, and collecting all terms of order $(M\omega)^2$, we obtain

$$\begin{aligned}h^{ij}=&\frac{2M\eta}{R}(M\omega)^2\Bigg\{(\Delta^2\chi_a+\Delta\chi_s)\Bigg[\left(-\frac{14}{3}n^{ij}+4\lambda^{ij}\right)(\boldsymbol{\lambda}\cdot\boldsymbol{N})\\&-\frac{28}{3}n^{(i}\lambda^{j)}(\boldsymbol{n}\cdot\boldsymbol{N})\Bigg]+\left(2\chi_a-\frac{8}{3}\Delta^2\chi_a-\frac{2}{3}\Delta\chi_s\right)\\&\times(\boldsymbol{n}\times\boldsymbol{N})^{(i}z^{j)}\Bigg\}^{\mathrm{TT}}+\frac{2M\eta^2}{R}(M\omega)^2\Bigg\{(19\chi_a+37\Delta\chi_s)\\&\times\left(\frac{1}{6}(\boldsymbol{N}\cdot\boldsymbol{\lambda})n^{ij}+\frac{1}{3}(\boldsymbol{N}\cdot\boldsymbol{n})n^{(i}\lambda^{j)}\right)-4(\chi_a+\Delta\chi_s)\\&\times(\boldsymbol{N}\cdot\boldsymbol{\lambda})\lambda^{ij}+(\chi_a-\Delta\chi_s)\Bigg[(\boldsymbol{n}\times\boldsymbol{N})^{(i}z^{j)}\Bigg(-\frac{7}{6}\\&-\frac{10}{3}(\boldsymbol{N}\cdot\boldsymbol{\lambda})^2+\frac{7}{2}(\boldsymbol{N}\cdot\boldsymbol{n})^2\Bigg)-\frac{20}{3}(\boldsymbol{N}\cdot\boldsymbol{\lambda})(\boldsymbol{N}\cdot\boldsymbol{z})(\boldsymbol{n}\times\boldsymbol{N})^{(i}\lambda^{j)}\\&+\frac{7}{3}(\boldsymbol{N}\cdot\boldsymbol{n})(\boldsymbol{N}\cdot\boldsymbol{z})(\boldsymbol{n}\times\boldsymbol{N})^{(i}n^{j)}-\frac{10}{3}(\boldsymbol{N}\cdot\boldsymbol{n})(\boldsymbol{N}\cdot\boldsymbol{z})\\&\times(\boldsymbol{\lambda}\times\boldsymbol{N})^{(i}\lambda^{j)}-\frac{20}{3}(\boldsymbol{N}\cdot\boldsymbol{n})(\boldsymbol{N}\cdot\boldsymbol{\lambda})(\boldsymbol{z}\times\boldsymbol{N})^{(i}\lambda^{j)}\Bigg]\Bigg\}^{\mathrm{TT}}.\end{aligned} \tag{5.3}$$

Finally, we project out the amplitudes of the h_+ and $h_\times$ polarizations. This is done by taking

$$h_+=\frac{1}{2}(p_ip_j-q_iq_j)h^{ij}, \tag{5.4a}$$

$$h_\times=\frac{1}{2}(p_iq_j+q_ip_j)h^{ij}. \tag{5.4b}$$

Note that the TT projection in Eq. (5.1) is subsumed in this operation (cf. Sec. VII A of [9]). The standard convention [9] is to use the unit triad composed of $\boldsymbol{N}$ (the direction to the observer), $\boldsymbol{p}$ (pointing from the descending node of the orbit to the ascending node), and $\boldsymbol{q}=\boldsymbol{N}\times\boldsymbol{p}$. The orbital phase ψ is measured from the ascending node, and the orbital inclination angle i is given by $\cos i=\boldsymbol{z}\cdot\boldsymbol{N}$. Thus we have

$$\boldsymbol{n}=\boldsymbol{p}\cos\psi+(\boldsymbol{q}\cos i+\boldsymbol{N}\sin i)\sin\psi, \tag{5.5a}$$

$$\boldsymbol{\lambda}=-\boldsymbol{p}\sin\psi+(\boldsymbol{q}\cos i+\boldsymbol{N}\sin i)\cos\psi, \tag{5.5b}$$

$$\boldsymbol{z}=-\boldsymbol{q}\sin i+\boldsymbol{N}\cos i. \tag{5.5c}$$

Following [22], we organize the amplitude contributions of the waveform polarizations according to post-Newtonian order and physical origin as

$$\begin{aligned}h_{+,\times}=&\frac{2M\eta}{R}x[H^{(0)}_{+,\times}+\cdots+x^{3/2}H^{(3/2)}_{+,\times}+x^{3/2}H^{(3/2,\mathrm{SO})}_{+,\times}\\&+x^2H^{(2)}_{+,\times}+x^2H^{(2,\mathrm{SO})}_{+,\times}+x^2H^{(2,\mathrm{SS})}_{+,\times}+\cdots],\end{aligned} \tag{5.6}$$

where $x=(M\omega)^{2/3}$. The 2PN spin-orbit contributions to the waveform polarizations are given by

$$\begin{aligned}H^{(2,\mathrm{SO})}_{+}=&\left\{\frac{-1}{24}[(109+15c^2)\chi_a+7(1+3c^2)\Delta\chi_s]\eta\right.\\&\left.+\frac{1}{4}(1+c^2)(\chi_a+\Delta\chi_s)\right\}\sin i\cos\psi\\&+\left\{\frac{9}{8}[(11+5c^2)\chi_a+(1+7c^2)\Delta\chi_s]\eta\right.\\&\left.-\frac{9}{4}(1+c^2)(\chi_a+\Delta\chi_s)\right\}\sin i\cos 3\psi\end{aligned} \tag{5.7a}$$

$$\begin{aligned}H^{(2,\mathrm{SO})}_{\times}=&\left\{\frac{-1}{24}[(127-3c^2)\chi_a+(25+3c^2)\Delta\chi_s]\eta\right.\\&\left.+\frac{1}{2}(\chi_a+\Delta\chi_s)\right\}c\sin i\sin\psi+\left\{\frac{9}{8}[(19-3c^2)\chi_a\right.\\&\left.+(5+3c^2)\Delta\chi_s]\eta-\frac{9}{2}(\chi_a+\Delta\chi_s)\right\}\\&\times c\sin i\sin 3\psi,\end{aligned} \tag{5.7b}$$

where $c\equiv\cos i$.

It is clear that these 2PN contributions to the waveform amplitudes do not contribute to the radiation reaction at 2PN order because the harmonics (ω and 3ω) average to zero when beat against the ''Newtonian'' terms $H^{(0)}_{+,\times}$ at frequency 2ω [cf. Eqs. (3),(4) of Ref. [22]].

VI. SUMMARY

We have calculated all the non-precessional 2PN spin-orbit effects on the gravitational waveforms of compact bodies in quasicircular orbit, and have shown that there is no spin-orbit radiation reaction effect at 2PN order. Our calculation was greatly simplified over previous spinning-body post-Newtonian efforts [13,12] by the use of a δ-function energy-momentum tensor for spinning particles. We have presented the waveform polarizations in ''ready-to-use'' form (cf. [22]).

Note that terms of $O(\eta)$ contribute significantly to Eqs. (5.7). These are the terms that could not have been obtained by the black-hole perturbation approach. Their presence leads us to expect that the 2.5PN radiation reaction will also contain significant terms of $O(\eta)$ which are not found in [17,23,20].

In this paper, we treated the bodies only to linear order in their spins (i.e., considered only spin-orbit effects). Spin-spin effects can be treated by a more complicated calculation using the MST energy-momentum tensor. However, because spin-spin effects appear at 2PN order, consistency would require that one also include the effects of the bodies' quadrupole moments [24].

ACKNOWLEDGMENTS

We thank Kip Thorne for helpful comments on the manuscript. B.J.O. thanks Alan Wiseman for many stimulating discussions. B.J.O. was supported in part by NSF Grants AST-9417371 and INT-9417348, NASA Grants NAGW-4268 and NAG5-4351, and by the NSF. H.T. was supported by the Japanese Society for the Promotion of Science. A.O. was supported in part by Monbusho Grant-in-Aid for Scientific Research No. 09440106.

APPENDIX

In this appendix, we derive the 1.5PN spin-orbit multipoles $I^{ij}_{(\mathrm{SO})}$ and $J^{ijk}_{(\mathrm{SO})}$ obtained in [12].

The general expressions are quite simple, requiring Eqs. (4.1) and (4.2) only to lowest order:

$$I^{ij}_{(\mathrm{SO})}=\int d^3x\hat{x}^{ij}(T^{00}_{(\mathrm{SO})}+T^{aa}_{(\mathrm{SO})})-\frac{4}{3}\frac{d}{dt}\int d^3x\hat{x}^{ija}T^{0a}_{(\mathrm{SO})}, \tag{A1}$$

$$J^{ijk}_{(\mathrm{SO})}=\epsilon^{ab\langle i}\int d^3x\hat{x}^{jk\rangle a}T^{0b}_{(\mathrm{SO})}. \tag{A2}$$

Using Eqs. (2.8) and (3.5) only to lowest order, it is straightforward to evaluate these expressions in the N-body case as

$$I^{ij}_{(\mathrm{SO})}=\sum\left[4x^i_A(\boldsymbol{v}_A\times\boldsymbol{S}_A)^j-\frac{4}{3}\frac{d}{dt}\{x^i_A(\boldsymbol{x}_A\times\boldsymbol{S}_A)^j\}\right]^{\mathrm{STF}}, \tag{A3}$$

$$J^{ijk}_{(\mathrm{SO})}=\sum 2[x^{ij}_A S^k_A]^{\mathrm{STF}}. \tag{A4}$$

Using the transformation (4.15) to the relative coordinate, we find the two-body forms of the multipoles to be

$$I^{ij}_{(\mathrm{SO})}=\frac{8}{3}M^2\eta^2[2x^i(\boldsymbol{v}\times\boldsymbol{\chi}_s)^j-v^i(\boldsymbol{x}\times\boldsymbol{\chi}_s)^j]^{\mathrm{STF}}, \tag{A5}$$

$$J^{ijk}_{(\mathrm{SO})}=4M^2\eta^2[x^{ij}\chi^k_s]^{\mathrm{STF}}. \tag{A6}$$

These results agree with those obtained using the fluid body energy-momentum tensor [12,9].

[1] K. S. Thorne, in *Black Holes and Relativistic Stars*, edited by R. M. Wald (University of Chicago Press, Chicago, in press); gr-qc/9706079; see also references therein.

[2] B. Barish *et al.*, LIGO Advanced Research and Development Program Proposal, Caltech/MIT, 1996 (unpublished).

[3] R. Narayan, T. Piran, and A. Shemi, Astrophys. J. **379**, L17 (1991).

[4] E. S. Phinney, Astrophys. J. **380**, L17 (1991).

[5] A. V. Tutukov and L. R. Yungelson, Mon. Not. R. Astron. Soc. **260**, 675 (1993).

[6] V. M. Lipunov, K. A. Postnov, and M. E. Prokhorov, Mon. Not. R. Astron. Soc. **288**, 245 (1997).

[7] B. S. Sathyaprakash, in *Relativistic Gravitation and Gravitational Radiation*, edited by J.-P. LaSota and J.-A. Marck (Cambridge University Press, Cambridge, England, 1997), p. 361. See also references therein.

[8] L. Blanchet, T. Damour, and B. R. Iyer, Phys. Rev. D **51**, 5360 (1995).

[9] C. M. Will and A. G. Wiseman, Phys. Rev. D **54**, 4813 (1996).

[10] L. Blanchet, Phys. Rev. D **54**, 1417 (1996).

[11] T. A. Apostolatos, C. Cutler, G. J. Sussman, and K. S. Thorne, Phys. Rev. D **49**, 6274 (1994).

[12] L. E. Kidder, Phys. Rev. D **52**, 821 (1995).

[13] L. E. Kidder, C. M. Will, and A. G. Wiseman, Phys. Rev. D **47**, R4183 (1993).

[14] S. A. Teukolsky, Astrophys. J. **185**, 635 (1973).

[15] Y. Mino, M. Sasaki, M. Shibata, H. Tagoshi, and T. Tanaka, Prog. Theor. Phys. Suppl. **128**, 1 (1997).

[16] Y. Mino, M. Shibata, and T. Tanaka, Phys. Rev. D **53**, 622 (1996).

[17] T. Tanaka, Y. Mino, M. Sasaki, and M. Shibata, Phys. Rev. D **54**, 3762 (1996).

[18] W. G. Dixon, in *Isolated Gravitating Systems in General Relativity*, edited by J. Ehlers (North-Holland, Amsterdam, 1979), p. 156. See also references therein.

[19] H. T. Cho, ''Post-Newtonian approximation for spinning particles,'' gr-qc/9703071.

[20] H. Tagoshi, M. Shibata, T. Tanaka, and M. Sasaki, Phys. Rev. D **54**, 1439 (1996).

[21] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[22] L. Blanchet, B. R. Iyer, C. M. Will, and A. G. Wiseman, Class. Quantum Grav. **13**, 575 (1996).

[23] M. Shibata, M. Sasaki, H. Tagoshi, and T. Tanaka, Phys. Rev. D **51**, 1646 (1995).

[24] E. Poisson, Phys. Rev. D **57**, 5287 (1998).