Hadronic Light-by-Light Contribution to Muon $g - 2$ in Chiral Perturbation Theory

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We compute the hadronic light-by-light scattering contributions to the muon anomalous magnetic moment, a_μ^{LL}(had), in chiral perturbation theory that are enhanced by large logarithms and a factor of N_C. They depend on a low-energy constant constrained by $\eta \to \mu^+\mu^-$ and $\pi^0 \to e^+e^-$ branching ratios. However, the dependence of a_μ^{LL}(had) on nonlogarithmically enhanced effects cannot be constrained except through the measurement of the anomalous moment itself.

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The recently reported measurement of the muon anomalous magnetic moment a_μ by the E821 Collaboration [1] has generated considerable excitement about possible evidence for new physics. The interpretation of the result, however, depends in part on a reliable treatment of hadronic contributions to a_μ which arise at two- and three-loop order. While the vacuum polarization contribution can be constrained by e^+e^- experiments and τ decays and appears to be under adequate theoretical control [2], recent analysis [3,4] of the hadronic light-by-light scattering contribution a_μ^{LL}(had) has uncovered a sign error in previous calculations [5,6] of the dominant, pseudoscalar pole term. The resulting sign change reduces the 2.6σ deviation of a_μ from the standard model prediction reported in [1] by 1 standard deviation, thereby modifying considerably the original interpretation of the result.

The commonly quoted values for a_μ^{LL}(had) (after incorporating the corrected overall sign) rely on model treatments of the off-shell $\pi^0\gamma^*\gamma^*$, $\eta\gamma^*\gamma^*$, and $\eta'\gamma^*\gamma^*$ interactions. While the amplitude for pseudoscalar decay into two real photons is dictated by the chiral anomaly, the off-shell amplitudes relevant for a_μ^{LL}(had) are affected by nonperturbative strong interactions whose effects cannot yet be computed from first principles in QCD. Similarly, the contributions from other hadronic intermediate states besides the π^0, η, and η' cannot be computed reliably at present. The analysis of a_μ^{LL}(had) falls naturally under the purview of chiral perturbation theory (χPT), which provides systematic, model-independent framework for parametrizing presently incalculable hadronic effects. The static quantity a_μ^{LL}(had) has an expansion in powers of p/Λ, where p is a small mass of order m_μ or m_π and Λ is a hadronic scale, typically taken to be $\sim 4\pi F_\pi \sim 1$ GeV (we treat m_μ and m_π to be of the same order and take both to be small compared with Λ). The coefficients appearing in the expansion depend in part on *a priori* unknown "low-energy constants" (LEC's), which parametrize the effects of nonperturbative short distance physics. In principle, the LEC's may be determined from an appropriate set of experimental measurements.

In this Letter, we perform a χPT calculation of a_μ^{LL}(had) including all the logarithmically enhanced contributions to a_μ^{LL}(had) that arise at order $N_C\alpha^3 p^2/\Lambda^2$, where N_C is the number of quark colors. We identify the dependence of a_μ^{LL}(had) on the large logarithms of Λ/p as well as on the relevant LEC's. The result for the large $\ln^2$ term—which is determined entirely by gauge invariance and the chiral anomaly—was given in Refs. [3,4] and was used to uncover the sign error in previous model calculations. However, only part of the large ln term is fixed by symmetry considerations. It receives an additional contribution involving χ, a LEC entering the rate for pseudoscalar decay into leptons. The χ-dependent piece was also computed in Ref. [4]. An analytic calculation of the χ-independent large ln term was performed by the authors of Ref. [7], who used a model for the off-shell $P\gamma^*\gamma^*$ form factor to regulate the two-loop amplitude and assumed m_μ was almost equal to m_π. The results of the latter calculation also contain a model prediction for the nonlogarithmic $\mathcal{O}(N_C\alpha^3 p^2/\Lambda^2)$ contribution to a_μ^{LL}(had).

In χPT, the sum of terms enhanced by large logarithms through $\mathcal{O}(N_C\alpha^3 p^2/\Lambda^2)$ is known, since existing measurements for $\eta \to \mu^+\mu^-$ and $\pi^0 \to e^+e^-$ branching ratios fix the value of χ. However, the uncertainty in a_μ^{LL}(had) from the error in χ is significant: $\pm 60 \times 10^{-11}$, which is roughly the same size as the present theoretical uncertainty in the leading-order vacuum polarization contributions to a_μ. In principle, an improved measurement of the $\pi^0 \to e^+e^-$ branching ratio could reduce this source of uncertainty.

A more serious consideration involves the contributions to a_μ^{LL}(had) that are beyond the order at which we compute. These include effects of order $\mathcal{O}(\alpha^3 p^2/\Lambda^2)$ which are not enhanced by a factor of N_C and order $\mathcal{O}(N_C\alpha^3 p^2/\Lambda^2)$ terms that are not enhanced by large logarithms. We parametrize all these effects in terms of $\tilde{C}$. For simplicity we will refer to $\tilde{C}$ as a low-energy constant even though it has nonanalytic dependence on the quark masses. At present, neither the magnitude nor the sign of $\tilde{C}$ can be determined in a model-independent way without reliance on the measurement of a_μ itself. The presence of this LEC renders the interpretation of a_μ in terms of new physics problematic, since the size of the $\tilde{C}$-dependent contribution could be as large as the present experimental error in a_μ [1].

To leading order in N_C, the lowest-order chiral contributions to $a_\mu^{LL}(\text{had})$, enhanced by large logarithms, arise from the graphs of Fig. 1. Taking m_π and m_μ as being of $\mathcal{O}(p)$, the leading, large logarithmic contributions arise at order $N_C\alpha^3 p^2/\Lambda^2$. The two-loop graphs (Fig. 1a) contain an overall, superficial cubic divergence as well as a linearly divergent one-loop subgraph involving two photons and a muon line. The latter must be regulated by adding the appropriate one-loop counterterm (ct) (Fig. 1b). The one-loop graphs also contain an insertion of $\chi(\mu)$. The sum of these graphs contains a residual divergence, which must be removed by the appropriate magnetic moment ct (Fig. 1c). Associated with this ct is a finite piece which, as discussed above, can be fixed only in a model-independent way by the measurement of a_μ itself. Additional contributions also arise from the graphs such as those containing a charged pion loop. Although subdominant in N_C counting the contribution of the charged pion loop diagrams is leading order in p/Λ. The order $\mathcal{O}(\alpha^3 N_C^0)$ term arising from the three-loop graphs with a charged pion loop—which we denote by $a_\mu^{LL}(\text{had})_{\text{l.o.}}$—has been computed in Refs. [5,8]. The result is finite and contains no large logarithms. We will include this contribution when we compare the theoretical prediction for $a_\mu^{LL}(\text{had})$ with experiment.

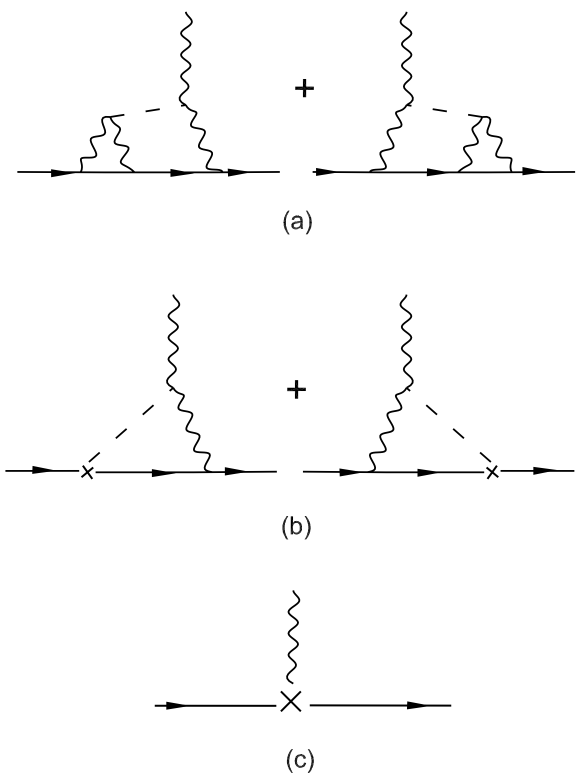

FIG. 1. Hadronic light-by-light contributions to muon anomalous magnetic moment, $a_\mu^{LL}(\text{had})$, obtained from insertion of the WZW interaction at the $\pi^0\gamma\gamma$ vertices. The $\times$ in (b) indicates the insertion of the low-energy constant χ plus its counterterm. The $\times$ in (c) denotes the magnetic moment coupling C plus its counterterm. The solid, dashed, and wavy lines denote the μ, π^0, and γ, respectively.

At order $\alpha^3 p^2/\Lambda^2$ there will be contributions to $a_\mu^{LL}(\text{had})$ from higher dimension operators inserted at the vertices in the charged pion loop graphs and from four-loop graphs containing an additional hadronic loop. Some of these should contain large logarithms. However, these are suppressed by a factor of N_C compared to the logarithmically enhanced pieces that we compute. As noted earlier we absorb these and many other subdominant contributions into $\tilde{C}$ and do not discuss them further here.

As inputs for amplitudes of Fig. 1, we require the Wess-Zumino-Witten (WZW) $P\gamma\gamma$ interaction Lagrangian [9] as well as the leading-order operator contributing to the decays $P \to \ell^+\ell^-$ [10]:

$$\mathcal{L}_{P\ell^+\ell^-} = -\frac{N_C\alpha^2\chi}{48\pi^2 F_\pi}\bar{\mu}\gamma^\lambda\gamma_5\mu\left(\partial_\lambda\pi^0 + \frac{1}{\sqrt{3}}\partial_\lambda\eta\right), \tag{1}$$

where $F_\pi = 92.4$ MeV is the pion decay constant. Note that in contrast to the conventions of Ref. [10], we have made the N_C dependence of the LEC χ explicit for the sake of clarity.

In computing the loop amplitudes involving these operators, it is important to employ a regulator which maintains the consistent power counting of the chiral expansion. To that end, we employ dimensional regularization, where we continue only momenta (and not Dirac matrices) into $d = 4 - 2\epsilon$ dimensions. The relation between bare and renormalized couplings is $\chi^0 = \chi(\mu) - 6/\epsilon$. Using this relation and adding the amplitudes for Figs. 1a and 1b, we obtain the divergent part of the two-loop amplitude

$$\mathcal{M} = -e\left(\frac{\alpha}{\pi}\right)^3\left(\frac{N_C}{3}\right)^2\left(\frac{m_\mu}{F_\pi}\right)^2\left(\frac{1}{32\pi^2}\right) \times\left[\frac{3}{2\epsilon^2} - \left(\frac{\chi(\mu)}{2} + \frac{3}{4}\right)\frac{1}{\epsilon}\right]\bar{u}\,\frac{i\sigma_{\alpha\beta}q^\alpha\varepsilon^\beta}{2m_\mu}\,u,$$

where q^α and ε^β are the photon momentum and polarization, respectively. We remove this divergence using a magnetic moment counterterm. The bare coupling C_0 and renormalized coupling $C(\mu)$ are related by

$$\mathcal{M}_0 = e\left(\frac{\alpha}{\pi}\right)^3\left(\frac{N_c^2 m_\mu^2}{3^2\cdot 32\pi^2 F_\pi^2}\right)C_0\bar{u}\,\frac{i\sigma_{\alpha\beta}q^\alpha\varepsilon^\beta}{2m_\mu}\,u, \tag{2}$$

$$C_0 = C(\mu) + \left[\frac{3}{2\epsilon^2} - \left(\frac{\chi(\mu)}{2} + \frac{3}{4}\right)\frac{1}{\epsilon}\right]. \tag{3}$$

The light-by-light contribution to the anomalous magnetic moment $a_\mu^{LL}(\text{had})$ is a physical quantity and has no dependence on the subtraction point μ. The μ dependence of the diagrams cancels that of the couplings $C(\mu)$ and $\chi(\mu)$. To obtain the μ dependence of the couplings, we require that the bare Green's functions corresponding to the sum of Figs. 1a and 1c and to the $P\ell^+\ell^-$ one-loop subgraphs, respectively, be independent of the subtraction scale. Doing so leads to a coupled set of renormalization group equations for $\chi(\mu)$ and $C(\mu)$:

$$\mu \frac{d\chi}{d\mu} = -12, \qquad \mu \frac{dC}{d\mu} = -3 - \chi. \tag{4}$$

The solution is

$$\chi(\mu) = 12 \ln(\mu_0/\mu) + \chi(\mu_0), \tag{5}$$

$$C(\mu) = 6 \ln^2(\mu_0/\mu) + [\chi(\mu_0) + 3] \times \ln(\mu_0/\mu) + C(\mu_0). \tag{6}$$

At a scale $\mu_0 = \Lambda \sim 1$ GeV, the constants $C(\mu_0)$ and $\chi(\mu_0)$ contain no large logarithms of the form $\ln^k(\Lambda/p)$ $(k = 1, 2)$ where p is around m_μ or m_π. For μ of $\mathcal{O}(p)$, however, the Feynman diagrams contain no such large logarithms, and they live entirely in $C(\mu)$ and $\chi(\mu)$. Hence, the resulting expression for a_μ^{LL}(had) is, in the $\overline{MS}$ scheme,

$$a_\mu^{LL}(\text{had}) = a_\mu^{LL}(\text{had})_{\text{l.o.}} + \frac{3}{16}\left(\frac{\alpha}{\pi}\right)^3\left(\frac{m_\mu}{F_\pi}\right)^2\left(\frac{N_C}{3\pi}\right)^2\left\{\ln^2\left(\frac{\Lambda}{\mu}\right) + \left[-f(r) + \frac{1}{2} + \frac{1}{6}\chi(\Lambda)\right]\ln\left(\frac{\Lambda}{\mu}\right) + \tilde{C}\right\}, \tag{7}$$

where μ is of order p and could be set equal to either m_μ or m_π. Recall that $\Lambda \sim 4\pi F_\pi \sim 1$ GeV. The function $f(r)$, with $r = m_\pi^2/m_\mu^2$, arises from the one-loop diagram with a coupling proportional to $\chi(\mu)$ (Fig. 1b). Since we compute only the terms enhanced by large logarithms, to get f we replace $\chi(\mu)$ by $12 \ln(\Lambda/\mu)$, yielding

$$f(r) = \ln\left(\frac{m_\mu^2}{\mu^2}\right) + \frac{1}{6} r^2 \ln r - \frac{1}{6}(2r + 13) + \frac{1}{3}(2 + r)\sqrt{r(4 - r)} \cos^{-1}\left(\frac{\sqrt{r}}{2}\right). \tag{8}$$

Note that we have absorbed all the remaining terms, including those proportional to $C(\Lambda)$ and $\chi(\Lambda)$ not enhanced by large logarithms, into $\tilde{C}$.

The logarithmically enhanced hadronic light-by-light contributions to a_μ are renormalization scheme independent. However, the values of $\chi(\Lambda)$, $f(r)$, and the constant appearing in the renormalization group equation for C [leading to the factor of 1/2 in the ln term in Eq. (7)] depend on one's choice of scheme. This scheme dependence cancels in the sum of their contributions to a_μ^{LL}(had). For our calculations, we adopted a scheme in which the loop integrals were evaluated in d dimensions with $d > 4$ (corresponding to $\epsilon < 0$), while the Dirac matrices and photon polarization indices were taken as four dimensional. For this choice we have $\eta_{\mu\nu}\,\text{Tr}(\gamma^\mu\gamma^\nu) = 16$ instead of $4d$. Moreover, the value of $\chi(\Lambda)$ in this scheme is the same as that in [10] but is four less than the $\chi(\Lambda)$ used in [11]. An alternative, equally convenient scheme is again to treat the Dirac matrices, epsilon tensors, and photon polarizations as four dimensional, take $d < 4$ $(\epsilon > 0)$, and rewrite the $\gamma^\lambda\gamma_5$ in Eq. (1) in terms of $\epsilon^{\mu\alpha\nu\lambda}\gamma_\mu\gamma_\beta\gamma_\mu\eta_\alpha^\beta$ where the metric tensor $\eta_{\alpha\beta}$ is d dimensional. In this scheme the value of $\chi(\Lambda)$ is still the same as in [10], but $f(r) \to f(r) + 3/2$ and the -3 in the renormalization group equation for C becomes -12. Notice that the total value for the logarithmically enhanced contribution to a_μ^{LL}(had) is unchanged.

As a check on the result in Eq. (7), one may compute the one- and two-loop amplitudes with the insertion of $\chi(\Lambda)$ in Fig. 1b and $C(\Lambda)$ in Fig. 1c. In this case, all of the large logarithms arise from the Feynman amplitudes and not from the operator coefficients. Using an explicit calculation, we have verified in the limit $m_\pi \to 0$ that this procedure exactly reproduces the expression in Eq. (7). We note that the $\ln^2$ term and the term proportional to χ agree with the expression in Ref. [4].

Chiral perturbation theory can be used for the $\eta \to \mu^+\mu^-$ amplitude [10], and the LEC $\chi(\Lambda)$ can be deduced from the measured $\eta \to \mu^+\mu^-$ branching ratio. This yields [11] $\chi(1\text{ GeV}) = -14^{+4}_{-5}$ or -39^{+5}_{-4}, where we have scaled the results in Ref. [11] from $\Lambda = m_\rho$ up to $\Lambda = 1$ GeV and subtracted four. Note that because the $\eta \to \mu^+\mu^-$ branching ratio is a quadratic function of χ, two different values for this LEC may be extracted from experiment.

Our calculation involved only the use of chiral $SU(2)_L \times SU(2)_R$ while this extraction of the LEC involves the use of chiral $SU(3)_L \times SU(3)_R$. Since one expects chiral perturbation theory to work better when only the pions are treated as light, it is desirable to have an extraction of χ that relies only on chiral $SU(2)_L \times SU(2)_R$. This can be done using the measured $\pi^0 \to e^+e^-$ branching ratio which yields [11] $\chi(1\text{ GeV}) = -29^{+25}_{-16}$ or $+74^{+16}_{-25}$. Unfortunately, the errors on the extracted $\chi(1\text{ GeV})$ are very large in this case. A more precise determination of the $\pi^0 \to e^+e^-$ branching ratio could reduce the theoretical uncertainty in $\chi(1\text{ GeV})$.

Model calculations for a_μ^{LL}(had) differ from our analysis typically through insertion of form factors at the $P\gamma^*\gamma^*$ vertices obtained from the WZW interaction. For example, one widely followed model employs form factors based on a vector meson dominance picture. This approach—known as resonance saturation—may also be used to obtain χ, giving [11] $\chi(1\text{ GeV})_{\text{res sat}} \simeq -17$. In a similar vein, one may analyze this LEC at leading order in N_C, where it depends on a sum over an infinite tower of vector meson resonances [12]. Using a model-dependent form factor for the sum over vector resonances that at short distances is consistent with the properties of QCD gives $\chi(1\text{ GeV})_{N_C,\text{res sat}} = -16 \pm 5$. This result provides some quantitative support for phenomenological models since it is close to the value $\chi(1\text{ GeV}) \simeq -14$ obtained from experiment. However, nonperturbative contributions to a_μ^{LL}(had) not included in such models (e.g., the full tower of higher mass meson poles) remain to be estimated.

Using $\chi(1\text{ GeV}) = -14^{+4}_{-5}$ as input, setting $\mu = m_\mu$, and adding the large $\ln^2$ and ln terms in Eq. (7), we obtain $a_\mu^{LL}(\mathrm{had})_{\log} = (57^{+50}_{-60}) \times 10^{-11}$. We emphasize that inclusion of *both* the $-f(r) + 1/2$ and $\chi/6$ parts of the $\ln(\Lambda/\mu)$ term is crucial to obtaining an accurate numerical result for $a_\mu^{LL}(\mathrm{had})_{\log}$. If, for example, one were to keep only the dependence on $\chi(\Lambda)$, one would find substantial cancellations between the $\ln^2$ and ln contributions. The presence of the calculable $-f(r) + 1/2$ term, however, substantially mitigates these cancellations.

We observe that the central value in $a_\mu^{LL}(\mathrm{had})_{\log}$ is roughly the same size as model calculations for the π^0 contribution, and that the uncertainty is about a third the size of the present experimental error in a_μ [1]. After the full E821 data set is analyzed, the uncertainty in $a_\mu^{LL}(\mathrm{had})_{\log}$ will be comparable to the anticipated experimental error. Using the other value of χ obtained from the $\eta \to \mu^+\mu^-$ branching ratio, $\chi(1\ \mathrm{GeV}) = -39^{+5}_{-4}$, leads to $a_\mu^{LL}(\mathrm{had})_{\log} = (-190^{+60}_{-50}) \times 10^{-11}$. Although there exists a strong theoretical prejudice in favor of the first solution $[a_\mu^{LL}(\mathrm{had})_{\log} = (57^{+50}_{-60}) \times 10^{-11}]$ based on both the resonance saturation model for χ and $a_\mu^{LL}(\mathrm{had})$ as well as consistency between the values of χ obtained from the $\eta \to \mu^+\mu^-$ and $\pi^0 \to e^+e^-$ branching ratios, one cannot rule out the second value for $a_\mu^{LL}(\mathrm{had})_{\log}$.

Adding in the $O(N_C^0\alpha^3)$ charged pion loop contribution [5,8], $a_\mu^{LL}(\mathrm{had})_{\mathrm{l.o.}} = -44.6 \times 10^{-11}$, gives the following χPT expression for $a_\mu^{LL}(\mathrm{had})$:

$$a_\mu^{LL}(\mathrm{had})_{\chi\mathrm{PT}} = (13^{+50}_{-60} + 31\tilde{C}) \times 10^{-11}. \qquad (9)$$

The largest uncertainty in the expression for $a_\mu^{LL}(\mathrm{had})$ above arises from the subdominant terms that have not been calculated and are parametrized by the LEC $\tilde{C}$. As noted above, this constant includes the effects of nonlogarithmically enhanced two-loop contributions, heavy mesons such as the η and η' which have been integrated out, and other nonperturbative dynamics. On general grounds, one could expect its natural size to be of order unity. A comparison of Eq. (9) with the results of model calculations is consistent with this expectation. For example, the model calculation of Ref. [13] corresponds roughly to $\tilde{C} \simeq 2$. Rigorously speaking, however, the precise value—as well as the sign—of $\tilde{C}$ is unknown. An uncertainty $\Delta\tilde{C} = \pm 1$ corresponds to $\Delta a_\mu^{LL}(\mathrm{had})_{\chi\mathrm{PT}} = \pm 31 \times 10^{-11}$. One should not, however, treat this as an estimate of the theoretical uncertainty in $a_\mu^{LL}(\mathrm{had})$. A value of $\tilde{C}$ equal to $+3$ or -3, for example, would not be unusual.

Alternatively, one may use the experimental result for a_μ to determine $\tilde{C}$. To that end, we use the updated results for hadronic vacuum polarization contributions [2], the QED and electroweak loop contributions in Ref. [14] and the value for $a_\mu^{LL}(\mathrm{had})_{\chi\mathrm{PT}}$ given in Eq. (9). From the E821 result for a_μ we obtain $\tilde{C} = 7 \pm 5 \pm 3 \pm 2$, where the first uncertainty arises from the experimental error in a_μ, the second corresponds to the theoretical QED, electroweak, and hadronic vacuum polarization errors, and the final uncertainty arises from the error in χ. In the future, the first uncertainty will be considerably reduced upon complete analysis of the full E821 data set. The value of $\tilde{C}$ is consistent with unity, though it could be considerably larger, given the other experimental and theoretical inputs used. Taking the second solution for χ and $a_\mu^{LL}(\mathrm{had})$ gives $\tilde{C} = 16 \pm 5 \pm 3 \pm 2$.

At present there is no indication that the hadronic LEC $\tilde{C}$ differs substantially from its natural size and, thus, no reason to discern effects of new physics, such as loops containing supersymmetric particles [14], from the a_μ result. In principle, a systematic calculation of some of the effects arising at higher order—such as terms of $\mathcal{O}(\alpha^3 N_C^0 p^2/\Lambda^2)$ enhanced by large logarithms—could modify this conclusion. Indeed, the leading order χPT result for the hadronic vacuum polarization contribution to a_μ underpredicts the total effect by an order of magnitude, and we cannot rule out a similar situation in the case of $a_\mu^{LL}(\mathrm{had})$. On the other hand, should the full E821 data imply a value for $\tilde{C}$ which differs significantly from ± 1 (e.g., by an order of magnitude), one might argue that there is evidence of new physics. Ultimately, however, the most convincing analysis will require a first principles QCD calculation of $a_\mu^{LL}(\mathrm{had})$.

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