Model for the Electric Dipole Moment of the Neutron*

P. BABU† AND MAHIKO SUZUKI
California Institute of Technology, Pasadena, California
(Received 8 May 1967)

A model of the electric dipole moment of the neutron is proposed and evaluated with the help of current algebra and the hypothesis of partially conserved axial-vector current.

TWO recent experiments[1,2] on $K_L\to 2\pi^0$ have clearly indicated that an appreciable amount of $\Delta I=\frac{3}{2}$ amplitude contributes to the CP-violating weak interactions. In this paper we develop a phenomenological model of the electric dipole moment of the neutron, incorporating the result of these measurements.

Our model consists of the assumption that the electric dipole moment arises from the tadpole mechanism of the octet pseudoscalar mesons. It does not necessarily lead to an effective $\Delta I=\frac{1}{2}$ rule for $K_L\to 2\pi$ decays, as will be discussed later. The tadpole diagrams relevant for the electric dipole moment would involve the π^0 (and η) tadpole. These tadpole diagrams are evaluated in terms of the photo π^0 (η) production amplitude and annihilation strength of π^0 (η) into the vacuum through the $\Delta Y=0$ part of the CP-violating interaction, $H_w^{(-)}$.

The π^0 (η) photoproduction amplitude can be written as[3,4]

$$F=\left(\frac{m_N^2}{4E_1E_2k\omega}\right)^{1/2}\bar{u}_1(\alpha M_\alpha+\beta M_\beta+\gamma M_\gamma+\delta M_\delta)u_2\,,\quad(1)$$

where

$$M_\alpha=-\tfrac{1}{2}i\gamma_5\gamma_\mu\gamma_\nu F_{\mu\nu}\,,\quad(2a)$$

$$M_\beta=2i\gamma_5 q_\mu P_\nu F_{\mu\nu}\,,\quad(2b)$$

$$M_\gamma=\gamma_5 q_\mu\gamma_\nu F_{\mu\nu}\,,\quad(2c)$$

$$M_\delta=2(\gamma_5 P_\mu\gamma_\nu F_{\mu\nu}-\tfrac{1}{2}im_N\gamma_5\gamma_\mu\gamma_\nu F_{\mu\nu})\,,\quad(2d)$$

$$F_{\mu\nu}=k_\mu\epsilon_\nu-k_\nu\epsilon_\mu\,,\quad(3a)$$

$$P=(p_1+p_2)/2\,,\quad(3b)$$

where p_1, p_2, k, and q are the four-momenta of the initial and final neutrons, the photon, and the pseudoscalar meson, respectively, ϵ being the photon polarization. In the limit of $q_\mu\to 0$, only M_α is nonzero, while M_β, M_γ, and M_δ vanish.

The electric dipole moment is defined by

$$d_E=F_3(0)\,,\quad(4a)$$

where F_3 is given by

$$\int\langle p_2|T(H_w^{(-)}(x),j_\mu(0))|p_1\rangle d^4x$$
$$=\frac{1}{i}\left(\frac{m_N^2}{E_1E_2}\right)^{1/2}\bar{u}_2[i\gamma_\mu F_1(k^2)-i\sigma_{\mu\nu}k_\nu F_2(k^2)-\gamma_5\sigma_{\mu\nu}k_\nu F_3(k^2)$$
$$+(k_\mu\gamma_5-i\gamma_5\gamma_\mu k^2/2m_N)F_4(k^2)]u_1.\quad(4b)$$

The matrix element

$$\int\langle p_2|T(H_w^{(-)}(0),j_\mu(x))|p_1\rangle e^{ik\cdot x}d^4x\quad(5)$$

involves four invariant variables $q^2=(p_2-p_1-k)^2$, k^2, $\nu=(P\cdot q)/m_N$, and $\nu_1=-(q\cdot k)/2m_N$. We write down the dispersion relation in q^2 with fixed ν, ν_1, and k^2, and then put $k^2=0$ and $q^2=\nu=\nu_1=0$ corresponding to $q_\mu=0$. Picking up only the one-pseudoscalar-meson intermediate state, we get

$$F_3{}^j(0)=\sum_i\int\lambda_i(q'^2)\alpha_i{}^j(\nu=\nu_1=0,\,q'^2)\delta(m_i^2-q'^2)/q'^2dq'^2\,,$$
$$=\sum_i\lambda_i(m_i^2)\alpha_i{}^j(\nu=\nu_1=0,\,q'^2=m_i^2)/m_i^2\,,\quad(6)$$

where i runs over π^0 and η, j runs over the isovector and isoscalar photon, and λ_i is the pseudoscalar meson annihilation strength defined by

$$\lambda_i=2\omega\langle i,q|H_w^{(-)}|0\rangle.\quad(7)$$

It should be noted that we take the tadpole model in the dispersion theoretical picture.

Because of the Fubini-Rossetti-Furlan sum rule,[4] $\alpha(\nu=\nu_1=0,\,m_i^2)$ approaches zero in the limit of $m_i^2\to 0$. However, we are interested in the case of physical m_i and could, in principle, evaluate it using the experimental data. But the analysis of Adler and Gilman[5] has shown that there are large uncertainties in the presently available data. We could also obtain an estimate of $\alpha_i{}^j/m_i^2$ by using the prescription of Adler[6] for the off-

* Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

† On deputation from Tata Institute of Fundamental Research, Bombay, India.

[1] J. M. Gaillard, F. Krienen, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. Ratcliffe, P. Day, A. G. Parhan, B. T. Payne, A. C. Sherwood, H. Faissner, and H. Reithler, Phys. Rev. Letters 18, 20 (1967).
[2] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters 18, 25 (1967).
[3] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).
[4] S. Fubini, C. Rossetti, and F. Furlan, Nuovo Cimento **43A**, 161 (1966).
[5] S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).
[6] S. L. Adler, Phys. Rev. **140**, B736 (1965); and Ref. 5.

mass-shell extrapolation and assuming that $\alpha_i{}^j$ is in fact zero for zero pseudoscalar meson mass. In this method, we obtain an expression for $\alpha_i{}^j/m_i{}^2$ which has the form[7]

$$\alpha_i{}^j/m_i{}^2=\frac{1}{\pi}\int\sum_l\frac{\mathrm{Im}\alpha_i{}^j(l)}{\nu}\left\{\frac{1-(q/q')^l}{m_i{}^2}\right\}d\nu \qquad (8)$$

to the lowest significant order in $m_i{}^2$, where $\alpha_i{}^j(l)$ stands for the contribution from the lth partial wave to the pseudoscalar meson-nucleon system and $\mathbf{q}$ and $\mathbf{q}'$ stand for the three-momentum of the meson corresponding to the physical and zero-mass pseudoscalar meson, respectively, for a given ν. For explicit calculation, we use the values given by Adler and Gilman for $\mathrm{Im}\alpha_i{}^j(l)$ and find that

$$\alpha_\pi{}^3/m_\pi{}^2=0.019em_\pi{}^{-4}, \qquad (9)$$

where almost all the contribution comes from the N_{33}^* resonance. For $\alpha_\pi{}^8/m_\pi{}^2$, $\alpha_\eta{}^3/m_\eta{}^2$, and $\alpha_\eta{}^8/m_\eta{}^2$, N_{33}^* cannot contribute and hence it is reasonable to assume that these terms are negligible compared to $\alpha_\pi{}^3/m_\pi{}^2$.[8] It is important to bear in mind that these estimates are dependent on the extrapolation procedure.

We thus obtain from Eqs. (6) and (9) an expression for the electric dipole moment of the neutron:

$$d_E=0.019\lambda_\pi em_\pi{}^{-4}. \qquad (10)$$

The pion annihilation strength λ_π depends entirely on the structure of $H_w{}^{(-)}$. In order to relate λ_π to the $K_L\to 2\pi$ decay, we consider the CP-violating 3π vertex. As in the case of d_E, we assume that the pion tadpole is dominant in this process. The annihilation strength λ_π is related to the $3\pi^0$ vertex by

$$(8\omega_1\omega_2\omega_3)^{1/2}\langle q_1q_2q_3|H_w{}^{(-)}|0\rangle=\lambda_\pi(A_0/m_\pi{}^2), \qquad (11)$$

where $-(32\pi m_\pi)^{-1}A_0$ is the $\pi^0\pi^0$ scattering length. As no reliable experimental determination of A_0 is available, we use the value obtained from PCAC (hypothesis of partially conserved axial-vector current), which is[9,10]

$$A_0/m_\pi{}^2=-(g_{\pi N}/m_Ng_A)^2. \qquad (12)$$

Combining Eqs. (11) and (12) with Eq. (10), we obtain

$$d_E=-0.019em_\pi{}^{-2}(m_Ng_A/g_{\pi N})^2M(\pi^0\pi^0\pi^0), \qquad (13)$$

where $M(\pi^0\pi^0\pi^0)$ is the (CP-violating) invariant $3\pi^0$ matrix element given by the left-hand side of Eq. (11).

We multiply and divide the right-hand side by $M[K_L(\pi^+\pi^-)_{I=2}]$, where $M[K_L(\pi^+\pi^-)_{I=2}]$ stands for the invariant decay matrix element of $K_L\to\pi^+\pi^-$ in the $I=2$ state. Explicitly,

$$d_E=-0.019em_\pi{}^{-2}(m_Ng_A/g_{\pi N})^2\times\{M(\pi^0\pi^0\pi^0)/M[K_L(\pi^+\pi^-)_{I=2}]\}\times M[K_L(\pi^+\pi^-)_{I=2}]. \qquad (14)$$

We can obtain the value of $M[K_L(\pi^+\pi^-)_{I=2}]$ from the experimental information regarding the decay rate of the $\pi^+\pi^-$ mode and the branching ratio of the $\pi^+\pi^-$ and $2\pi^0$ modes. We obtain[2]

$$M[K_L(\pi^+\pi^-)_{I=2}]\simeq-0.5\times M(K_L\pi^+\pi^-) \qquad (15a)$$

or

$$\simeq1.2\times M(K_L\pi^+\pi^-). \qquad (15b)$$

We have two possibilities due to the fact that the information regarding branching ratio $(K_L\to\pi^+\pi^-)/(K_L\to\pi^0\pi^0)$ leaves the relative sign of the matrix elements $K_L\to\pi\pi$ unspecified [Eq. (15b) corresponds to approximate $\Delta I=\frac{3}{2}$ rule for the K_L decay]. Substituting the known values of m_π, m_N, g_A, $g_{\pi N}$, and $|M(K_L\pi^+\pi^-)|$, and making use of $M[K_L(\pi^+\pi^-)_{I=2}]\approx M[K_2(\pi^+\pi^-)_{I=2}]$,[11] we obtain

$$|d_E|=0.27M(\pi^0\pi^0\pi^0)/M[K_2(\pi^+\pi^-)_{I=2}]\times10^{-23}\,e\text{ cm}, \qquad (16a)$$

or

$$=0.65M(\pi^0\pi^0\pi^0)/M[K_2(\pi^+\pi^-)_{I=2}]\times10^{-23}\,e\text{ cm}. \qquad (16b)$$

Now we assume that $SU(3)$ symmetry is approximately good,[12] and try to evaluate the ratio $M(\pi^0\pi^0\pi^0)/M[K_2(\pi^+\pi^-)_{I=2}]$ under certain assumptions about the $SU(3)$ properties of $H_w{}^{(-)}$. Decomposing $M(\pi^0\pi^0\pi^0)$ and $M[K_2(\pi^+\pi^-)_{I=2}]$ into $SU(3)$ amplitudes, we get[13]

$$M(\pi^0\pi^0\pi^0)=\frac{\sqrt{30}}{10}C(8,0,1)T_8+\cdots, \qquad (17a)$$

$$M[K_2(\pi^+\pi^-)_{I=2}]=\sqrt{2}\left\{-\frac{\sqrt{30}}{30}C(64,1,\tfrac{5}{2})T_{64}-\frac{\sqrt{105}}{315}C(64,1,\tfrac{3}{2})T_{64}-\frac{\sqrt{21}}{42}C(27,1,\tfrac{3}{2})T_{27}+\frac{\sqrt{3}}{36}C(10,1,\tfrac{3}{2})(T_{10}+T_{\overline{10}})\right\}, \qquad (17b)$$

---

[7] Despite large cancellations, we can still expect that the one-pion intermediate state is dominant because of the pion mass square in the denominator. There is no experimental evidence for a resonance or the like in the corresponding channel of 3π.

[8] The electric dipole moment of the neutron, therefore, would be predominantly isoscalar. However, it is almost impossible to verify it.

[9] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); N. N. Khuri, Phys. Rev. **153**, 1477 (1967).

[10] A different extrapolation procedure adopted by A. P. Balachandran, M. G. Gundzik, and F. Nicodemi (unpublished) gives A_0 to be zero. If their value is closer to the actual value, our model would not be applicable.

[11] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964). We also assume that the contributions from $H_w{}^{(-)}$ and the CP-conserving $H_w{}^{(+)}$ to the $M[K_L(2\pi)_{I=0}]$ do not exactly cancel each other. Combining this assumption with the observed $|\eta_{+-}|/|\eta_{00}|$, we are led to this approximate equality.

[12] For the $\Delta I=\frac{1}{2}$ part of the amplitude, such an assumption may not be justified because of the fact that the term with $T_3{}^3$ breaking could get enhanced due to the octet enhancement mechanism. But we are making this assumption only for the $\Delta I=\frac{3}{2}$ (and $\frac{5}{2}$) part of the amplitude.

[13] For the Clebsch-Gordan coefficients of three octets, see, for example, Y. Dothan and H. Harari, Nuovo Cimento Suppl. **3**, 48 (1965).

where $C(i,Y,I)$ is the coefficient of the irreducible representation of $SU(3)$ contained in $H_w^{(-)}$, and Y and I specifying the particular component. We have retained only the octet part for the 3π vertex, assuming octet enhancement.

We determine the octet enhancement factor for the $K_2 \rightarrow 2\pi$ decay matrix element by the following method. We find that the strong-interaction dynamics of the three-meson matrix elements for the CP-conserving and CP-violating processes are the same if we assume that the CP-conserving Hamiltonian $H_w^{(+)}$ and the $SU(3)$-breaking T_3^3 type medium strong interaction can be approximated to act at the same space-time point and that the space-time structure of the spurions can be disregarded (see Fig. 1). With this assumption, the octet enhancement factor for the process $K_1 \rightarrow 2\pi$ would be the same as that for the CP-violating three-meson matrix element. The octet enhancement factor for the matrix element $K_1 \rightarrow 2\pi$ (which is approximately the same as $K_s \rightarrow 2\pi$) can be determined by combining the assumption about the current$\times$current form of $H_w^{(+)}$ with the known value of the ratio of the decay rates $K_s \rightarrow \pi^+\pi^-/K^+ \rightarrow \pi^+\pi^0$. We obtain[14]

$$|T_8/T_{27}| = 8.6. \qquad (18a)$$

Since channels other than the octet are not enhanced, we would also expect

$$|T_{10}|, |T_{64}| \simeq |T_{27}|. \qquad (18b)$$

Combining Eqs. (16), (17), and (18), we can estimate the value of $|d_E|$ in the specific theories of $H_w^{(-)}$. As an example, we evaluate d_E in Zachariasen and Zweig's model.[15] In this model

$$C(64,1,\tfrac{5}{2}) = C(64,1,\tfrac{3}{2}) = C(10,1,\tfrac{3}{2}) = 0, \qquad (19a)$$

$$C(27,1,\tfrac{3}{2}) = (\tfrac{1}{3}\sqrt{5}) \cos\theta \sin\theta \times C(8,0,1), \qquad (19b)$$

and therefore,

$$|d_E| = 1.1(\cos\theta \sin\theta)^{-1} \times 10^{-22}\, e \text{ cm}, \qquad (20a)$$

[14] This large octet enhancement factor determined for the $K_2 \rightarrow 2\pi$ matrix element would suggest an effective $\Delta I = \frac{1}{2}$ rule for the $K_2 \rightarrow 2\pi$ matrix element and hence also for the $K_L \rightarrow 2\pi$ decay process, contrary to the observed $(K_L \rightarrow \pi^+\pi^-)/(K_L \rightarrow \pi^0\pi^0)$ branching ratio. This can, however, be explained by the smallness or the absence of the octet term in $H_w^{(-)}$. In fact, this feature is obtained in the Zachariasen-Zweig model (Ref. 15) which we consider below.

[15] F. Zachariasen and G. Zweig, Phys. Rev. Letters **14**, 794 (1965).

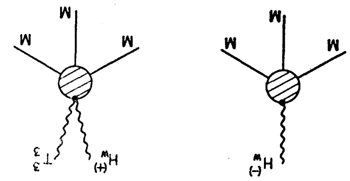

FIG. 1. Diagrammatic representation of the model used to relate the octet enhancement factor of the CP-conserving and CP-violating three-meson matrix elements.

or

$$= 2.7(\cos\theta \sin\theta)^{-1} \times 10^{-22}\, e \text{ cm}. \qquad (20b)$$

Substituting the value of θ chosen by Zachariasen and Zweig, i.e., $\theta = 0.19$, we obtain

$$|d_E| = 6 \times 10^{-22}\, e \text{ cm}, \qquad (21a)$$

or

$$= 14 \times 10^{-22}\, e \text{ cm}. \qquad (21b)$$

It is interesting to note that in this theory, even if θ is taken as an unknown parameter, there is a lower bound on the magnitude of d_E corresponding to $\theta = \pi/4$ and possibility 20(a), which is

$$|d_E|_{\min} = 2.2 \times 10^{-22}\, e \text{ cm}. \qquad (22)$$

Although the experimental upper bound on d_E is of the order of $5 \times 10^{-20}\, e$ cm,[16] experiments now being conducted[17] presumably will be able to measure d_E to an accuracy of $10^{-22}\, e$ cm in the near future, thus checking the validity of this model.

Our model of the electric dipole moment is quite different from so-far existing models[18] involving a cutoff associated with perturbation calculations or using crude dimensional considerations. It applies to theories in which CP violation takes place only in weak interactions and $H_w^{(-)}$ has the specified $SU(3)$ properties.

We would like to thank Professor F. Zachariasen for helpful discussions.

[16] J. H. Smith, E. M. Purcell, and N. F. Ramsey, Phys. Rev. **108**, 120 (1957).

[17] P. D. Miller, N. F. Ramsey, J. K. Baird, and W. B. Dress, Bull. Am. Phys. Soc. **12**, 419 (1967); C. G. Shull and R. Nathans, *ibid.* 12, 508 (1967); W. Dress, P. D. Miller, J. K. Baird, and N. F. Ramsey, *ibid.* 12, 650 (1967).

[18] N. T. Meister and P. K. Radha, Phys. Rev. **135**, B769 (1964); F. Salzman and G. Salzman, Phys. Letters **15**, 91 (1965); D. G. Boulware, Nuovo Cimento **40A**, 1041 (1965).