Regge Poles and Inelastic Scattering at High Energies*

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The Regge pole hypothesis is applied to some inelastic processes at high energies. Particular attention is given to production of pion-nucleon resonances N^* in the reaction $N+N \to N+N^*$. Data on production of the $I=1/2$ resonances are interpreted in terms of exchange of the "Pomeranchuk" Regge pole which is believed to be responsible for the diffraction peaks in elastic reactions, and production of the 3-3 resonance is interpreted in terms of exchange of the pion Regge pole. The prediction is obtained that production and decay of the $I=1/2$ resonances provides secondary pions and neutrons with energies that rise proportionally to the incident proton energy—a result of interest in connection with the secondary beams in future accelerators.

I. INTRODUCTION

IT has recently been proposed that, at high energies, Regge pole terms[1] dominate two-body scattering amplitudes involving strongly interacting particles.[2–8] A brief discussion of the production of unstable isobars, as in the reaction $N+N \to N+N^*$, has been given from the same point of view.[8] In the present paper we present a more detailed discussion of the consequences of Regge pole terms for inelastic events and, in particular, isobar production at high energies. The relation of the Regge pole hypothesis to diffraction dissociation[9,10] and the single-pion exchange model[11–13] is described, and comparison with existing data is made.

In Sec. II the Regge formalism needed at high energies is briefly introduced and discussed. Section III deals with its application to $I=1/2$ isobar production, Sec. IV with production of the 3-3 pion-nucleon resonance. In Sec. V predictions are obtained for the high-energy tail of pions and neutrons emerging from a proton-proton collision—a matter of importance in estimating secondary beams which can be extracted from future proton accelerators with energy >30 BeV. It is found that $I=1/2$ isobar production can provide pions with as much as 2/3 of the initial proton energy, almost independently of the proton energy. Finally, in Sec. VI, some speculations about the relation of the diffraction or Pomeranchuk Regge term to conservation laws and the "maximal strength" of strong interactions are briefly discussed.

II. REGGE FORMALISM AT HIGH ENERGIES

A familiar way to think about scattering is to represent it as a one-pion exchange (if allowed by quantum numbers), plus an exchange of a two-pion state with definite spin and isotopic spin, and so on.

In the Regge pole hypothesis we again think of scattering as a sum of terms, in each of which a definite isotopic spin, G parity, baryon number, strangeness, etc., are exchanged. Each term, however, now includes exchange of *all* particle combinations with the appropriate quantum numbers; for example, exchange of 2π, 4π, $N\bar{N}$, $\cdots$, would all contribute to a Regge term where $I=0$ and $G=+$ are transferred. Furthermore, each term now represents a coherent sum over exchange of *all* physical spins. This coherent set of physical spin terms is equivalent to exchange of a single spin $\alpha_i(t)$ which can take on noninteger values and varies continuously with momentum transfer t. A number of different $\alpha_i(t)$ may be associated with the exchange of each set of quantum numbers. Consider, for example, two-body scattering of spinless particles. We assume each particle has mass m, and employ the usual Mandelstam variables $s=(p_1+p_2)^2$, $t=(p_3+p_1)^2$, $u=(p_4+p_1)^2$. According to the Regge pole hypothesis the amplitude

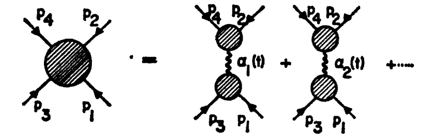

FIG. 1. Analysis of scattering via a sum of intermediate states of angular momenta $\alpha_i(t)$.

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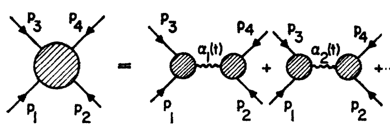

FIG. 2. Analysis of scattering in the s channel by exchanges of Regge terms $\alpha_i(t)$.

for scattering in the t channel can be represented by

$$A(t, \cos\theta_t)=\sum_i \frac{\beta_i(t)}{\sin\pi\alpha_i(t)} P_{\alpha_i(t)}(-\cos\theta_t), \tag{2.1}$$

plus exchange terms in $(+\cos\theta_t)$, plus integral terms that are of less importance to us for high-energy applications. In the t channel, t has the significance of center-of-mass energy squared, θ_t is the center-of-mass scattering angle, and

$$\cos\theta_t=-1+s/(2m^2-t/2). \tag{2.2}$$

Scattering in the s or u channel can be represented in a similar way.

At each t the ith term represents the reaction proceeding through an intermediate state of angular momentum $\alpha_i(t)$ (Fig. 1). In the unphysical limit $|\cos\theta_t|\rightarrow\infty$ at fixed t, the relation $P_\alpha(-x)\sim x^\alpha$ can be used, and each term of Eq. (2.1) becomes proportional to[14]

$$\frac{\beta_i(t)}{\sin\pi\alpha_i(t)}\left(\frac{s}{2m^2-t/2}\right)^{\alpha_i(t)}. \tag{2.3}$$

We now continue the amplitude from $t\geqq 4m^2$ to the region $t\leqq 0$, $s\geqq 4m^2$, corresponding to physical scattering in the s channel. The full representation (2.1) requires some modification in this region, but the expression (2.3) for large s can be taken over without change. Thus, we represent scattering in the s channel as a sum over exchanges of angular momenta $\alpha_i(t)$ (Fig. 2). Each $\alpha_i(t)$ is expected to be real and satisfy $d\alpha_i/dt\geqq 0$ in this region.

The differential cross section for the s channel at sufficient large s is

$$d\sigma/dt\propto |A|^2/s^2. \tag{2.4}$$

Now, $d\sigma/dt$ can be expanded into a sum of Regge terms. In view of (2.3), the highest $\alpha_i(t)$ (labeled α_1) dominates at large s and the representation simplifies to

$$d\sigma/dt=F_1(t)(s/2m^2)^{2\alpha_1(t)-2}. \tag{2.5}$$

If other Regge powers α_2, $\cdots$ are nearly as large as α_1, there will be other appreciable terms at intermediate energies behaving as $s^{\alpha_1+\alpha_2-2}$, etc.

[14] The next term in the expansion of $P_\alpha(-x)$ is of order $x^{\alpha-2}$, but there is also an $s^{\alpha-1}$ correction since $x=[s/(2m^2-t/2)]\times[1-(2m^2-t/2)/s]$. Thus, if the leading power in a reaction (the Pomeranchuk trajectory, for example) is α_i, then $P_{\alpha_1}(-x)$ must be treated more carefully than in (2.3) if one wishes to include other Regge poles with $\alpha\lesssim\alpha_1-1$ (the ABC trajectory, for example).

In the s channel the center-of-mass scattering angle is

$$\cos\theta_s=1+t/(s/2-2m^2), \tag{2.6}$$

so at $\theta_s\sim 90°$ and high energies one has $t\sim -s/2$. Referring back to the angle in the t channel, Eq. (2.2), we see that $\cos\theta_t$ is rather small when $t\lesssim -s/2$. Therefore, the asymptotic expression (2.5), which originated in an expansion for large $\cos\theta_t$, is most reliable at $0\leq\theta_s<90°$. An alternative representation could be derived starting in the u rather than the t channel. Scattering in the s channel would then be expressed as a sum of exchanges of new Regge terms with angular momenta $a_j(u)$ (see Fig. 3), and the cross section could be expressed as

$$d\sigma/du=\phi_1(u)(s/2m^2)^{2a_1(u)-2}+\cdots. \tag{2.7}$$

The new representation (2.7) is most reliable at $90°<\theta_s\leqq 180°$. Our detailed discussion in Secs. III–VI will concentrate upon small-angle behavior where (2.5) is relevant, but Eq. (2.7) would be useful for studying the influence of nucleon or 3-3 resonance exchange on backward π-N scattering, for example.[8]

When the restriction to spinless particles is removed the expression for the amplitude [Eq. (2.1)] becomes more complicated but (2.5) or (2.7) remains valid. Similarly, when the masses differ, the kinematical expressions (2.2) and (2.6) become more complicated, but we can still use (2.5) or (2.7). We shall take the arbitrary normalization mass m in $s/2m^2$ [Eqs. (2.5) and (2.7)] to be the nucleon mass throughout this paper. Furthermore, each Regge term $\alpha_i(t)$ [or $a_j(u)$] is associated with definite isotopic spin, strangeness, etc. in the t (or u) channel.

Finally, we remove the restriction to the two-body processes. We conjecture that the following procedure is valid:

(1). For an arbitrary inelastic process the incoming and outgoing particles are lumped together into two groups (1,2) of incoming and two groups (3,4) of outgoing particles:

$$1+2\rightarrow 3+4. \tag{2.8}$$

Each group has a rest energy m_i in its own center of mass.

(2). Process (2.8) is treated like a two-body reaction, and represented in the Regge form (2.5) or (2.7). This conjecture allows us to apply methods developed for the two-body case[2–8] to an arbitrary inelastic reaction.

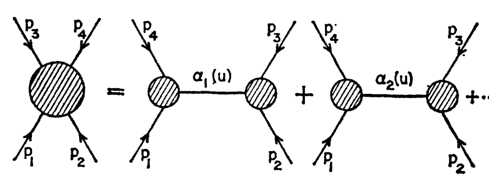

FIG. 3. Analysis of scattering in the s channel by exchanges of Regge terms $a_j(u)$.

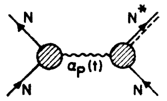

FIG. 4. Diagram for the exchange of the leading Regge term in N-N $I=1/2$ isobar production.

For a check of relations like (2.5) or (2.7), it is necessary to keep the internal arrangement of each group of particles fixed ($m_i=$ const, etc.) while varying the total energy s and momentum transfer t or u. This is easiest, both experimentally and theoretically, for a nearly elastic process in which 1, 2, and 3 are single particles while 4 is an unstable isobar, and the present paper will be confined to this special case. We, in fact, treat the isobars like stable particles of definite mass. In a future paper more highly inelastic processes will be discussed.

III. PRODUCTION OF $I=1/2$ PION-NUCLEON ISOBARS

Consider the reaction

$$N+N\rightarrow N+N^* \tag{3.1}$$

where N^* is one of the $I=1/2$ pion-nucleon isobars N_2^* or N_3^* (600- or 900-MeV resonance). In accordance with our Regge pole hypothesis, the cross section at sufficiently large energy will have the form

$$d\sigma/dt=F_{NN^*P}(t)(s/2M^2)^{2\alpha(t)-2}, \tag{3.2}$$

where α is the highest Regge power consistent with the quantum numbers that can be exchanged. The assignment $I=1/2$ for N^* permits the exchange of the quantum numbers of the vacuum, $I=0$, $S=0$, etc., so at small t the highest Regge power of all—the Pomeranchuk pole α_P responsible for diffraction—is present (Fig. 4).

The breakup of a diffraction-scattered particle into its components has often been called "diffraction dissociation."[9,10] The technical description was usually put in a wave-mechanical form, the generalization of which to particles such as protons was not clear. Regge poles now provide a technical description of scattering at high energies, and as a natural extension of the customary terminology we *define* diffraction dissociation to mean any inelastic process in which the Pomeranchuk pole is exchanged.

Although reaction (3.1) comes under our definition of diffraction dissociation, special attention must be paid to the fact that the $D_{3/2}$ resonance has opposite parity from the nucleon, and both the $D_{3/2}$ and $F_{5/2}$ resonances have different spins than the nucleon. These distinctions become important in the limit $t\rightarrow 0$ where exchange of the Pomeranchuk pole tends towards pure diffraction. It has previously been pointed out[8] that $F_{NN_3^*P}(t)$ contains a factor $(\alpha_P-1)^2$ which vanishes at $\alpha_P(t=0)=1$, because the exchange of a spin one Pomeranchuk pole cannot convert a spin 1/2 proton into a spin 5/2 resonance at $t=0$.[15] Angular momentum conservation does allow spin 1/2 or 3/2 states to be reached, but only the former has the coherence with the initial state[16] that one expects in the usual descriptions[9,10] of diffraction dissociation. On these grounds we believe (contrary to reference 8) that $F_{NN_2^*P}(t=0)$ will also turn out to be small, though we have no detailed proof for this at the present time.

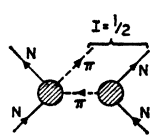

FIG. 5. Peripheral model used by Drell and Hiida (reference 17) to explain the peaking in N-N inelastic scattering.

A word of caution must be inserted concerning the use of (3.2) at small t. This formula is the result of an expansion of $P_\alpha(\cos\theta_t)$ at large $\cos\theta_t$, as explained in the previous section. There we treated equal masses and found for that case, that $\cos\theta_t$ is large when s is large and $0°\leqq\theta_s<90°$. In isobar production we deal with two masses, M for the proton and M^* for the isobar, and find

$$\cos\theta_t=\frac{-t^{1/2}(s+t/2-3M^2/2-M^{*2}/2)}{\{(t/4-M^2)[t-(M^*-M)^2][t-(M^*+M)^2]\}^{1/2}}. \tag{3.3}$$

In forward scattering ($\theta_s=0$), t does not quite vanish because of the mass change, but has the value

$$t\sim -M^2(M^{*2}-M^2)^2/s^2$$

at high energies. There $\cos\theta_t$ has magnitude one at $\theta_s=0$; $|t|$ must be increased by a substantial factor before $\cos\theta_t$ becomes large and (3.2) can be applied. For the typical values $M^{*2}\sim 2.5$ (BeV)^2 and $E_{\rm lab}\sim 15$ BeV, for example, one can begin to apply (3.2) at $|t|\gtrsim 0.05$ (BeV)^2.

It is of interest to compare our approach with the "peripheral model" which has also been invoked to treat reaction (3.1). Drell and Hiida[17] suggested the diagram of Fig. 5 in the spirit of the peripheral model. Since the πN state on the right side of the diagram has $I=1/2$, the $\pi N\rightarrow\pi N$ vertex on the left side can proceed by diffraction scattering, which is put in phenomenologically. By adding final-state interactions for the $I=1/2$ πN states, Drell and Hiida can single out the isobar energies as playing a dominant role. Thus, the Drell-Hiida model has features in common with ours, but with somewhat different emphasis. Our model emphasizes the role of $\alpha_P(t)$ in giving diffraction-like behavior. In the Drell-Hiida model, a specific contribution to the N-N^*-Pomeranchuk vertex is stressed.[17a]

[15] M. Gell-Mann (private communication) states that the similar factor α_P^2, which appears in production of either $I=1/2$ isobar,[8] just cancels the ghost at $\alpha_P=0$ leaving a finite, nonzero amplitude at that point.

[16] We are indebted to Dr. J. Levinger for a helpful discussion of this point.

[17] S. D. Drell and K. Hiida, Phys. Rev. Letters 7, 199 (1961).

[17a] *Note added in proof.* In a recent study [Imperial College preprint "High Energy Quasi-Elastic Proton-Proton Scattering and Final State Interaction"], M. M. Islam considers in detail the phenomenological $\pi N\rightarrow\pi N$ vertex and the final state interactions. He concludes that the Drell-Hiids diagram does not explain the data.

TABLE I. Calculation of the functions $F_{NNP}(t)$ and $F_{NN_2^*P}(t)$.

	Elastic scattering					$T=1/2$, $J=3/2$ isobar production				
s (BeV^2)	$-t$ (BeV^2)	$d\sigma/d\Omega_l$ (exptl) (mb/sr)	F_{NNP} for $\alpha_P'(0)=$ $1/60m_\pi^2$	$1/52m_\pi^2$	$1/45m_\pi^2$	$-t$ (BeV^2)	$d\sigma/d\Omega_l$ (exptl) (mb/sr)	$F_{NN_2^*P}$ for $\alpha_P'(0)=$ $1/60m_\pi^2$	$1/52m_\pi^2$	$1/45m_\pi^2$
26.2	0.524	82	12.6	19.7	39.7	0.495	5	0.7	1.0	2.1
31.55	0.783	20	8.3	17.9	38.0	0.749	4.2	2.0	3.1	6.6
35.15	0.978	7.6	7.7	20.9	45.5	0.936	1.75	1.7	5.3	8.1
38.8	1.206	1.28	4.7	14.3	37.2	1.161	0.42	1.2	3.6	8.6
42.85	1.474	0.40	6.3	13.3	36.9	1.42	0.12	1.1	3.7	9.6
50.6	2.071	0.14	6.6	17.6	48.1	2.01	0.05	1.4	5.8	15.7

At high energies the elastic proton-proton cross section obeys a relation similar to (3.2), with F_{NN^*P} replaced by F_{NNP}. For momentum transfers in the range of BeV the recent elastic data from CERN on high-energy p-p collisions[18] provide the possibility of a relatively unique determination of the Pomeranchuk-Regge trajectory. Within experimental error, we find

$$\alpha_P(t)=\alpha_P't+1,\quad -1.2\ \mathrm{BeV}^2\leq t<0, \qquad (3.4)$$

where the slope α_P' can take values

$$1/45m_\pi^2\geqq\alpha_P'\geqq 1/60m_\pi^2.$$

For more negative t the same data indicate a smaller slope.

A good check of our theoretical expression (3.2) would require measurements at fixed t and different high s, to determine $\alpha(t)$; once $\alpha(t)$ was known $F_{NN^*P}(t)$ would follow from the variation of $(d\sigma/dt)_{NN\to NN^*}$ with t. The Regge power α should be the same for elastic scattering and diffraction dissociation. The data are insufficient to carry out this check at present, so we shall follow the less ambitious program of assuming the same $\alpha(t)$ for both processes and showing that $F_{NNP}(t)$ and $F_{NN^*P}(t)$ vary only slowly with t, most of the variation of both elastic and inelastic scattering in this region being associated with the Regge power.

The existing data on reaction (3.1) at high energies are derived from experiments[19] in which fast outgoing protons of varying energies are detected at fixed angle for a given incoming proton energy. An elastic peak is observed, followed by a continuum of lower outgoing proton energies associated with inelastic scattering. The continuum shows two distinct bumps at the energies expected if the target is converted into one or the other of the $I=1/2$ π-N isobars; one takes the difference between the bumps and the continuous background as the cross section for these reactions. In this way Cocconi *et al.*[19] obtained the laboratory differential cross sections $d\sigma/d\Omega_l$, which we reproduce in Table I, for elastic p-p scattering and for production of the $I=1/2$ isobars. The two isobars overlap to some extent and are rather hard to separate, but in a rough sense they contribute constant proportions—we have taken 50% for each—to the isobar peaks in the region investigated in reference 19. Using these data and the Pomeranchuk-Regge trajectories of Fig. 6 we have calculated the functions F_{NNP} and $F_{NN_2^*P}$ of Table I. Our conclusions for their variation in the range -2 BeV$^2<t<-0.5$ BeV^2 can be formulated as follows:

(a) For the Pomeranchuk-Regge trajectory with slope $\alpha_P'(0)=1/60m_\pi^2$ (Fig. 6), $F_{NNP}(t)$ decreases with increasing $|t|$ by a factor of 2, while $F_{NN_2^*P}(t)$ increases by 2.

(b) Within experimental error, the trajectories with $\alpha_P'(0)=1/52m_\pi^2$ and $1/45m_\pi^2$ (Fig. 6) are consistent with $F_{NNP}(t)\simeq$const and $F_{NN_2^*P}(t)$ increasing by a factor of 3.5 and 5, respectively.

(c) The proton momentum spectra of Fig. 2 of reference 19 strongly suggest that for the same trajectories the variation of the function $F_{NN_3^*P}(t)$, corresponding to production of the $T=1/2$ $F_{5/2}$ isobar, is of the same order of magnitude.

Clearly, in all these cases, the variation with t of F_{NNP}, $F_{NN_2^*P}$, and $F_{NN_3^*P}$ is much slower than the corresponding variation of the differential elastic and isobar production cross sections. This behavior is not surprising

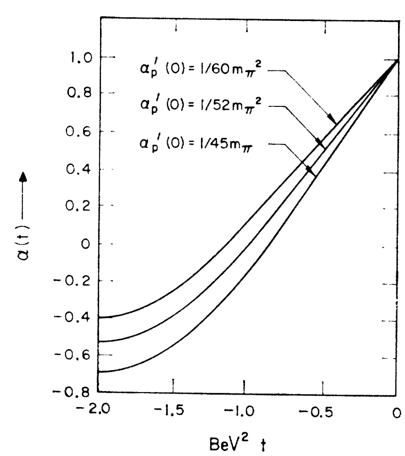

FIG. 6. The Pomeranchuk-Regge trajectories used in the calculation of Sec. III.

[18] E. Taylor (private communication).

[19] G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters 7, 450 (1961).

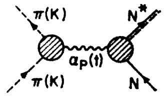

FIG. 7. Diagrams for the exchange of the leading Regge term in $\pi(K)+N\rightarrow\pi(K)+N^*$.

since there is no known mechanism for making $F(t)$ fall off so rapidly, whereas the factor $(s/2M^2)^{2\alpha(t)-2}$ naturally provides an exponential falloff in t similar to the observations.

What variation of $F(t)$ does occur is consistent with the expectation that $F_{NN^*P}(t)$ becomes very small as $t\rightarrow 0$. In further support of this expectation, recent data from Brookhaven[20] at incoming proton kinetic energies 1.3 to 2.9 BeV and lower $|t|$ give a ratio of $I=1/2$ isobar production to elastic scattering which decreases as $|t|$ decreases (however, at such low energies other trajectories which have nothing to do with diffraction may be important).

As already emphasized by Drell and Hiida,[17] π-N and K-N reactions should exhibit the same "nearly elastic" bumps as N-N reactions. The mechanism is again exchange of the Pomeranchuk trajectory (Fig. 7). In addition to the position of the bumps, their magnitude can be predicted because the residues $\beta(t)$ of Eq. (2.1), and therefore $F(t)$, can be factored[21,22] into products of vertices. For example, the amplitude for $\pi+N\rightarrow\pi+N^*$ factors into a (NN^* Pomeranchuk) vertex and a ($\pi\pi$ Pomeranchuk) vertex, leading to

$$F_{\pi N^*P}(t)=G_{\pi\pi P}(t)G_{NN^*P}(t). \tag{3.5}$$

In the same way we have

$$\begin{aligned} &F_{\pi NP}(t)=G_{\pi\pi P}(t)G_{NNP}(t);\quad F_{NNP}(t)=G_{NNP}{}^2(t);\\ &F_{NN^*P}(t)=G_{NNP}(t)G_{NN^*P}(t), \end{aligned} \tag{3.6}$$

etc., which imply for the spin-averaged differential cross sections[23]

$$\frac{[d\sigma(s,t)/dt]_{\pi N\rightarrow\pi N^*}}{[d\sigma(s,t)/dt]_{NN\rightarrow NN^*}}=\frac{[d\sigma(s,t)/dt]_{\pi N\rightarrow\pi N}}{[d\sigma(s,t)/dt]_{NN\rightarrow NN}}, \tag{3.7}$$

and similar relations for K-N scattering.

IV. PRODUCTION OF THE 3-3 PION-NUCLEON ISOBAR

Consider the reaction

$$N+N\rightarrow N+N_1^*, \tag{4.1}$$

where N_1^* is the 3-3 isobar. The reaction requires exchange of isotopic spin 1. Therefore, the Pomeranchuk trajectory cannot contribute, the reaction has a lower Regge power than production of the $I=1/2$ isobars, and it falls off more rapidly with increasing energy. Among recognized Regge trajectories, those responsible for the ρ and π mesons provide the highest powers associated with isotopic spin 1. It is believed[3] that $\alpha_\rho>\alpha_\pi$ at small $|t|$. But the coefficient $F_{NN_1^*\pi}(t)$ for exchange of the pion Regge term varies rapidly at small $|t|$ and plays an important role. In conventional terms, the pion pole at $t=m_\pi^2$ is much nearer the physical region than the singularities associated with ρ exchange. In terms of Regge poles $\alpha_\pi(t=m_\pi^2)=0$ and $\alpha_\pi(t)$ varies with t, so the factor $[\sin\pi\alpha(t)]^{-1}$ of (2.1) behaves like $[\pi(t-m_\pi^2)d\alpha_\pi/dt]^{-1}$ at small $|t|$, providing the same large factor as the conventional pion pole term. Furthermore, the pion-nucleon coupling constant is much larger than the effective ρ-nucleon coupling[24] or, in other words, the residue at the pion pole is greater than the residue at the ρ pole. For these reasons the pion trajectory is expected to dominate the ρ trajectory at small $|t|$ up to quite high energy.[25]

It is again instructive to compare the present theory with the peripheral model. Selleri[13] has treated reaction (4.1) in terms of one-pion exchange. One-pion exchange differs from our formalism in two respects:

(a) It amounts to treating the pion as an elementary particle with constant exponent $\alpha(t)=0$ in (4.5).[8]

(b) Only the pion pole contribution to $F_{NN_1^*}(t)$ is included.

At small $|t|$ the pion pole is nearby, justifying (b), and the Regge exponent α_π is still nearly zero, making it hard to distinguish a composite pion with variable $\alpha_\pi(t)$ from an elementary pion with $\alpha_\pi=0$. If s and $|t|$ are not too large, then, we reproduce Selleri's predictions. Distinctions appear at large s, where with $\alpha_\pi(t)<0$ the Regge factor $(s/2M^2)^{2\alpha_\pi(t)-2}$ leads to a more rapid falloff with increasing energy than does one-pion exchange. At large $|t|$ it is of course expected that the peripheral model gives an incomplete description of $F_{NN_1^*}(t)$, but it is still of great interest to see whether the data are consistent with constant $\alpha=0$.

There is recent data from Brookhaven[20] for inelastic p-p scattering at incoming proton kinetic energies 2.1 and 2.9 BeV and various laboratory scattering angles from 2.72° to 17.75°; in all these data the production of the 3-3 isobar is exhibited very clearly. At higher energies (>12 BeV) the inelastic p-p scattering data exhibit no indication of 3-3 isobar production. The experiment of Cocconi *et al.*[19] establishes an upper limit on the 3-3 isobar cross section at these energies:

$$(d\sigma/d\Omega_l)_{NN\rightarrow NN_1^*}\lesssim 0.05(d\sigma/d\Omega_l)_{NN\rightarrow NN}. \tag{4.2}$$

The large 3-3 production below 3 BeV combined with

[20] M. A. R. Kemp, G. B. Chadwick, G. B. Collins, P. T. Duke, T. Fujii, N. C. Hien, and F. Turkot, Bull. Am. Phys. Soc. 7, 46 (1962) and private communications from T. Fujii and F. Turkot.
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[24] This follows from analysis of nucleon electromagnetic form factors and pion-nucleon scattering, as given for example by J. Bowcock, W. N. Cottingham, and D. Lurié, Nuovo Cimento 16, 918 (1960) and Phys. Rev. Letters 5, 386 (1960).
[25] The cross section for pion exchange contains the familiar factor t due to the pseudoscalarity of the pion, as Dr. Y. Hara reminded us. However, this factor probably reduces pion exchange to the magnitude of ρ exchange only in the small region $|t|\ll m_\pi^2$.

the 5% limit above 12 BeV indicate that the leading trajectory here has a lower spin than the Pomeranchuk trajectory, but they are consistent with either variable $\alpha_\pi(t)$ or constant $\alpha_\pi=0$. Experiments which trace in detail how 3-3 production disappears as the kinetic energy is raised above 3 BeV may prove helpful.

Of course, $\pi+N\rightarrow\pi+N_1^*$ and $K+N\rightarrow K+N_1^*$ cannot proceed by one-pion exchange. If our interpretation that the pion trajectory dominates $N+N\rightarrow N+N_1^*$ is correct, 3-3 isobar production should be less prominent in pion- and kaon-induced reactions of several BeV.

V. PRODUCTION OF ENERGETIC PIONS

In plans for proton accelerators present and future, knowledge of the type of secondary pion beam which can be extracted is important. Collisions of cosmic-ray protons with nuclei provide an estimate of the number of secondary pions, average energy of secondaries, distribution of transverse momenta, and so forth, in advance of the construction of an accelerator, but they do not provide reliable information on rare events, such as the emission of pion secondaries carrying most of the original cosmic-ray energy. Yet the high-energy tail of emitted pions is crucial for determining whether a proton accelerator produces a high-energy pion beam suitable for inducing πN reactions, decaying into energetic neutrinos, etc.

Production of $I=1/2$ πN isobars from the reaction $p+p\rightarrow p+N^*$ (Fig. 4) can be helpful in filling this gap in the cosmic-ray information. The isobars emphasized in Sec. III were produced from the target protons and emerged with low laboratory energy. We now want to emphasize the isobars produced from the fast proton and emerging with high laboratory energy.[26] In Sec. III we have determined $F_{NN^*P}(t)$ and $\alpha_P(t)$. With these quantities known, and Eq. (3.2) to tell us how to scale up in energy, the rate of production of fast isobars and their decay pions is easily calculated. This is only one of many processes leading to fast pions, but is significant because of its association with the leading Pomeranchuk trajectory and because there are only two fast particles competing for the available energy in the final state.

Thus, we consider the decay of $I=1/2$ isobars with laboratory momentum $p^*\gg M^*$, where M^* is the isobar mass (= total energy in the c.m. system of the isobar). These isobars are produced at very small lab angles and are expected to emerge with very small momentum transfer ($|t|<0.5$ BeV^2). However, the determination of $F_{NN_2^*P}(t)$ in Sec. III was based on data at $|t|>0.5$ BeV^2 and therefore some extrapolation is required. At $t=-0.5$ BeV^2, Table I indicates that

$$F_{NN_2^*P}(t)/F_{NNP}(t)\cong 1/20 \tag{5.1}$$

(the third resonance gives a similar value), while we gave arguments in Sec. III why $F_{NN^*P}(t)$ becomes very small as $t\rightarrow 0$ for both resonances. For purposes of obtaining a rough estimate we use only the second resonance and take (5.1) over the whole region of small $|t|$. The actual t variation can be determined by experiments at small angles with present energies.

Let, then, E_π be the energy of the decay pion in the lab system and E_π^*, p_π^*, and θ^* the energy, momentum, and angle of the pion in the c.m. system of the isobar. Clearly

$$E_\pi=\gamma(E_\pi^*+vp_\pi^*\cos\theta^*), \tag{5.2}$$

where v is the velocity of the isobar in the laboratory and $\gamma\equiv(1-v^2)^{-1/2}$. The quantities p_π^* and E_π^* depend on the mass M^* only, while v (and γ) depend on M^* and the energy of the incoming proton. Since the reaction is strongly peaked forward, the dependence on momentum transfer to the isobar can be ignored.

For given M^* and s, the distribution of pions emitted by the fast isobar per unit energy E_π will be

$$\frac{d\sigma(s,t_m,M^*)}{dE_\pi}\propto\int_{t_0}^{t_m}dt\,F_{NN^*P}(t)\left(\frac{s}{2M^2}\right)^{2\alpha_P(t)-2}\frac{dn}{dE_\pi}, \tag{5.3}$$

where $|t_m|$ is the square of the maximum momentum transfer under detection and for high s:

$$t_0\cong -M^2(M^{*2}-M^2)^2/s^2; \tag{5.4}$$

dn/dE_π denotes the number of isobar decays per unit energy interval. For $|t_m|\gtrsim$ BeV^2 and for very high s

$$\frac{d\sigma(s,M^*)}{dE_\pi}\propto F_{NN^*P}\frac{dn/dE_\pi}{\ln(s/2M^2)}. \tag{5.5}$$

Finally, to calculate dn/dE_π we assume that the isobar decays isotropically in its own rest system. In view of (5.2)[27]:

$$\frac{dn}{dE_\pi}=\frac{dn}{d\cos\theta^*}\cdot\frac{d\cos\theta^*}{dE_\pi}=\frac{1}{2\gamma vp_\pi^*}. \tag{5.6}$$

For fixed s and M^* this result gives a uniform distribution of pions between $E_{\pi,\max}=\gamma(E_\pi^*+vp_\pi^*)$ and $E_{\pi,\min}=\gamma(E_\pi^*-vp_\pi^*)$.

The π-N isobar under consideration is characterized by a significant width. Therefore, in (5.3) and (5.5) we will receive contributions from a continuum of M^*. For fixed, very high s and $|t_m|\gtrsim$ BeV^2:

$$\frac{d\sigma(s)}{dE_\pi}\propto\frac{F_{NN_2^*P}(0)}{\ln(s/2M^2)}\int_{M_2^*}^{M_1^*}dM^*\,D(M^*)(vp_\pi^*\gamma)^{-1}, \tag{5.7}$$

[26] These isobars decay into pions and nucleons with a wide range of energies in the laboratory, so they do not produce easily visible narrow peaks of the type discussed in Sec. III. Some experimental evidence (protons with ~2/3 of the total energy in two-pronged events) for the fast isobars has been found by D. R. O. Morrison, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucleaires de Salcay, Seine-et-Oise, 1961).

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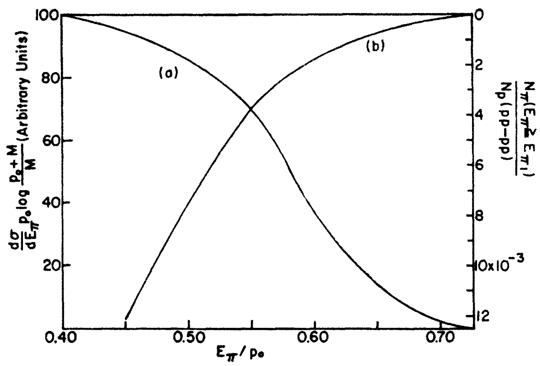

FIG. 8. Universal distributions for single-pion production from the decay of fast isobars in the reaction $p+p \to p+N_2^*$ at very high proton energies: (a) differential distribution; (b) ratio of pions with lab energies greater than E_π/p_0 to elastically scattered protons.

where $D(M^*)$ stands for the mass distribution of the $I=1/2$ $D_{3/2}$ isobar. We determine $D(M^*)$ from π-N scattering data and take[28]:

$$M_1^*=1.20 \text{ BeV}, \quad M_2^*=1.80 \text{ BeV}.$$

The shape of $D(M^*)$ used and the limits M_1^*, M_2^* are, within experimental error, in agreement with the data of reference 19. In this way we have calculated the differential and integral distributions of Fig. 8.

The following remarks can be made:

(a) Since the $I=1/2$ isobars produced in $p+p \to p+N^*$ have $I_z=+1/2$, 2/3 of the pions in Fig. 8 are π^+ and 1/3 are π^0.

(b) At each incident proton energy E_p, the ratio of π^+ with lab energies $E_\pi > \frac{2}{3}E_p$ to elastically scattered protons is about 1/3000, [if $F_{NN^*P}(0)$ really vanishes, the ratio will fall off logarithmically in s as the diffraction peak narrows]. At any lower E_π, pions are available in large numbers. The detailed numerical results are uncertain because $F_{NN^*P}(t)$ is not well known; the important point is that, in contrast to statistical considerations, the pion energies rise proportionally to the incident proton energy.

(c) With regard to getting the pions out of the accelerator, it should be noted that they will have transverse momenta of the same order as elastically scattered protons in the diffraction peak.

(d) Other reactions will produce many more low-energy pions, so Fig. 8 is of interest chiefly near the high-energy end of the spectrum.

(e) Fast neutrons are produced with the π^+'s. The upper energy limit of the neutrons is even greater than that of the pions.

(f) It is important to notice that other particles can be produced in the same way, with energies rising proportionally to the incident proton energy. For example, the reaction $p+p \to p+(KY)$, proceeding by exchange of the Pomeranchuk pole, will produce energetic K mesons and hyperons. The $F(t)$ functions for such reactions are expected to be rather small, but if the system happened to resonate in a state with the same quantum numbers as the nucleon $F(t)$ would not have to decrease as $t \to 0$.

VI. DISCUSSION

Diffraction has played an important role in our considerations. We would like to close upon a note of speculation, concerning the possible role of diffraction in determining basic properties of strong interactions.

At high energies the elastic-scattering amplitude is dominated by the Pomeranchuk Regge term

$$\frac{\beta(t)}{\sin\pi\alpha_P(t)}[P_{\alpha_P(t)}(-\cos\theta_t)+P_{\alpha_P(t)}(\cos\theta_t)] \quad (6.1)$$

(here the symmetric exchange term has been included). The imaginary part of (6.1) in the region of interest ($t \leq 0$, s large and positive), obtained[4] from properties of Legendre polynomials, is

$$-i\beta(t)P_{\alpha_P(t)}(\cos\theta_t). \quad (6.2)$$

In pure diffraction the forward-scattering amplitude is expected to be imaginary, and this can be accomplished by taking $\alpha_P(t=0)=1$. For then the imaginary part grows as $\cos\theta_t \sim s$ at large s, while in the real part the zero of $P_1(-\cos\theta_t)+P_1(\cos\theta_t)$ cancels the zero of $(\sin\pi\alpha_P)^{-1}$, leaving a residue which can be neglected. Thus $\alpha_P(0)$ gives a pure imaginary amplitude linear in s at $t=0$, corresponding to constant cross sections. It has already been remarked[3] that $\alpha=1$ is the highest power at $t \leq 0$ consistent with the Froissart limit—a limit derived from unitarity and the Mandelstam representation[29] which puts on a rigorous basis the plausible statement that exchange of objects with nonzero mass should not lead to cross sections increasing without bound as a power of the energy. We would like to remark that $\alpha=1$ is also the *lowest* power which makes (7.1) purely imaginary—in general, $P_\alpha \sim s^\alpha$ with a *complex* coefficient. Thus, we believe that the actual situation—$\alpha_P(0)$ is "as large as possible"—may be the only way in which diffraction can be expressed consistently by Regge poles.

A second characteristic of diffraction is that it represents coherence in scattering. Maximum coherence—i.e., the maximum value of $\alpha(0)$—occurs for exchange of the quantum numbers $S=0$, $I=0$, etc. The effect this is likely to have on the ordering of trajectories, favoring trajectories with low quantum numbers, has been described elsewhere.[2,3] Here we note that the exchanged quantum numbers would have to vanish for maximum coherence at $t=0$, *even if* these quantum numbers were not conserved. It is not easy to see how the association of definite quantum numbers with the Pomeranchuk trajectory could be assured if these quantum numbers were not conserved at all t and s, so perhaps the conser-

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vation laws are also related to the exigencies of expressing diffraction consistently by Regge poles.

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Overlapping Final-State Interactions

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A simple model for a production process with a three-body final state is constructed. Two pairs of particles in the final state are assumed to interact, perhaps strongly, and the structure of the resulting interaction is examined. The dynamics of the model is defined by a dispersion relation, which yields a singular integral equation. This equation is solved in the case of large mass of the particle which is common to the two interacting pairs.

I. INTRODUCTION

THE effects of interactions in the final state on inelastic processes have been appreciated for a long time.[1] For example, such interactions form the basis of much of the current experimental search for resonances. They also are the foundation of various theoretical studies, such as the isobar model[2] and its modifications.[3,4]

If there are three or more particles in the final state, then several pairs can interact. In many of the theoretical analyses of production processes the assumption has been made that the interaction of only one pair is important (e.g., references 2–4)—the remaining final-state interactions may be inherently weak, or else weak in the relevant kinematical regions. (This assumption may be justified in certain other cases as well.[1])

There have also been discussions in which several pairs were assumed to interact. Some of these discussions deal with the regions near the production threshold, where the kinetic energies are small, and the final-state interactions assumed weak.[5–8] The solutions then were based, essentially, on a perturbation expansion. (This approach was also considered[5,8] for the process $K \rightarrow 3\pi$.) Other discussions, in which several interacting pairs were considered, relate to the statistical model[9] and to the isoscalar nucleon structure.[10,11] In these discussions the form of the amplitude was assumed rather than derived. As a final example, in which several interacting pairs are considered, we mention production in the Lee model. For this problem an exact solution has been obtained.[12]

In this paper we examine a very limited problem involving overlapping final-state interactions of particles with finite masses. By *overlapping final-state interactions* we mean the two-body interactions of two pairs of particles, if the two pairs have one particle in common. Thus, for a three-body final state, two interactions are overlapping whenever they operate simultaneously, whereas in the case of four or more particles one can have simultaneous interactions of two independent pairs.

Our approach is based on integral equations which

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