Department of the Navy
Bureau of Ordnance
Contract NOrd-16200
Task 1

AN EXPERIMENTAL DETERMINATION OF DYNAMIC COEFFICIENTS FOR THE BASIC FINNER MISSILE BY MEANS OF THE TRANSLATIONAL DYNAMIC BALANCE

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INTRODUCTION

Report number E-73.3, published by this Laboratory in June 1957, (Ref. 1) presents certain dynamic coefficients for a model of the Basic Finner Missile (Fig. 1) which had been measured on the angular dynamic balance in the High Speed Water Tunnel at this Laboratory. Several of the desired coefficients, specifically

Y_r'	coefficient of rotary force derivative
$Y_{\dot{v}}'$	virtual inertia coefficient (lateral acceleration)
N_r'	coefficient of rotary moment derivative
$N_{\dot{v}}'$	virtual moment of inertia coefficient (lateral acceleration)

remained undetermined at that time.

By employing the translational dynamic balance and its associated internal moment balance, it had been hoped that the missing values for these coefficients would be supplied. Only partial success has been achieved, insofar as numerical results are concerned, at contract expiration time.

The coefficient of static force derivative, Y_v', and the virtual inertia coefficient, $Y_{\dot{v}}'$, have been measured as part of this investigation. These coefficients have been designated Z_w' and $Z_{\dot{w}}'$ in this report to comply with the new direction of model motion with respect to the tunnel coordinate system. Since the first of these, Z_w', had already been determined in the angular dynamic measurements, only the presentation of a value for $Z_{\dot{w}}'$ is new. This coefficient had appeared in linear combination with the coefficient of rotary force derivative; hence the latter important quantity also is now uniquely determined.

In addition to the force reactions, the moments arising from transverse velocity and acceleration components were also measured, but under conditions of undetermined deflection of the model-spindle assembly. For this reason the moment coefficients have not been presented here, nor have the experimental procedures used to obtain them been included. Instead, a detailed discussion of both the apparatus and the experimental procedures has been planned for reference 3.

APPARATUS

The hydrodynamic measurements were made using the translational dynamic balance (Figs. 2, 3) mounted in the working section of the High Speed Water Tunnel. This balance is shown schematically in Figure 4. From the diagram it can be seen that the balance assembly consists of two major components, the driving platform and the model assembly, the two being coupled by a linear spring system of known and controllable stiffness. A method is provided for measuring the amplitudes of both the driving platform and the model assembly, as well as for determining the phase angle relating the motion of one to that of the other. The effective mass and damping of parts of the system, as revealed by the observed steady-state motion, can then be related to the desired hydrodynamic coefficients.

In actual practice, the motion of the driving platform is imparted to it by means of a motor-driven cam. A large, geared flywheel assures freedom from short period speed variations, while long period variations are controlled by varying the input voltage to the driving motor. This voltage is supplied by a saturable reactor controlled by the difference between the output voltage of a d-c tachometer generator mounted on the flywheel shaft and that of an adjustable reference source. At one end of the shaft is mounted a 2-pole a-c tachometer generator providing a sinusoidal output voltage accurately proportional to the angular velocity of the shaft. The phase of the output voltage can be controlled by rotating the outer case of the generator to any desired position. This latter assembly was originally installed to provide a controllable voltage against which to compare the output of a strain gage moment balance located within the model, but was subsequently used as a supplementary method of analyzing the motions of the driving and model platforms.

At the other end of the main cam shaft is located the contactor assembly. It consists of a secondary cam which closes an electrical circuit at any desired angular position of the main cam. This angle is then displayed on a mechanical counter face for the observer to record. The contactor assembly is part of the trigger circuit for a repeating electronic flash lamp which illuminates a pair of targets, one carried by the driving platform and the other by the model platform. A white line on the target card is lined up with the cross-hairs of a cathetometer by the observer making the run. The contactor position is recorded, as are the cathetometer readings showing the instantaneous positions of both the driver and model. Readings of this type are taken at 24 equal angular intervals to permit a Fourier analysis to be made of the motions of both elements.

An alternate and somewhat faster method employs variable linear differential transformers (Schaevitz gages) to measure the amplitudes and phase angles of the two platforms. To do this, the output of the a-c tachometer generator is made equal and opposite to the a-c output of the Schaevitz gage on the platform being measured. The resulting error signal is then displayed along the vertical axis on the face of an oscilloscope tube, while the a-c generator signal alone is fed to the horizontal sweep. The resulting Lissajous figure can be used to obtain phase and amplitude null balance. Control of the output voltage is achieved by means of precision potentiometers, and control of the phase angle is effected by rotating the outer case of the a-c generator. A more complete description of the electronic equipment will be presented in reference 3.

The entire assembly is mounted on the High Speed Water Tunnel with the support spindle projecting into the tunnel water. By controlling the clearance space between the spindle and the hole through which it passes, it was possible to limit the leakage of the water from the tunnel. In addition, provision for air-pressurizing an annular space around the shaft reduced the leakage to about one gallon per hour when the pressure within the working section was held near the working value of one atmosphere, absolute.

THEORETICAL ANALYSIS

The hydrodynamic forces and moments acting on a submerged body are assumed to be equal to the sum of the separate reactions arising from displacement, velocity, and acceleration in each degree of freedom. For small, effective instantaneous angles of attack, a body executing longitudinal and transverse motion (along the z, or vertical body axis) is acted upon by total hydrodynamic force and moment reactions which can be written:

$$Z = Z_w w + Z_{\dot{w}} \dot{w} + Z_u u + Z_{\dot{u}} \dot{u} \tag{1}$$

$$M = M_w w + M_{\dot{w}} \dot{w} + M_u u + M_{\dot{u}} \dot{u} \tag{2}$$

where the subscripts indicate partial derivatives in accordance with the procedure *outlined* in reference 2, and where

Z normal component of hydrodynamic force (in the positive direction of the z fixed body axis)

M hydrodynamic pitching moment about the y-axis (positive in accordance with the right-hand screw rule)

$w = \dot{z}$ velocity of origin of body axis in the direction of the z (vertical) body axis, positive downward.

$\dot{w} = \ddot{z}$ acceleration of origin of body axis in the direction of the z (vertical) body axis, positive downward.

$u = \dot{x}$ velocity of origin of body axis in the direction of the x (longitudinal) body axis, positive forward.

$\dot{u} = \ddot{x}$ acceleration of origin of body axis in the direction of the x (longitudinal) body axis, positive forward.

It is to be noted that equations (1) and (2) contain no angular velocity terms, since the model was constrained to permit translation only.

Because the body possesses quasi rotational symmetry, longitudinal velocity and acceleration cannot contribute transverse reactions; therefore

$$Z = Z_w w + Z_{\dot{w}} \dot{w} \tag{3}$$

$$M = M_w w + M_{\dot{w}} \dot{w} \tag{4}$$

Only force reactions will be considered here. Rewriting equation (3) in terms of the displacement z,

$$Z = Z_w \dot{z} + Z_{\dot{w}} \ddot{z} \quad . \tag{5}$$

Newton's law, written for the missile alone and accounting for $Z_{(s)}$ the force exerted on the missile by the spindle, can be written as

$$Z_{(s)} + Z = m_b \ddot{z} \tag{6}$$

in the absence of spring or damping forces acting on the mass of the model, m_b. If it can be assumed that the hydrodynamic contribution to the model can be considered equivalent to the addition of a spring, dashpot, and mass to the existing model, then the following equation of motion can be formulated:

$$Z_{(s)} = (m_b + m_f)\ \ddot{z} + b\dot{z} + K_d\ z \tag{7}$$

where

$Z_{(s)}$ = force exerted by the spindle on the model

m_b = mass of model

m_f = apparent mass of model due to hydrodynamic reactions

$m_b + m_f = m$ total effective mass of model assembly

b = damping coefficient due to hydrodynamic reactions

K_d = rate of coupling spring = 857.04 lb/ft

k = effective spring coefficient due to hydrodynamic reactions.

That the latter quantity k equals zero is easily verified by disconnecting the coupling spring (Fig. 4) and manually displacing the model spindle. No detectable spring-like restoring force exists at any velocity; therefore

$$Z_{(s)} = (m_b + m_f)\ \ddot{z} + b\,\dot{z} \tag{8}$$

Rewriting equations (6) and (8)

$$Z_{(s)} = m_b\ \ddot{z} - Z \tag{9}$$

$$Z_{(s)} = (m_b + m_f)\ \ddot{z} + b\,\dot{z} \tag{10}$$

Combining equations (9) and (10)

$$Z = - m_f\ \ddot{z} - b\,\dot{z} \tag{11}$$

Equating coefficients of like terms in equations (5) and (11) yields

$$Z_{\dot{w}} = - m_f \tag{12}$$

$$Z_w = - b\ . \tag{13}$$

These can be expressed in dimensionless form (see ref. 2)

$$Z_{\dot{w}} = Z_{\dot{w}}'\ (1/2\,\rho A\,d) \tag{14}$$

$$Z_w = Z_w'\ (1/2\,\rho A\,U) \tag{15}$$

where

ρ = mass density of the fluid

A = cross-sectional area (ℓ^2 is used in ref. 2)

d = diameter of missile (ℓ is used in ref. 2)

To obtain the steady-state solution, the equation of motion of the entire system is written in terms of the effective physical parameters introduced by the hydrodynamic reactions as well as those belonging to the mechanical system

$$m\,\ddot{z}_o + b\,\dot{z}_o + K_d(z_o - z_2) = 0 \tag{16}$$

where K_d is the spring constant of the coupling spring. Assume

$$z_o = A_o \sin \omega t \tag{17}$$

$$z_2 = A_2 \sin (\omega t + \delta) \tag{18}$$

δ being the phase angle between the driver and the model platform motions and A_o and A_2 the amplitudes of the model and driving platforms, respectively. Substituting

$$\dot{z}_o = A_o\,\omega \cos \omega t \tag{19}$$

$$\ddot{z}_o = A_o\,\omega^2 \sin \omega t \tag{20}$$

into the equations of motion and equating coefficients of like terms

$$- m A_o \omega^2 + K_d A_o = A_2 \cos \delta \tag{21}$$

$$b A_o \omega = K_d A_2 \sin \delta \tag{22}$$

or

$$m = \frac{K_d}{\omega^2}\left[1 - \frac{A_2}{A_o}\cos \delta\right] \tag{23}$$

$$b = \frac{K_d}{\omega}\left[\frac{A_2}{A_o}\sin \delta\right] \tag{24}$$

DATA SUMMARY

The following tables show the data used to compute the desired coefficients.

I. Amplitude Ratio $\frac{A_2}{A_o}$

Frequency - cycles per second	Velocity - feet per second				
	In Air	0	5	10	15
3.0	.928	.912	.896	.908	.911
3.5	.912	.875	.872	.870	.892
4.0	.876	.834	.827	.832	.847
4.5	.848	.775	.786	.783	.900
5.0	.791	.713	.722	.729	.757
6.0	.704	.589	.595	.605	.630
6.5	.644	.538	.524	.541	.564
7.0	.594	.503	.475	.488	.503

II. Phase Angle $\delta = \delta_2 - \delta_0$, in degrees

Frequency - cycles per second	Velocity - feet per second				
	In Air	0	5	10	15
3.0	0.4	0.8	2.2	4.0	6.3
3.5	0.4	0.8	2.8	4.7	7.4
4.0	0.6	0.9	3.2	5.7	9.9
4.5	0.5	1.0	3.7	7.0	11.7
5.0	0.5	1.4	4.3	7.0	11.7
6.0	0.6	2.3	5.6	10.1	16.5
6.5	0.7	3.4	6.6	12.6	19.6
7.0	0.8	5.0	8.1	15.9	23.7

SUMMARY CF RESULTS

The quantities listed below are the arithmetic mean values of the coefficients for frequencies ranging from 3 to 7 cycles per second, and velocities from 0 to 15 feet per second. Because of the limited range of velocities tested (due to structural limitations of the model support system) no consistent dependence of the coefficients on either velocity or frequency could be established:

(1) $$Z_w' = \frac{Z_w}{1/2\, \rho A U} = -13.5$$

(2) $$Z_{\dot{w}}' = \frac{Z_{\dot{w}}}{1/2\, \rho D A} = -17.0 \; .$$

Further experimental work would be required to detect the effect of scale on the values of these coefficients. It should be noted here that the sign of the coefficient $Z_{\dot{w}}'$ was incorrectly reported as positive in reference 1.

REFERENCES

1. Kiceniuk, Taras, "An Experimental Determination of Dynamic Coefficients for the Basic Finner Missile by Means of the Angular Dynamic Balance", California Institute of Technology, Hydrodynamics Laboratory Report No. E-73.3, June 1957.

2. "Nomenclature for Treating the Motion of a Submerged Body through a Fluid", The Society of Naval Architects and Marine Engineers, New York, Technical and Research Bulletin No. 1-5.

3. Kiceniuk, Taras, and Shapiro, H., "An Experimental Method for Determining the Dynamic Coefficients of Submerged Bodies", California Institute of Technology, Hydrodynamics Laboratory Report No. E-35.5. (To be published.)

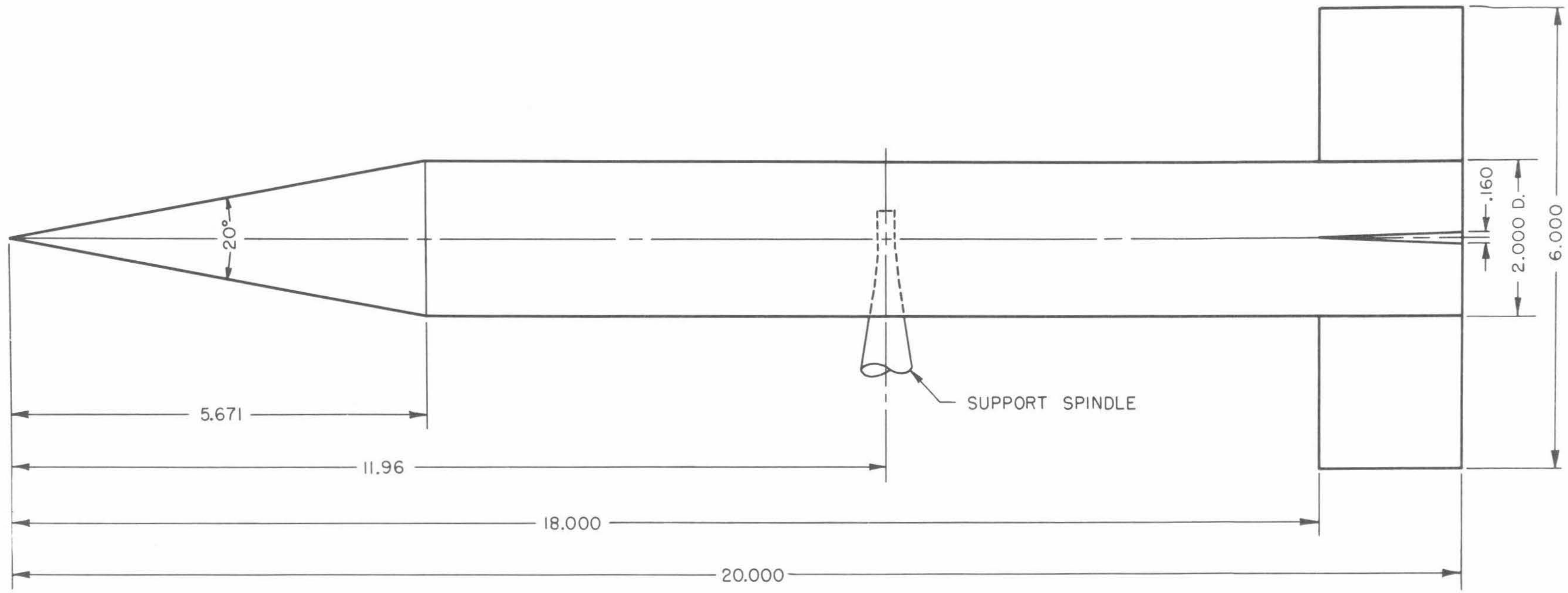


Fig. 1. Important geometrical dimensions of the Basic Finner Missile.

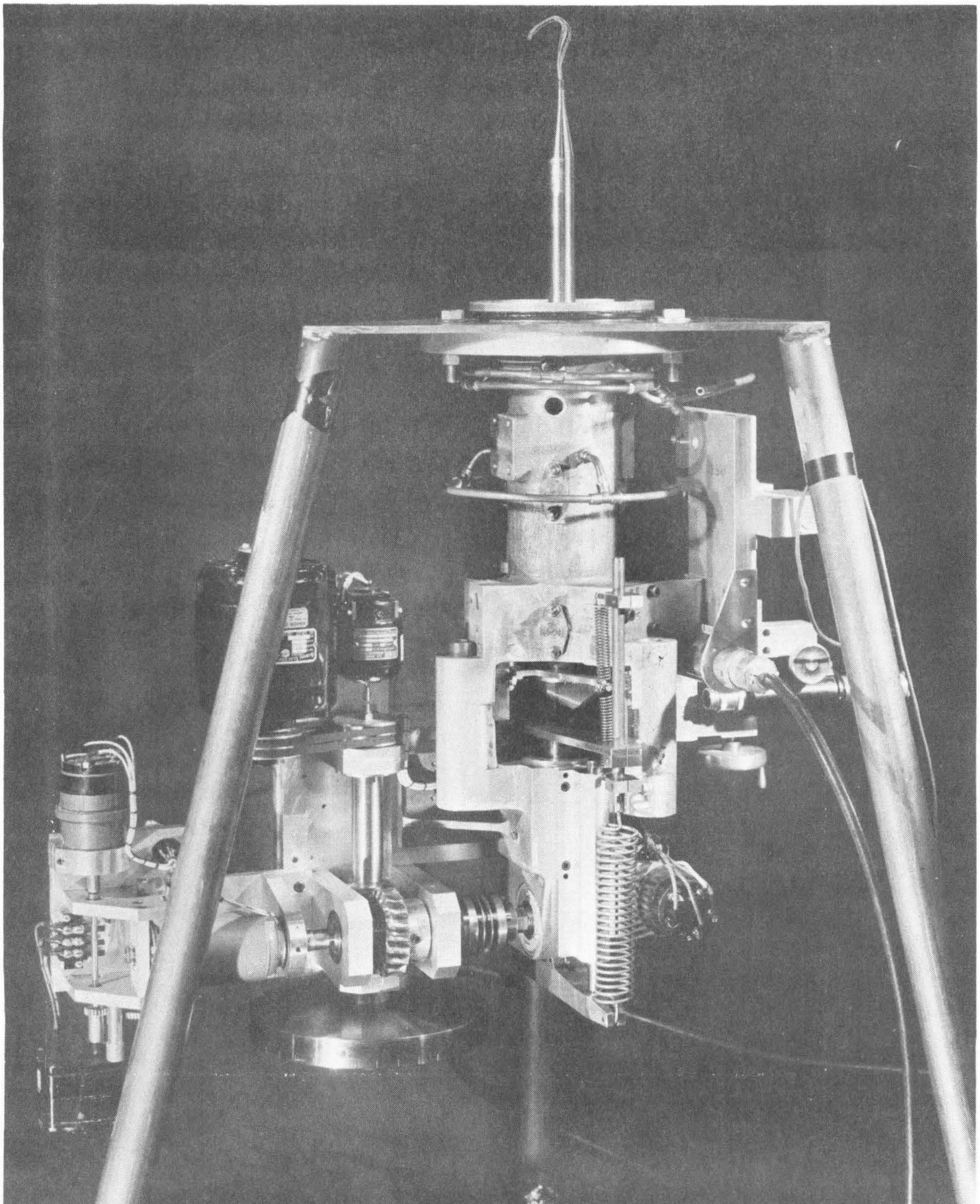


Fig. 2. Translational dynamic balance on calibrating stand.

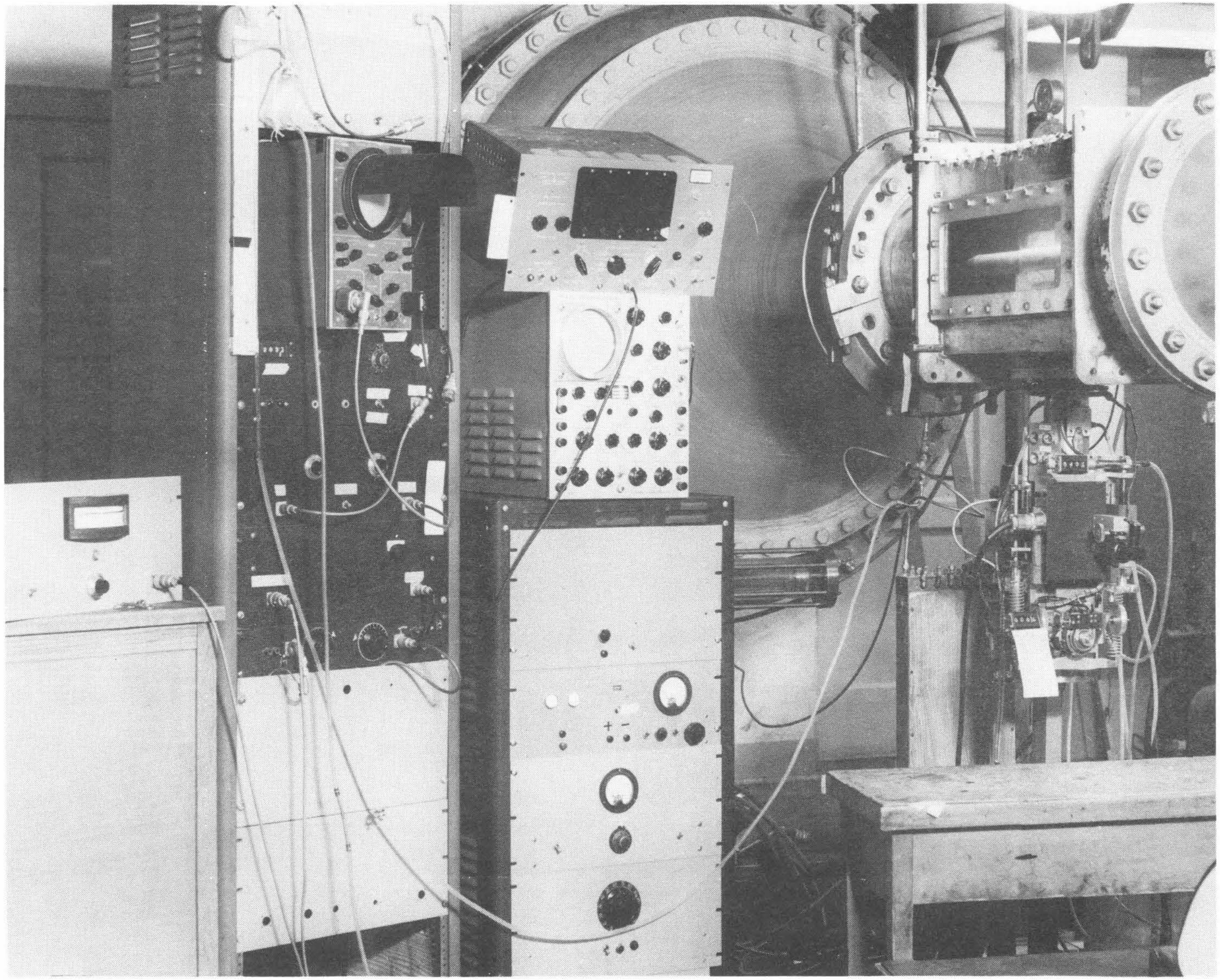
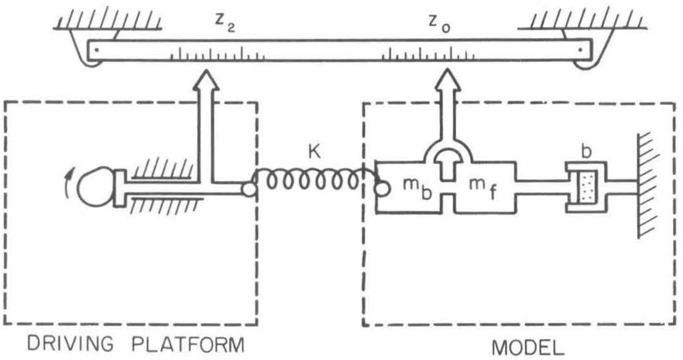


Fig. 3. Translational dynamic balance in place at the working section of the High Speed Water Tunnel.



K	DRIVING SPRING RATE
m_b	MASS OF MODEL
m_f	VIRTUAL MASS
b	EFFECTIVE DAMPING RATE DUE TO HYDRODYNAMIC REACTIONS
z_2, z_0	DISPLACEMENT OF PLATFORM AND MODEL
A_2, A_0	MAXIMUM AMPLITUDE OF PLATFORM AND MODEL
δ_2, δ_0	PHASE CONSTANT OF PLATFORM AND MODEL

$$z_2 = A_2 \text{ SIN } (\omega t + \delta_2)$$
$$z_0 = A_0 \text{ SIN } (\omega t + \delta_0)$$

Fig. 4. Schematic diagram of the translational dynamic balance.