value will be defined as the saturation boundary, and analogously the minimum negative value in the case of a downward overflow. The most easy way to detect an overflow within a RNS-filter consists in introducing a "redundant" modulus (m_{n+1}); the redundant residue number systems formed in this way, more generally, are used for error-detection, too (cf. [5]).

It may be shown (see, e.g., [4], [5]) that an overflow after an addition has occurred if and only if the MR-coeff. a_{n+1} belonging to the sum is nonzero. Since the sign of the represented integer may be derived immediately by means of this coefficient in symmetric RNS, as has been pointed out above, assigning a saturation-value with the correct sign consequently will be quite easy.

However, in conventional RNS there are some problems to distinguish between positive and negative overflows. These problems arise from the fact that in these "unsymmetric" RNS negative integers usually are represented in some complement-form, i.e., $-X$ will be replaced by $M-X$ for $X>0$ (with M defined in (1)). In other words, all integers $X \geqslant [M/2]$ are considered as negative numbers. Accordingly, in a redundant RNS all integers between $M_T := Mm_{n+1}$ and $M_T-[M/2]$ rank as negative numbers. Consequently they lie within the range that originally is provided for the overflow-region ($X \geqslant M$), i.e. a MRC erroneously indicates an overflow here; in a similar manner a sum Z satisfying $[M/2] \leqslant Z < M$ will not be detected as an overflow.

To avoid such errors, one either has to inspect more than one MR-coefficient or the value $[M/2]$ has to be added to the sum to be examined. (By this addition the negative integers are shifted cyclic into the range from 0 to $[M/2]-1$, the positive into the range from $[M/2]$ to $M-1$, respectively.) Then, in the latter case, not only every overflow will be detected correctly, but also it will be possible to decide, whether there is a positive or a negative overflow. After having carried out the MRC, obviously the value $[M/2]$ has to be subtracted (if no overflow has occurred), i.e., (upto) two extra additions are required here in comparison with the symmetric RNS.

IV. Scaling with Overflow Detection

For efficient use of RNS in recursive digital filters, it is desirable to combine overflow-detection and scaling to form one operation (e.g., at the filter output, after temporarily increasing the internal wordlength; see [4]). This is possible in symmetric RNS without additional effort compared with usual overflow-detection, because scaling may be performed at the same time within the same MRC. This combination may be attained in conventional RNS, too, but again at the expense of two extra additions:

> To achieve correct overflow-detection, one first has to add the value $[M/2]$ again. After that, a complete MRC is effected (including scaling by the factor S). Is the MR-coefficient a_{n+1} equals zero, i.e., if no overflow has occurred, then finally the value $[M/(2S)]$ has to be subtracted to get the correct scaled value.

Using this new method (which has not been given in [4]), it now results a rounded quotient as is always the case in symmetric RNS; there is no longer a truncated quotient as is usually the case in conventional RNS. In order to show that this procedure works, one only has to prove the following identity:

$$[(X+[M/2])/S]-[M/(2S)]=[X/S+1/2]. \qquad (4)$$

This holds for arbitrary integers X, M, and S satisfying the conditions that S divides M and that M and S are both even or both odd; the proof is quite easy.

V. Conclusion

A new definition of symmetric RNS has been presented. This special kind of RNS offers some advantages over conventional RNS in processing negative numbers. This is true in particular for scaling and overflow-detection (after additions). If scaling is used in recursive digital filters, then the rounding of quotients in symmetric RNS produces less noise than the truncation in conventional RNS. Moreover, a new method for overflow-detection with simultaneous scaling (which was first shown for symmetric RNS only in [4]) has been given here for conventional RNS, too.

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Corrections to "On Error-Spectrum Shaping in State-Space Digital Filters"[1]

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In the above paper,[1] the following corrections should be made:

1) In Eqn. (29),

$$\int_0^{2G_p} \cdots$$

should be replaced with

$$\int_0^{2\pi} \cdots.$$

2) In Eqn. (45), S_{jj} should be replaced with K_{jj}.

3) The sentence after Eqn. (7) should read: "Thus the kth noise source is shaped by a transfer function with a real zero at $z=a_{kk}^{(i)}$."

4) In Eqn. (1b) $x(n)$ should be replaced with $\mathbf{x}(n)$.

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