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by
Din-Yu Hsieh and Milton S. Plesset

Engineering Division
CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

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Abstract

In the first portion of this paper a review is given of the theory of the propagation of sound in a homogeneous gas taking into account the effect of heat conduction. This consideration is preliminary to the treatment in the second portion of the paper of the propagation of sound in a liquid with a homogeneous and isotropic distribution of gas bubbles. Again the effect of heat conduction is included. If f is the ratio of gas volume in the mixture to liquid volume, it is shown for the range of values of f of general interest that the acoustic condensations and rarefactions of the gaseous portion of the medium are essentially isothermal. It is also found that the attenuation of an acoustic disturbance by heat conduction is quite small.

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Introduction

The thermodynamic aspects of the propagation of sound in a liquid containing gas bubbles are considered in this paper. In particular, the velocity of sound is determined as a function of the liquid-gas mixture ratio. The gas bubbles are supposed to be small and uniformly distributed so that the resulting medium is homogeneous and isotropic. In the solution of the problem, the effect of heat conduction must be included since the present concern is with the thermodynamic behavior of the medium. The effect of viscosity is, however, not included for the sake of simplicity. The results of this analysis are not believed to be entirely new but it is hoped that the presentation is straightforward and clear. A brief discussion is also given of the propagation of sound in a uniform gas, and the results of this analysis have certainly been known for a long time. It is included here partly for completeness and partly to facilitate comparison with the behavior in the liquid with bubbles.

Propagation of Sound in a Gas

The dynamic behavior of a gas is determined by the conservation equations for mass, momentum, and energy together with the equation of state. The conservation equations are

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \underset{\sim}{v} + \underset{\sim}{v} \cdot \nabla \rho = 0 , \tag{1}$$

$$\frac{\partial \underset{\sim}{v}}{\partial t} + (\underset{\sim}{v} \cdot \nabla) \underset{\sim}{v} = - \frac{1}{\rho} \nabla P , \tag{2}$$

$$\kappa \Delta T = \rho C_V \left[\frac{\partial T}{\partial t} + \underset{\sim}{v} \cdot \nabla T \right] + P (\nabla \cdot \underset{\sim}{v}) \tag{3}$$

where ρ is the density, P the pressure, $\underset{\sim}{v}$ the fluid velocity,

T the (absolute) temperature, κ the thermal conductivity, and C_V the specific heat at constant volume for unit mass (1 gm) of the gas. Viscosity has been neglected. The gas will be assumed to be perfect so that its equation of state is

$$P = \frac{R}{m} \rho T , \tag{4}$$

where R is the universal gas constant and m is the gram molecular weight of the gas.

The linearization implied in acoustics is carried out by supposing that the disturbance from equilibrium is small. If one writes

$$P = P_o (1 + p) , \tag{5}$$

$$\rho = \rho_o (1 + \sigma) , \tag{6}$$

$$T = T_o (1 + \theta) , \tag{7}$$

then p, σ, θ, as well as $\underset{\sim}{v}$ will be treated as small quantities all of the same order, and the subscript zero denotes equilibrium quantities. The linearized equations are as follows:

$$\frac{\partial \sigma}{\partial t} = - \nabla \cdot \underset{\sim}{v} , \tag{1'}$$

$$\frac{\partial \underset{\sim}{v}}{\partial t} = - \frac{P_o}{\rho_o} \nabla p , \tag{2'}$$

$$\Delta \theta = \frac{1}{D} \frac{\partial \theta}{\partial t} + \frac{P_o}{\kappa T_o} \nabla \cdot \underset{\sim}{v} , \tag{3'}$$

and

$$p = \sigma + \theta , \tag{4'}$$

where D is the coefficient of thermal diffusion defined as $\kappa / \rho C_V$.

Elimination of $\underset{\sim}{v}$ from Eqs. (1') and (2') gives

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{P_o}{\rho_o} \quad \Delta p \ , \tag{8}$$

and Eq. (3') may be written

$$\Delta \theta = \frac{1}{D} \frac{\partial \theta}{\partial t} - \frac{P_o}{\kappa T_o} \frac{\partial \sigma}{\partial t} \tag{9}$$

by use of Eq. (1'). One also has from Eqs. (4') and (8)

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 \sigma}{\partial t^2} + \frac{\partial^2 \theta}{\partial t^2} = \frac{P_o}{\rho_o} \Delta p + \frac{\partial^2 \theta}{\partial t^2} \ , \tag{10}$$

and from Eq. (9)

$$\frac{\partial}{\partial t} (\Delta \theta) = \frac{1}{D} \frac{\partial^2 \theta}{\partial t^2} - \frac{P_o}{\kappa T_o} \frac{\partial^2 \sigma}{\partial t^2} \ ,$$

$$= \frac{1}{D} \left[\frac{\partial^2 p}{\partial t^2} - \frac{P_o}{\rho_o} \Delta p \right] - \frac{P_o}{\kappa T_o} \frac{P_o}{\rho_o} \Delta p \ ,$$

or

$$\frac{\partial}{\partial t} (\Delta \theta) = \frac{1}{D} \frac{\partial^2 p}{\partial t^2} - \frac{P_o}{\rho_o} \left[\frac{1}{D} + \frac{P_o}{\kappa T_o} \right] \Delta p \ . \tag{11}$$

Equation (11) now gives

$$\frac{\partial^2}{\partial t^2} (\Delta \theta) = \frac{1}{D} \frac{\partial}{\partial t} \frac{\partial^2 p}{\partial t^2} - \frac{P_o}{\rho_o} \left[\frac{1}{D} + \frac{P_o}{\kappa T_o} \right] \frac{\partial}{\partial t} \Delta p \ ,$$

and Eq. (10) gives

$$\frac{\partial^2}{\partial t^2} (\Delta \theta) = \frac{\partial^2}{\partial t^2} (\Delta p) - \frac{P_o}{\rho_o} \Delta(\Delta p)$$

so that these last two relations lead to the following:

$$\Delta \left[\frac{\partial^2 p}{\partial t^2} - \frac{P_o}{\rho_o} \Delta p \right] = \frac{1}{D} \frac{\partial}{\partial t} \left[\frac{\partial^2 p}{\partial t^2} - \frac{P_o}{\rho_o} \left(1 + \frac{D P_o}{\kappa T_o} \right) \Delta p \right] . \tag{12}$$

It may be noted that

$$1 + \frac{D P_o}{\kappa T_o} = 1 + \frac{R}{M C_V} = \gamma ,$$

where γ is the ratio of the specific heat at constant pressure, C_P, to the specific heat at constant volume C_V. Equation (12) may then be written

$$\Delta \left[\frac{\partial^2 p}{\partial t^2} - c_i^2 \Delta p \right] = \frac{1}{D} \frac{\partial}{\partial t} \left[\frac{\partial^2 p}{\partial t^2} - c_a^2 \Delta p \right] , \tag{12'}$$

where $c_i^2 = P_o/\rho_o$ so that c_i is the isothermal sound speed, and $c_a^2 = \gamma P_o/\rho_o$ so that c_a is the adiabatic sound speed.

It may be remarked that the isothermal limit is obtained if one says in Eq. (3') that

$$\left| \frac{1}{D} \frac{\partial \theta}{\partial t} \right| << \left| \frac{P_o}{\kappa T_o} \nabla \cdot \underset{\sim}{v} \right| = \left| \frac{P_o}{\kappa T_o} \frac{\partial \sigma}{\partial t} \right| ,$$

for then Eqs. (4') and (10) give

$$\frac{\partial^2 p}{\partial t^2} - c_i^2 \Delta p = 0 . \tag{13}$$

On the other hand the adiabatic limit is obtained if one has

$$\left| \Delta \theta \right| << \left| \frac{P_o}{\kappa T_o} \nabla \cdot \underset{\sim}{v} \right|$$

so that the heat conduction Eq. (9) becomes approximately

$$\frac{1}{D} \frac{\partial \theta}{\partial t} = - \frac{P_o}{\kappa T_o} \nabla \cdot \underset{\sim}{v} = \frac{P_o}{\kappa T_o} \frac{\partial \sigma}{\partial t} .$$

Equations (10) and (8) then gives

$$\frac{\partial^2 p}{\partial t^2} = \frac{P_o}{\rho_o} \Delta p + \frac{D P_o}{\kappa T_o} \frac{\partial^2 \sigma}{\partial t^2} = \frac{\gamma P_o}{\rho_o} \Delta p = c_a^2 \Delta p . \tag{14}$$

The complete equation (12') describes an acoustic disturbance which is not only propagated but is also attenuated. This attenuation may be illustrated very simply by considering a plane wave with

$$p = A e^{i(kx-\omega t)} = A e^{ik(x-ct)} . \tag{15}$$

Then one gets from Eq. (12')

$$-k^2(-\omega^2+k^2 c_i^2) = -\frac{i\omega}{D}(-\omega^2+k^2 c_a^2) , \tag{16}$$

or

$$k^4 - \frac{1}{c_i^2}\left(\omega^2 + \frac{i\omega c_a^2}{D}\right)k^2 + \frac{i\omega^3}{c_i^2 D} = 0 . \tag{16'}$$

Another form of this relation is

$$-k^2(-k^2c^2+k^2c_i^2) = -\frac{ikc}{D}(-k^2c^2+k^2c_a^2) , \tag{17}$$

or

$$c^3 + iDkc^2 - c_a^2 c - iDkc_i^2 = 0 . \tag{17'}$$

It is easy to see that the terms in D in Eq. (17') are negligible over a wide range of wave number k. For example, for air $D \cong 0.29\ \mathrm{cm}^2/\mathrm{sec}$ and both c_i and c_a are for air of the order of magnitude of 3×10^4 cm/sec. If one considers further a range of angular frequency from, say, 10/sec to 2×10^4/sec, so that k ranges from approximately $2 \times 10^{-3}\ \mathrm{cm}^{-1}$ to $4\ \mathrm{cm}^{-1}$, then Dk runs from about 5×10^{-4} cm/sec to the order of 1 cm/sec which is negligible compared with c_i or c_a. In the range for which Dk is small compared with c_i or c_a it follows at once from Eq. (17') that the propagation velocity c of the wave is very closely the adiabatic velocity c_a.

The result that the velocity of sound is determined by the adiabatic response of the gas has been known for a long time and also has been correctly attributed to the small value for the thermal diffusivity. A statement

is often made, however, which is quite misleading, of the following kind: "The condensations and rarefactions of the air concerned in the propagation of sound take place with such rapidity that the heat and cold produced have not time to pass away and therefore the thermodynamic processes are essentially adiabatic."[1] This statement is completely justified if radiation only is considered as the mechanism of heat transfer. This was the procedure used by Stokes.[2] When the effect of heat conduction is considered, as has been carried through here, it is evident from (17') that an increase in the rapidity of the process, i.e., an increase in frequency, tends to give the isothermal rather than the adiabatic behavior. Such a result is easily understood physically. An increase in frequency corresponds to a decrease in wavelength, and the region that undergoes condensation and rarefaction is measured by the wavelength. Although the time for heat to be conducted away becomes smaller with increasing frequency, the decrease in the extent of the affected region is a more important effect as is to be expected from the well-known behavior of the fundamental solutions of the heat equation.

The attenuation of the plane wave by heat conduction may be estimated in the following way. Since

$$c \cong c_a ,$$

one may write

$$k = \frac{\omega}{c_a} (1 + \eta) \tag{18}$$

where $|\eta| \ll 1$. If now the form of k given by (18) is substituted in

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[1] Rayleigh, "Theory of Sound", vol. II, p. 19, Dover Publications (New York, 1945).

[2] Op. cit., p. 24.

Eq. (16') and if terms $0(\eta^2)$ are neglected, then one obtains

$$\frac{\omega^4}{c_a^4}(1+4\eta) - \frac{\omega^2}{c_a^2 c_i^2}\left(\omega^2 + i\omega\frac{c_a^2}{D}\right)(1+2\eta) + \frac{i\omega^3}{c_i^2 D} = 0 . \tag{19}$$

Since $Dk \ll c$ so that $D\omega \ll c_a^2$ and $D\omega \ll c_i^2$, Eq. (19) may be approximated by

$$-\frac{2i\omega^3}{c_i^2 D}\eta + \frac{\omega^4}{c_a^2}\left(\frac{1}{c_a^2} - \frac{1}{c_i^2}\right) = 0 ,$$

or

$$\eta = \frac{iD\omega}{2c_a^4}\left(c_a^2 - c_i^2\right) . \tag{20}$$

In this approximation, therefore, the plane wave of Eq. (15) is given by

$$p = e^{-\frac{D\omega^2}{2c_a^5}(c_a^2 - c_i^2)x} \; e^{i\frac{\omega}{c_a}(x - c_a t)} .$$

The attenuation factor of Eq. (20) is the familiar one.[3] This attenuation is relatively slight. For example, in air the amplitude of a wave of frequency $\omega/2\pi$ (in $\sec^{-1}$) is decreased by the factor $1/e$ in the distance in units of the wavelength given by $10^{11}/6\omega$.

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[3] Op. cit., p. 322.

Propagation of Sound in a Liquid Containing Gas Bubbles

The medium to be studied is a liquid containing gas bubbles which are supposed to be uniformly distributed. The bubbles should be sufficiently small compared with the wavelength of the acoustic disturbance so that the medium may be considered homogeneous and isotropic. If one considers any element of this mixed fluid with total mass $M = M_1 + M_2$ and with total volume $V = V_1 + V_2$, where the subscript 1 refers to the liquid and the subscript 2 to the gas, then the density of the mixed fluid is

$$\rho = \frac{M_1 + M_2}{V_1 + V_2} . \tag{21}$$

The mass ratio is

$$g = \frac{M_2}{M_1} , \tag{22}$$

and the volume ratio is

$$f = \frac{V_2}{V_1} . \tag{23}$$

In ordinary mixtures $g << 1$ while f need not be small. The mixture density ρ may be expressed in terms of the liquid density ρ_1 and the gas density ρ_2 as follows:

$$\frac{1}{\rho} = \frac{1}{1+g} \left(\frac{1}{\rho_1} + \frac{g}{\rho_2} \right) . \tag{24}$$

The continuity equation and the equation of conservation of momentum for the mixture are given by Eqs. (1) and (2) above where ρ now denotes the mixture density. The gas is assumed to be perfect so that its equation of state is

$$P = \frac{R}{m_2} \rho_2 T \tag{25}$$

where m_2 is the gram molecular weight of the gas. The equation of state for the liquid will be left in the general form

$$P = \varphi(\rho_1, T) \quad . \tag{26}$$

The energy equation may be written in the form

$$\frac{M_1 C_1 + M_2 C_2}{V_1 + V_2} \frac{\partial T}{\partial t} = \kappa \Delta T + P(\nabla \cdot \underset{\sim}{v}) \quad , \tag{27}$$

where C_1 is the specific heat per unit mass of the liquid and C_2 is the specific heat at constant volume per unit mass of the gas. The constant κ is the average coefficient of heat conduction for the mixture which may be taken to be

$$\kappa = \frac{\kappa_1 V_1 + \kappa_2 V_2}{V_1 + V_2} = \frac{\kappa_1 + \kappa_2 f}{1 + f} \quad . \tag{28}$$

One may also define the effective average specific heat, C, for the mixture to be

$$C = \frac{1}{\rho} \frac{M_1 C_1 + M_2 C_2}{V_1 + V_2} = \frac{C_1 + g\, C_2}{1 + g} \quad . \tag{29}$$

The heat conduction equation (27) then may be written as

$$\rho\, C \frac{\partial T}{\partial t} = \kappa \Delta T + P(\nabla \cdot \underset{\sim}{v}) \quad . \tag{27'}$$

The linearization is carried out by writing

$$\rho = \rho_o(1 + \sigma) \;; \qquad \rho_1 = \rho_1^{(o)}(1 + \sigma_1) \;; \qquad \rho_2 = \rho_2^{(o)}(1 + \sigma_2) \;;$$

$$P = P_o(1 + p) \;; \qquad T = T_o(1 + \theta) \;;$$

where σ, σ_1, σ_2, p, and θ together with $\underset{\sim}{v}$ are to be treated as small quantities. One then gets from Eq. (24) the approximate relation

$$\frac{\sigma}{\rho_o} \cong \frac{g}{(1+g)\,\rho_2^{(o)}} \left(\frac{\sigma_1}{f} + \sigma_2 \right)$$

Now $\sigma_1 << \sigma_2$ and, since volume ratios f which are not small are of present interest, one may suppose that $|\sigma_1/f| << \sigma_2$ so that this further approximation may be made to give

$$\frac{\sigma}{\rho_o} \cong \frac{g}{(1+g)\,\rho_2^{(o)}}\,\sigma_2 = \frac{f}{(1+g)\,\rho_1^{(o)}}\,\sigma_2 \tag{30}$$

Equation (30) makes it possible to proceed without an explicit expression for the equation of state of the liquid.

The linearized equations which are obtained are the following:

$$\frac{\partial \underset{\sim}{v}}{\partial t} = -\frac{P_o}{\rho_o}\,\nabla p \;; \tag{31}$$

$$\frac{\partial \sigma}{\partial t} = -\nabla \cdot \underset{\sim}{v} \;; \tag{32}$$

$$p = \sigma_2 + \theta \cong \delta\sigma + \theta \;; \tag{33}$$

and

$$\Delta\theta = \frac{1}{D_m}\,\frac{\partial\theta}{\partial t} + \frac{P_o}{\kappa\, T_o}\,\nabla \cdot \underset{\sim}{v} \tag{34}$$

In Eq. (33), the parameter δ is given by

$$\delta = \frac{(1+g)\,\rho_2^{(o)}}{g\,\rho_o} \tag{35}$$

and in Eq. (34) D_m is the effective thermal diffusivity of the mixture:

$$D_m = \frac{\kappa}{\rho_o\, C} \tag{36}$$

As in the previous section these equations lead in a straightforward way to the following relation:

$$\Delta\left[\frac{\partial^2 p}{\partial t^2} - \frac{P_o\delta}{\rho_o}\Delta p\right] = \frac{1}{D_m}\frac{\partial}{\partial t}\left[\frac{\partial^2 p}{\partial t^2} - \frac{P_o\delta}{\rho_o}\left(1 + \frac{P_o}{C\delta\rho_o T_o}\right)\Delta p\right] \tag{37}$$

It may be noted that

$$\frac{P_o}{C\delta\rho_o T_o} = \frac{g}{(1+g)}\frac{P_o}{\rho_2^{(o)} T_o C} = \frac{g}{(1+g)}\frac{R}{m_2 C_2}\frac{C_2}{C}$$

$$= \frac{(\gamma-1)\,g\,C_2}{C_1 + g\,C_2}\ , \tag{38}$$

where γ is the ratio of specific heats for the gas. It may be seen from Eq. (38) that ordinarily $P_o/(C\delta\rho_o T_o) << 1$. For example, for the combination of air and water one has $C_1 \cong 4.2\times10^7$ ergs/gm ^oC, $C_2 \cong 7.2\times10^6$ ergs/gm ^oC, $\gamma \cong 1.4$, and in addition one usually has $g << 1$. If the quantity $P_o/(C\delta\rho_o T_o)$ is neglected, then one gets from Eq. (37)

$$\frac{\partial^2 p}{\partial t^2} - \frac{P_o\delta}{\rho_o}\Delta p = 0 \tag{39}$$

so that the velocity of sound propagation is essentially $(P_o\delta/\rho_o)^{1/2}$.

It must be kept in mind that this result for the sound velocity requires the validity of the approximation of Eq. (30) which has been used in Eq. (33). This approximation means that the volume ratio f must be large enough so that the gas compressibility plays the dominant role in the over-all compressibility of the mixture; at the same time f cannot be so large that the mass ratio becomes comparable with unity.

The velocity of sound for the fluid mixture may be derived formally in another way. If c is the sound velocity of the mixture, c_1 and c_2 the sound velocities for the liquid and the gas respectively, then one has

$$\frac{1}{c^2} = \frac{d\rho}{dp} = -\left(\frac{M_1+M_2}{V_1+V_2}\right)\frac{1}{V_1+V_2}\frac{d}{dp}(V_1+V_2) ,$$

$$= \rho\left(\frac{M_1}{V_1+V_2}\frac{1}{\rho_1^2}\frac{d\rho_1}{dp} + \frac{M_2}{V_1+V_2}\frac{1}{\rho_2^2}\frac{d\rho_2}{dp}\right);$$

or

$$\frac{1}{c^2} = \frac{\rho}{\rho_2+g\rho_1}\left(\frac{g}{f}\frac{1}{c_1^2} + \frac{f}{c_2^2}\right) , \tag{40}$$

where $1/c_1^2 = d\rho_1/dP$ and $1/c_2^2 = d\rho_2/dP$. Ordinarily $c_1^2 > c_2^2$, so that, if it is assumed that $g << f^2$, a good approximation to Eq. (40) is

$$c^2 \cong \frac{\rho_2+g\rho_1}{f\rho}c_2^2 . \tag{41}$$

It follows from the identity $f = g\rho_1/\rho_2$ and from Eq. (24) that

$$\frac{\rho_2+g\rho_1}{f\rho} = \frac{(1+g)}{g}\frac{\rho_2^2}{\rho^2}$$

and Eq. (41) then becomes

$$c^2 \cong \frac{(1+g)}{g}\frac{\rho_2^2}{\rho^2}c_2^2 . \tag{41'}$$

Now the preceding analysis which led to Eq. (39) gave the result

$$c^2 = \frac{P_o\delta}{\rho_o} = \frac{(1+g)}{g}\frac{\rho_2^{(o)}}{\rho_o^2}P_o$$

or, dropping the index (o), one has

$$c^2 = \frac{(1+g)}{g} \frac{\rho_2^2}{\rho^2} \frac{P}{\rho_2} \tag{42}$$

Now in Eq. (41') c_2^2 stands for $dP/d\rho_2$ and the comparison of Eq. (41') with Eq. (42) shows that in the present approximation

$$c_2^2 = \frac{dP}{d\rho_2} = \frac{P}{\rho_2}$$

It follows that the appropriate derivative, $dP/d\rho_2$, must be the <u>isothermal</u> one. It is in the sense of this result that the gas phase of the mixture may be described as behaving isothermally for the condensations and rarefactions of acoustic waves.

The attenuation coefficient for an acoustic wave travelling through the gas-liquid mixture may be immediately calculated in the same way as in the treatment of the previous section for a homogeneous fluid. One takes, as before, a plane wave of the form

$$p = A\, e^{i(kx-\omega t)}$$

with the notation

$$c^2 = \frac{P_o \delta}{\rho_o}, \qquad \alpha = \frac{P_o}{C \delta \rho_o T_o}, \tag{43}$$

one finds from Eq. (37)

$$-k^2 \left[-\omega^2 + k^2 c^2 \right] = -\frac{i\omega}{D_m} \left[-\omega^2 + k^2 c^2 (1+\alpha) \right] \tag{44}$$

One may write

$$k = \frac{\omega}{c} (1+\eta), \tag{45}$$

where it is assumed that $|\eta| << 1$. It has already been remarked that $\alpha << 1$. A good approximation to Eq. (44) is, therefore

$$- 2 \frac{\omega^2}{c^2} \eta \cong - \frac{i}{D_m} (\alpha + 2\eta)$$

so that

$$\eta \cong \frac{1}{2} \left[\frac{-\alpha + i \alpha D_m \omega / c^2}{1 + (D_m \omega / c^2)^2} \right] \tag{46}$$

The real part of η, as given in Eq. (46), represents a correction of $O(\alpha)$ to the propagation velocity c, and the imaginary part is the attenuation coefficient.

A numerical example will serve to illustrate the behavior of a mixture and the example chosen is an air-water mixture at standard conditions with $f = 1/10$.

Then

$$\frac{\rho_2}{\rho_1} \cong \frac{1}{800} ; \qquad g = \frac{\rho_2}{\rho_1} f \cong \frac{1}{8000} ;$$

and

$$\rho_o = \frac{\rho_1^{(o)}}{1 + f} + \frac{\rho_2^{(o)} f}{1 + f} \cong \frac{\rho_1^{(o)}}{1 + f} \cong 0.9 \text{ gm/cm}^3 .$$

One then has

$$\delta = \frac{1 + g}{g} \frac{\rho_2^{(o)}}{\rho_o} \cong 11 ,$$

and

$$c^2 = \frac{P_o \delta}{\rho_o} \cong \frac{1}{80} c_2^2$$

so that

$$c \cong \frac{1}{9} c_2 \cong 3 \times 10^3 \text{ cm/sec} ,$$

and

$$\alpha = \frac{P_o}{C \delta \rho_o T_o} \cong 10^{-5} ,$$

so that α is indeed small. Finally

$$C = \frac{C_1 + g C_2}{1 + g} \cong C_1 \cong 4 \times 10^7 \text{ ergs/gm}^{\circ}\text{C} ;$$

$$\kappa = \frac{\kappa_1 + \kappa_2 f}{1 + f} \cong \kappa_1 \cong 6 \times 10^4 \text{ ergs/cm sec}^{\circ}\text{C} ;$$

and

$$D_m = \frac{\kappa}{\rho_o C} \cong \frac{\kappa_1}{\rho_1^{(o)} C_1} \cong 1.4 \times 10^{-3} \text{ cm}^2/\text{sec} .$$

As ω runs, say, from 10/sec to 2×10^4/sec, $D_m \omega / c^2$ ranges from 10^{-8} to 2×10^{-5}. If η is written as $\eta_1 + i \eta_2$, then

$$\eta_2 \cong \frac{\alpha}{2} \frac{D_m \omega}{c^2} \cong 7 \times 10^{-16} \omega$$

where ω is in sec^{-1}. It may be noted that this attenuation coefficient is much smaller than the value for air alone which was found to be $6 \times 10^{-11} \omega$.

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