Finally, by examining the typical LC admittance $sC+1/(sL)$ which may be rewritten as

$$[(js^2C+j/L)/(js)]$$

we see that the admittance of an LC network may be written in terms of $CX-$networks as

$$Y_{LC}(s)=Y_{CX-}(js^2)/(js).$$

This allows, we believe, a more straightforward derivation of the properties of LC and RC networks than by proceeding from LC to RC, or by using arguments based on energy.

IV. Example

A single capacitor in parallel with a negative-valued reactance (and, hence, a positive susceptance) has the admittance $sC+jB$, where B is positive. The zero is on the $j\omega$ axis at $\omega=-B/C$, and the slope of the susceptance is positive, being equal to C.

V. Conclusion

We have derived the properties of $CX-$networks. The argument is "elementary." Also, we have related these results to the properties of LC and RC networks.

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A Note on General Parallel QMF Banks

P. P. VAIDYANATHAN

Abstract —**Two issues concerning alias-free, parallel, quadrature mirror filter (QMF) banks are addressed in this correspondence. First, a property concerning alias-free analysis/synthesis systems is established; second, a scheme is proposed, by which a synthesis bank can be modified in order to take care of aliasing errors caused by linear channel-distortion in a simple manner. Applications of the stated results are outlined.**

I. Introduction

Quadrature mirror filter (QMF) banks have received the attention of a number of researchers in the past [1]–[9], and their importance in communications and sub-band coding is well known [2]. Fig. 1 shows a parallel r-channel QMF bank, the adjective "parallel" being meant to distinguish it from tree-structured QMF banks [2], [3]. The transfer functions $H_0(z), H_1(z),\cdots,H_{r-1}(z)$ represent the analysis-bank filters, and $F_0(z), F_1(z),\cdots,F_{r-1}(z)$ represent the synthesis-bank filters. The filtered signals are decimated and transmitted; at the receiver end, the signals are interpolated, filtered, and recombined as shown.

In general, the reconstructed signal $\hat{x}(n)$ suffers from aliasing, amplitude, and phase distortions. Smith and Barnwell have shown [5] how these distortions can be formulated in a unified manner. For the two-channel case, the theory and filter-design procedures are well understood [2]–[4], and methods for perfect cancelation of aliasing have been well known. For the case of r channels, if r is a power of two, tree structures can be used with perfect alias-free properties; for arbitrary r, interesting and valuable filter-design methods have been reported [6], [7], with the synthesis bank chosen so as to approximately cancel aliasing. For the special case of uniform DFT r-channel parallel structures, schemes for exact cancelation of aliasing have been reported in [8] and [9], but the resulting synthesis bank has higher order than the structures in [6] and [7].

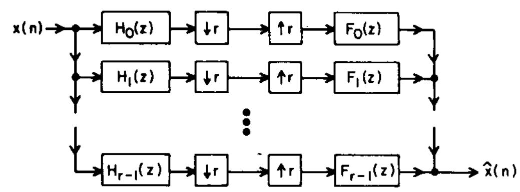

Fig. 1. A parallel QMF bank.

In this correspondence, we consider parallel QMF banks as in Fig. 1, with no specific assumptions about the nature of $H_k(z)$ and $F_k(z)$. They could be FIR or IIR; the analysis bank may or may not be of the uniform DFT type [2], and r may not be a power of 2. By employing the properties of decimators and interpolators, one can arrive at the most general expression for the reconstructed signal $\hat{X}(z)$

$$\hat{X}(z)=\frac{1}{r}\sum_{l=0}^{r-1}X(zW^{-l})\sum_{k=0}^{r-1}H_k(zW^{-l})F_k(z) \qquad (1)$$

where $W=e^{-j2\pi/r}$. The terms corresponding to $l\neq 0$ in (1) represent the aliasing terms. The first question we wish to address here is this: For a given (but arbitrary) set of transfer functions $H_k(z), 0\leqslant k\leqslant r-1$, let the transfer functions $F_k(z), 0\leqslant k\leqslant r-1$, be chosen such that $\hat{x}(n)$ is free from aliasing, for arbitrary $x(n)$. If we now interchange the $F_k(z)$'s with the $H_k(z)$'s, i.e., if we let the analysis filters be $F_0(z), F_1(z),\cdots,F_{r-1}(z)$, and the synthesis filters be $H_0(z), H_1(z),\cdots,H_{r-1}(z)$, is the reconstructed signal still free from aliasing? We show that the answer is in the affirmative. It is possible to address this question, based on the concept of network-transposition for multirate systems [2], but we take a direct approach here. If $\hat{X}(z)$ is free from aliasing in Fig. 1, then the following set of $r-1$ equations hold, in view of (1):

$$\sum_{k=0}^{r-1}F_k(z)H_k(zW^{-l})=0, \qquad 1\leqslant l\leqslant r-1. \qquad (2)$$

Since (2) holds for *all* z, in particular it holds if z is replaced with zW^l. Thus, the following $r-1$ equations also hold:

$$\sum_{k=0}^{r-1}F_k(zW^l)H_k(z)=0, \qquad 1\leqslant l\leqslant r-1. \qquad (3)$$

As $W^r=1$, the above set can be rephrased as

$$\sum_{k=0}^{r-1}H_k(z)F_k(zW^{-l})=0, \qquad 1\leqslant l\leqslant r-1 \qquad (4)$$

which is precisely (2) with $F_k(z)$'s and $H_k(z)$'s interchanged. Thus, after the interchange is made, conditions necessary to cancel the aliasing still hold. Once aliasing is canceled, the

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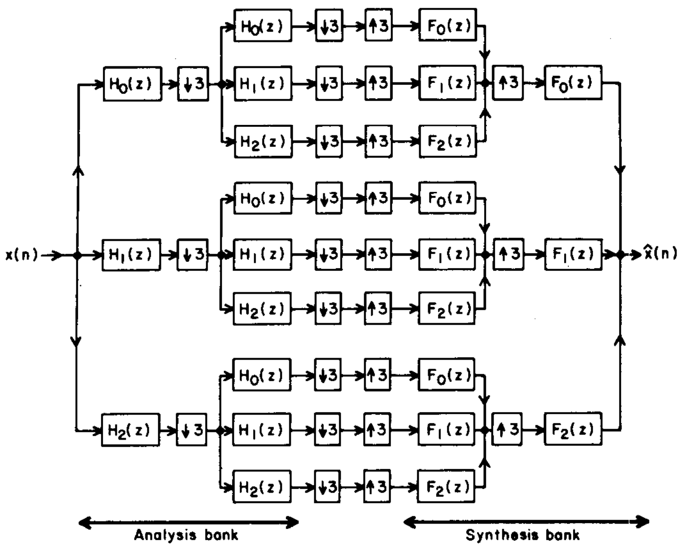

Fig. 2. The tree structure for a 9-channel QMF bank.

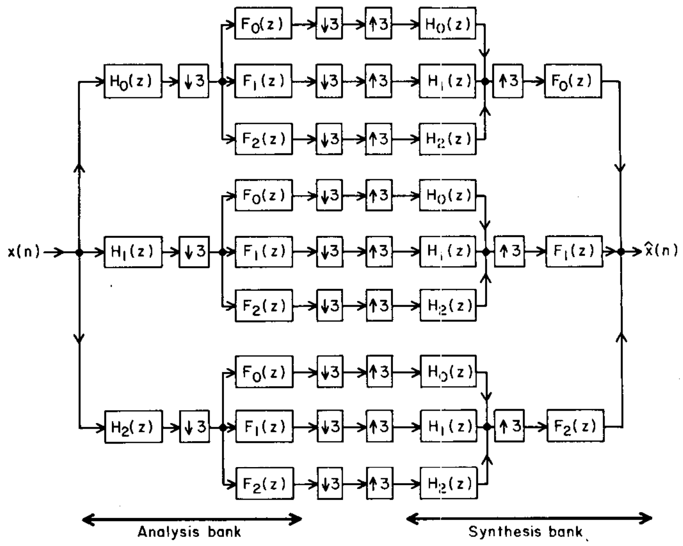

Fig. 3. A modified tree structure with more balanced complexity for analysis and synthesis banks.

transfer function for Fig. 1 is

$$\frac{\hat{X}(z)}{X(z)} = \sum_{k=0}^{r-1} F_k(z) H_k(z)/r. \tag{5}$$

The reconstructed signal continues to be given by (5) even after interchanging $H_k(z)$'s with the $F_k(z)$'s. We state these properties compactly as follows.

The Swapping Property: In the parallel QMF bank of Fig. 1, let aliasing be completely absent. Then, interchanging each analysis filter $H_k(z)$ with the corresponding $F_k(z)$ leaves the reconstructed signal $\hat{x}(n)$ unchanged.

Note that if $F_k(z)$ in Fig. 1 were such that aliasing is not perfectly canceled, then the above interchange of $F_k(z)$'s and $H_k(z)$'s does *not* leave the reconstructed signal unchanged.

A possible application of this property is in QMF structures with composite r that is not a power of 2. For a parallel structure with large r which is not a power of 2, aliasing can be perfectly canceled only by a choice of synthesis filters $F_k(z)$ having much higher order than the $H_k(z)$'s [8], [9]. (If $H_k(z)$'s are FIR with order $N-1$, the orders of $F_k(z)$'s are roughly proportional to $(r-1)(N-1)$.) It is thus beneficial to build up the QMF bank in the form of tree structures [2], with alias-free building blocks having small (prime) number of channels. Fig. 2 shows a nine-channel QMF bank in terms of three-channel building blocks. Now, such a tree structure is disbalanced in the sense that the synthesis filters still have higher order than the analysis filters (even though not as much as in a parallel structure). This inbalance in complexity, if undesirable, can be reduced by restructuring the filter bank as shown in Fig. 3. Such restructuring leaves the final output $\hat{x}(n)$ unchanged, as evidenced by repeated application of the above swapping property.

A second issue we wish to address in this correspondence is related to channel distortion. In general, the transmission medium between the analysis and synthesis banks introduces both linear and nonlinear distortions, due to various coding schemes [2] involved. If the distortion is dominantly linear, it can be represented by a channel transfer function, denoted by $C_k(z)$ for the kth channel. Even if $\hat{X}(z)$ is free from aliasing when channels are ideal, this is not so with $C_k(z)$'s present. Accordingly, for a given set of $H_k(z)$'s, a new set of synthesis filters $G_k(z)$ should be designed to cancel aliasing. Fig. 4 depicts this situation.

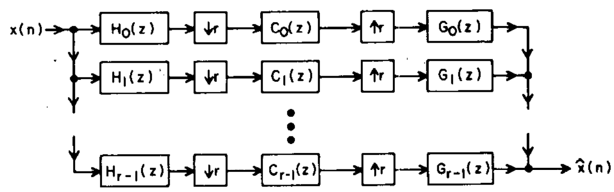

Fig. 4. The equivalent QMF structure with channel distortion.

Clearly, the output signal is given by

$$\hat{X}_1(z) = \frac{1}{r}\sum_{l=0}^{r-1} X(zW^{-l}) \sum_{k=0}^{r-1} H_k(zW^{-l}) C_k(z^r) G_k(z). \tag{6}$$

If $F_k(z)$ is such that (2) holds, it would appear that the choice $G_k(z) = F_k(z)/C_k(z^r)$ would result in $\hat{X}_1(z) = \hat{X}(z)$. Such a choice would result in an unstable synthesis bank if $C_k(z)$ had zeros on or outside the unit circle. In order to overcome this difficulty, note that if (2) holds, then so does the following equation (for arbitrary choice of $S(z)$ independent of k):

$$\sum_{k=0}^{r-1} S(z^r) F_k(z) H_k(zW^{-l}) = 0, \qquad 1 \leqslant l \leqslant r-1. \tag{7}$$

Thus, if $G_k(z)$ are chosen according to

$$G_k(z) = F_k(z) \prod_{\substack{l=0 \\ l \neq k}}^{r-1} C_l(z^r) \tag{8}$$

then (2), which holds, implies that (7) holds with

$$S(z) = \prod_{l=0}^{r-1} C_l(z). \tag{9}$$

Thus, if aliasing is absent in the ideal QMF structure of Fig. 1, then the structure with linear channel distortion is free of aliasing if the new synthesis filters are chosen according to (8). Such an alias-free system has the overall transfer function

$$\frac{\hat{X}_1(z)}{X(z)} = S(z^r) \sum_{k=0}^{r-1} F_k(z) H_k(z). \tag{10}$$

The order of $G_k(z)$ defined in (8) is typically much higher than that of $F_k(z)$. This difficulty can be avoided by carefully defining

$G_k(z)$ to minimize cost as follows. Let

$$C_k(z)=\frac{P_k(z)Q_k(z)}{D_k(z)} \tag{11}$$

where the polynomial $P_k(z)$ has all zeros inside the unit circle, and $Q_k(z)$ has zeros on and outside the unit circle. Assume, of course that the channels are stable, so that the polynomials $D_k(z)$ have all zeros inside the unit circle. Define $G_k(z)$ according to

$$G_k(z)=\frac{F_k(z)D_k(z^r)}{P_k(z^r)}V_k(z^r) \tag{12}$$

where

$$V_k(z)=\prod_{\substack{l=0\\ l\neq k}}^{r-1}Q_l(z). \tag{13}$$

This is equivalent to taking $S(z)$ to be the polynomial

$$S(z)=\prod_{l=0}^{r-1}Q_l(z) \tag{14}$$

rather than a typically higher order rational function as in (9). In fact, if there are common factors among $Q_l(z)$'s, this can be exploited to further reduce the order of $G_k(z)$ to some extent.

If the ideal system of Fig. 1 is alias-free and such that the overall transfer function (5) is allpass (which is a requirement if amplitude distortion is to be avoided), then (10) does not necessarily represent an allpass distortion because $S(z)$ is not guaranteed to be allpass. This situation can be corrected by defining $G_k(z)$ in a different manner, provided that $C_k(z)$'s do not have zeros on the unit circle (under this condition, $Q_k(z)$'s have all zeros strictly outside the unit circle). If we now define $G_k(z)$ to be

$$G_k(z)=F_k(z)\frac{D_k(z^r)}{P_k(z^r)\hat{Q}_k(z^r)}\prod_{\substack{l=0\\ l\neq k}}^{r-1}\frac{Q_l(z^r)}{\hat{Q}_l(z^r)} \tag{15}$$

where $\hat{Q}_l(z)$ is $Q_l(z)$ with coefficients in reversed order, then $G_k(z)$ are indeed stable, and this is equivalent to defining $S(z)$ in (7) as

$$S(z)=\prod_{l=0}^{r-1}\frac{Q_l(z)}{\hat{Q}_l(z)}. \tag{16}$$

Since $S(z)$ in (16) is (stable and) allpass, the overall transfer function (10) of the alias-free system is now allpass, if (5) is allpass. In this manner, aliasing caused due to linear channel distortion can be avoided, at the same time avoiding additional amplitude distortion.

II. Conclusions

Two important issues pertaining to alias-free parallel QMF structures have been analyzed in this correspondence. Some applications have been outlined; it is felt that the swapping property may have further applications in the efficient design of QMF banks, but details remain to be explored.

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Harmonic Response of Variable-Structure-Controlled Van der Pol Oscillators

HEBERTT SIRA-RAMIREZ

***Abstract*— A variable-structure-controlled (VSC) Van der Pol oscillator is shown to produce an ideal average response of perfect harmonic nature when its motions are made to slide on a circle defined in the interior of its limit cycle. The sinusoidal responses are robust with respect to parameter perturbations and external disturbances.**

I. Introduction

The variable-structure control of dynamic systems undergoing sliding motions on nonlinear manifolds offers a richer variety of design alternatives than those possible with linear hyperplanes. These possibilities are based on the fact that a larger class of static relationships can be synthesized, among the state variables, when nonlinear surface values are used in the switching logic controlling the system. The dynamic behavior of the controlled system is totally determined by the nature of the nonlinear surface, making the controlled system robust with respect to external disturbances and parameter perturbations.

Sliding surface reachability and invariance are essential ingredients of the sliding motion design for variable-structure systems (VSS's). These tasks are accomplished by opportune switchings among feedback laws, which guarantee state trajectories invariably directed towards the sliding surface. In the sliding regime, one of the outstanding characteristics of the controlled motion is that radically new properties are obtained compared with those of the individual structures responsible for its creation.

The reader is referred to several books (Utkin [1], Utkin [2], Itkis [3]) and survey articles (Utkin [4], Utkin [5]) for a complete account of the theory and its many practical applications.

Using the theory of variable-structure control [1], the possibility of creating harmonic limit cycles in controlled Van der Pol oscillators is explored. An ideally sinusoidal response is obtained

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