Reduction of relative intensity noise of the output field of semiconductor lasers due to propagation in dispersive optical fiber

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(Received 4 December 1995; accepted for publication 23 February 1996)

The effect of dispersive, linear propagation (e.g., in single-mode optical fiber) on the intensity noise from semiconductor lasers is investigated. Relations between the frequency and amplitude noise variations of semiconductor lasers are obtained from the laser rate equations and used to calculate the change in the relative intensity noise (RIN) spectrum that occurs during dispersive propagation. Propagation in fiber with positive dispersion ($D>0$) over moderate distances (several km for standard single-mode fiber at 1.55 μm) is found to reduce the RIN over a wide range of frequencies. Measurements with a 1.56 μm distributed feedback laser confirm the main theoretical results and demonstrate reductions in RIN of up to 11 dB with 4 km of standard fiber. © *1996 American Institute of Physics.* [S0003-6951(96)03418-3]

In the use of single-mode optical fibers to carry light from semiconductor lasers, fiber dispersion and its contribution to the modulation response of the laser-fiber-detector combination is usually an important consideration. This is especially true in the case of direct modulation of the laser, which causes the optical field exiting the laser (and entering the fiber) to be chirped. In cases where the system noise level is determined by the relative intensity noise (RIN) of the laser, it is also important to consider the effect of dispersion on the propagation of the noise.

In this letter we give a theory for the effect of linear propagation in dispersive optical fiber on the semiconductor laser intensity noise, and we show experimental results for a 1.56 μm distributed feedback (DFB) laser and various lengths of standard single-mode fiber. The theory and measurements both indicate that the RIN can be reduced over a wide range of frequencies by propagation in dispersive fiber. This reduction in the RIN can result in an increase in the signal-to-intensity noise ratio after propagation.

Direct modulation of a semiconductor laser leads to a modulation of the lasing frequency according to the well-known relation[1]

$$\Delta\omega(t) = -\frac{\alpha}{2P_0}\left\{\Delta\dot{P}(t) + \frac{\epsilon P_0}{\tau_{ph}}\Delta P(t)\right\}, \qquad (1)$$

where α is the linewidth enhancement factor ($\alpha<0$ for semiconductor lasers), P_0 is the CW photon density in the laser cavity, $\Delta P \ll P_0$ is the photon density variation (proportional to the output power variation), τ_{ph} is the photon lifetime, and ϵ is a parameter which describes photon-density-dependent compression of the gain.

As a frequency and amplitude modulated optical field propagates in dispersive fiber, some of the FM will be converted to AM and vice versa. When the relationship between FM and AM at the input to the fiber is described by Eq. (1), the transfer function $H_1(\Omega,z)$ for the propagation of small-signal intensity modulations with $e^{i\Omega t}$ time dependence through a fiber of length z will be[2]

$$H_1(\Omega,z) = \cos\,\theta(\Omega,z) + \alpha\,\sin\,\theta(\Omega,z) + i\alpha\frac{\epsilon P_0}{\tau_{ph}}\frac{1}{\Omega}\sin\,\theta(\Omega,z), \qquad (2)$$

where $\theta(\Omega,z) = -\frac{1}{2}\beta_0'' z\Omega^2$ is the phase distortion angle of the fiber at modulation frequency Ω, with $\beta_0'' = -\lambda^2 D/(2\pi c)$ characterizing the group velocity dispersion of the fiber. The available signal power (i.e., the electrical power in the signal after it is photodetected) will be changed after propagation by the factor $|H_1(\Omega,z)|^2$. Measuring $|H_1(\Omega,L)|^2$ for a fiber of known dispersion D and length L and then fitting to the parameters in Eq. (2) is one method of determining α and $\epsilon P_0/\tau_{ph}$ for the laser.[3]

Equations (1) and (2) were obtained for frequency variations driven by injection current modulation only. To find the corresponding results for noise-driven variations, we consider the laser rate equations. The equations describing small variations ΔN, ΔP, $\Delta\omega$, in the carrier density, photon density, and lasing frequency about a CW operating point N_0, P_0, ω_0, including gain compression and Langevin noise terms are

$$\Delta\dot{N} = \frac{\Delta I}{eV} - \left(\frac{1}{\tau} + v_g A P_0\right)\Delta N - \frac{1-\epsilon P_0}{\Gamma\tau_{ph}}\Delta P + \frac{F_1}{V}, \qquad (3)$$

$$\Delta\dot{P} = \Gamma v_g A P_0 \Delta N - \frac{\epsilon P_0}{\tau_{ph}}\Delta P + \frac{\Gamma F_2}{V}, \qquad (4)$$

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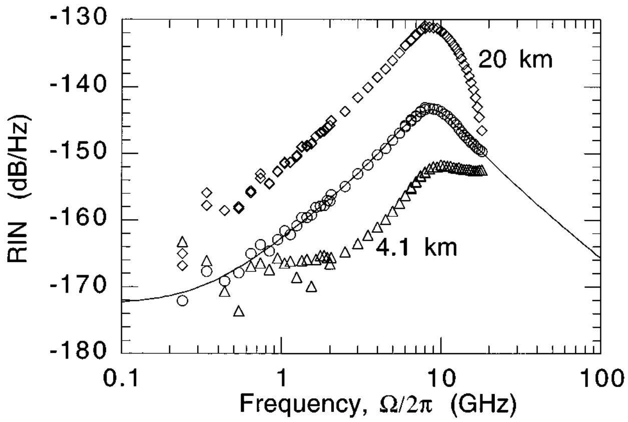

FIG. 1. Measured RIN at the laser (circles), and after propagation in fiber of length 4.1 km (triangles) and 20 km (diamonds). The solid curve is theoretical, as described in the text.

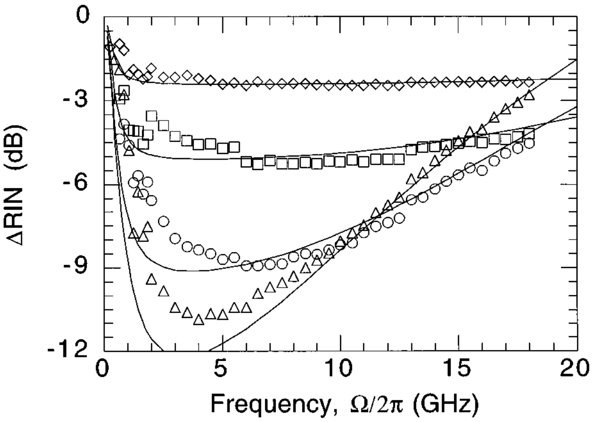

FIG. 2. Change in RIN after propagation in fiber of length 1.1 km (diamonds), 2.1 km (squares), 3.2 km (circles), and 4.1 km (triangles). The solid curves are for the theory given in the text.

$$\Delta\dot{\phi}=-\frac{\alpha}{2}\Gamma v_g A\Delta N+F_3, \tag{5}$$

where ΔI is the modulation current, and F_1, F_2, and F_3 are the Langevin noise source terms. Here τ is the differential carrier relaxation time, $v_g A P_0$ is the differential stimulated carrier recombination rate, V is the active region volume, and Γ is the optical confinement factor. The variation of the lasing frequency $\Delta\omega$ is equal to $\Delta\dot{\phi}$.

In the simple laser noise model[4,5] we will consider here, the noise sources F_2 and F_3 are associated with spontaneous emission into the lasing mode, while F_1 is associated with the decay of carriers. The correlation between these processes can be handled most easily by eliminating F_1 from Eq. (3) using $F_1=-F_2+F_{1'}$, where $F_{1'}$ is then the part of the noise F_1 due to carrier decay not involving spontaneous emission of a photon into the lasing mode. The sources $F_{1'}$, F_2, and F_3 are uncorrelated, and the spectral density of the photon density variation of an unmodulated laser is (from the equations above) the sum of the spectral densities of the photon density responses to each of these sources, that is,

$$S_{PP}(\Omega)=S_{PP}^{(1')}(\Omega)+S_{PP}^{(2)}(\Omega)+S_{PP}^{(3)}(\Omega) \tag{6}$$

and the RIN at the output of the laser will be

$$\mathrm{RIN}(\Omega)=10\log\left\{\frac{S_{PP}^{(1')}(\Omega)+S_{PP}^{(2)}(\Omega)+S_{PP}^{(3)}(\Omega)}{P_0^2}\right\}. \tag{7}$$

The laser response to the noise source $F_{1'}$ is essentially the same as that for the current modulation, since $F_{1'}$ (but not F_1) and $\Delta I/e$ enter the rate equations in the same way. The contribution of $F_{1'}$ to the relative intensity noise spectrum of the laser is found from Eq. (3) and Eq. (4) to be

$$\frac{S_{PP}^{(1')}(\Omega)}{P_0^2}=\frac{(v_g A P_0)^2}{(\Omega_0^2-\Omega^2)^2+\gamma_0^2\Omega^2}\frac{S_{1'1'}(\Omega)}{(P_0V/\Gamma)^2}, \tag{8}$$

where $\Omega_0\simeq\sqrt{v_g A P_0/\tau_{ph}}$ is the laser relaxation oscillation frequency and $\gamma_0=1/\tau+v_g A P_0+\epsilon P_0/\tau_{ph}$ is twice the relaxation oscillation damping rate. In the Markovian emission approximation, $F_{1'}$ is a white noise with one-sided spectral density $S_{1'1'}(\Omega)=2R_c$, where R_c is the number of carriers per second which decay by means other than by interaction with the lasing mode.

The relationship between frequency variation $\Delta\omega$ and photon density variation ΔP resulting from the noise source $F_{1'}$ is also the same as that resulting from current modulation, which was given in Eq. (1). As indicated in Ref. 1 this result can be easily obtained from Eq. (4) and Eq. (5), with F_2 and F_3 set to zero. Because Eq. (1) applies, the small-signal transfer function for the propagation of intensity noise variations due to $F_{1'}$ is $H_1(\Omega,z)$, and the contribution $S_{PP}^{(1')}/P_0^2$ makes to the RIN will become $|H_1(\Omega,z)|^2S_{PP}^{(1')}/P_0^2$ after propagation in the fiber. (Here it has also been used that $|H_1(0,z)|^2=1$.)

The main contribution to the RIN, however, usually comes from F_2, the noise associated with spontaneous emission into the lasing mode, not $F_{1'}$. The contribution of F_2 to the RIN at the output of the laser is found from Eq. (3) and Eq. (4) to be

$$\frac{S_{PP}^{(2)}(\Omega)}{P_0^2}=\frac{1/\tau^2+\Omega^2}{(\Omega_0^2-\Omega^2)^2+\gamma_0^2\Omega^2}\frac{S_{22}(\Omega)}{(P_0V/\Gamma)^2}, \tag{9}$$

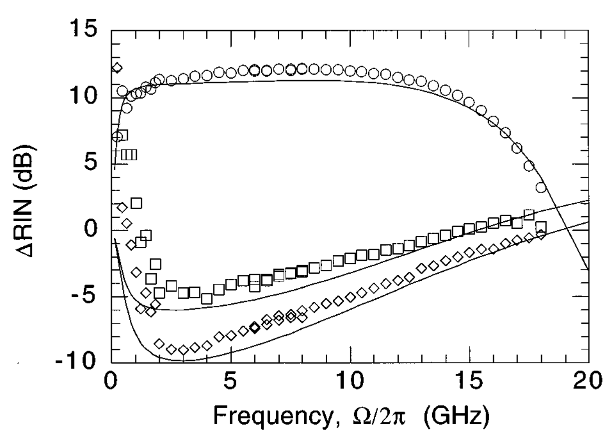

FIG. 3. Change in RIN after propagation in fiber of length 5.2 km (diamonds), 6.2 km (squares), and 20 km (circles). The solid curves are for the theory given in the text.

where $S_{22}(\Omega)=4n_{sp}KP_0(V/\Gamma)/\tau_{ph}$ is the spectral density of F_2. Here n_{sp} is the spontaneous emission factor and K is the Petermann enhancement factor.[6]

The relationship between the FM and AM responses to F_2 can be obtained by combining Eq. (3), Eq. (4), and Eq. (5) so as to eliminate F_2 and ΔN and then setting ΔI, F_1', and F_3 equal to zero to get

$$\Delta\dot{\omega}(t)+\frac{1}{\tau}\Delta\omega(t)=\frac{\alpha}{2}v_g A\left\{\Delta\dot{P}(t)+\frac{1}{\tau_{ph}}\Delta P(t)\right\}. \quad (10)$$

This important result, corresponding to Eq. (1) but now for variations driven by F_2 rather than by ΔI or F_1', is not dependent on the presence of gain compression in the laser. According to Eq. (10), the rate of change of the lasing frequency $\Delta\dot{\omega}$ will be proportional to ΔP for variations driven by F_2 in the range of frequencies $1/\tau\ll\Omega\ll1/\tau_{ph}$.

Using Eq. (10), the small-signal transfer function $H_2(\Omega,z)$ for propagation of intensity variations driven by F_2 with $e^{i\Omega t}$ time dependence is found[7] to be

$$H_2(\Omega,z)=\cos\,\theta(\Omega,z)+\alpha\frac{\Omega_0^2}{\Omega^2}\left\{\frac{1-i\Omega\tau_{ph}}{1+i/(\Omega\tau)}\right\}\sin\,\theta(\Omega,z), \quad (11)$$

where we used $v_gAP_0/\tau_{ph}\simeq\Omega_0^2$. For frequencies in the range $1/\tau\ll\Omega\ll1/\tau_{ph}$, the factor in braces { } approaches unity.

For small propagation distance, $z\ll2|\alpha\beta_0''\Omega_0^2|^{-1}$, and moderate frequencies, $\Omega\ll|\frac{1}{2}\beta_0''z|^{-1/2}$, the result in Eq. (11) can be simplified somewhat. In this case, the factor $|H_2(\Omega,z)|^2$ will become

$$|H_2(\Omega,z)|^2\simeq1-\alpha\beta_0''z\Omega_0^2\left(1-\frac{\tau_{ph}}{\tau}\right)\frac{\Omega^2}{1/\tau^2+\Omega^2} \quad (12)$$

so that for frequencies $\Omega\gg1/\tau$ there will then be a uniform reduction in the noise due to F_2 after propagation if $D>0$.

According to the rate equations, the photon density variation due to the noise F_3 is zero, so that

$$S_{PP}^{(3)}(\Omega)=0 \quad (13)$$

and therefore the contribution of F_3 to the RIN after propagation in the fiber must be treated as an additive noise rather than with a transfer function. There is a frequency variation due to F_3 at the output of the laser of

$$\Delta\omega(t)=F_3(t) \quad (14)$$

which leads to a contribution to the RIN after dispersive propagation of $4S_{33}(\Omega)\sin^2\theta(\Omega,z)/\Omega^2$, where $S_{33}(\Omega)=S_{22}(\Omega)/(2P_0V/\Gamma)^2$ is the spectral density of F_3. The RIN of the semiconductor laser emission after propagation in a dispersive fiber is then

$$\mathrm{RIN}(\Omega,z)=10\log\left\{|H_1(\Omega,z)|^2\frac{S_{PP}^{(1')}(\Omega)}{P_0^2}+|H_2(\Omega,z)|^2\times\frac{S_{PP}^{(2)}(\Omega)}{P_0^2}+\frac{4S_{33}(\Omega)}{\Omega^2}\sin^2\theta(\Omega,z)\right\}. \quad (15)$$

Measurements of the RIN of a packaged, fiber-coupled 1.56 μm DFB laser were made before and after propagation in various lengths of standard single-mode fiber ($D=17$ ps/nm/km), over the frequency range $0.24<\Omega/2\pi<18$ GHz. Figure 1 shows the RIN (not including shot noise) measured at the laser pigtail output and also after propagation through fibers of length 4.1 and 20 km. The solid curve is a fit to Eq. (7), using Eq. (9) and Eq. (13) but neglecting the contribution due to $S_{PP}^{(1')}(\Omega)$, which was expected to be small. This yielded the laser parameters $\tau=0.5$ ns, $1/\gamma_0=0.021$ ns, and $\Omega_0=2\pi\times8.6$ GHz.[8] The modulation response with and without the fiber was also measured and used as in Ref. 3 to obtain $\alpha=-7.9$ and $(\epsilon P_0/\tau_{ph})^{-1}=.036$ ns. The estimate of the photon lifetime obtained from $\tau_{ph}=(\gamma_0-1/\tau-\epsilon P_0/\tau_{ph})/\Omega_0^2$ with these parameters is 6 ps.

The change in RIN after propagation in fibers of length 1.1, 2.1, 3.2, and 4.1 km is shown in Fig. 2, and after 5.2, 6.2, and 20 km in Fig. 3. The solid curves, which show the change in RIN calculated as $\Delta\mathrm{RIN}(\Omega,z)=\mathrm{RIN}(\Omega,z)-\mathrm{RIN}(\Omega,0)$ using Eq. (15), Eq. (11), Eq. (9), and again neglecting the contribution due to $F_{1'}$, are in good agreement with the measured results. The laser parameters used in the calculations are those given above, except for $\alpha=-7.1$ and $\tau_{ph}=5.5$ ps chosen to better fit the data.

In short lengths of fiber (0 to 2 km), the measured reduction of the RIN after propagation was nearly uniform in frequency Ω as predicted by Eq. (12). Larger changes in RIN occurred at other fiber lengths, with reductions up to 11 dB seen at 4.1 km. Changes in RIN after propagation in much longer lengths of fiber (up to 70 km) were also measured and found to agree with the theoretical results. For these cases, the RIN was generally increased by the fiber, except for dips at specific frequencies where $|H_2(\Omega,z)|^2$ is nearly zero.

The work of two of the authors (W.K.M. and A.Y.) was supported by ARPA and the National Science Foundation.

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