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How to Recover a Qubit That Has Fallen Into a Black Hole

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We demonstrate an algorithm for the retrieval of a qubit, encoded in spin angular momentum, that has been dropped into a no-firewall unitary black hole. Retrieval is achieved analogously to quantum teleportation by collecting Hawking radiation and performing measurements on the black hole. Importantly, these methods only require the ability to perform measurements from outside the event horizon and to collect the Hawking radiation emitted after the state of interest is dropped into the black hole.

INTRODUCTION

Recovering the complete quantum state of a black hole from the Hawking radiation [1] into which it evaporates is notoriously difficult [2]. In this letter we tackle a simpler problem: recovering the quantum state of a single spin qubit that has fallen into an evaporating black hole.

Our protocol uses information about the spin state of the black hole before and after the qubit entered, as well as the state of the first Hawking particle to be radiated. The outline of the procedure is as follows:

- The initial spin state of the black hole is measured, putting the density matrix of the black hole in the form $\rho_B = \rho_B^{(\mathrm{int})} \otimes |j,m\rangle\langle j,m|$, where j, m are the quantum numbers for total and projected angular momentum, and $\rho_B^{(\mathrm{int})}$ characterizes the internal degrees of freedom. Perfect fidelity can be achieved only if $m = 0$; the experimenter can measure the spin along different axes until this outcome is attained.

- The qubit, a spin-1/2 particle in an arbitrary state $|\phi\rangle_A = \alpha\,|\uparrow\rangle_A + \beta\,|\downarrow\rangle_A$, is dropped into the hole.

- The first emitted Hawking particle is a photon that is part of a singlet Bell pair, the other photon of which falls into the hole.

- After the first Hawking photon is emitted, the black hole's spin state is again measured, so that the density matrix becomes $\rho'_B = \rho_B^{\prime(\mathrm{int})} \otimes |j',m'\rangle\langle j',m'|$. Dephasing of the hole's spin does not occur if the interactions between the hole's spin and its internal state are rotationally-invariant (conserve angular momentum).[1]

- The initial state of the qubit can then be reconstructed from the state of the observed Hawking photon.

This falls far short of a resolution to the information-loss problem [3–6], but it does provide a concrete illustration of how information can escape from a black hole in certain special circumstances. Moreover, whether or not the Page time [7] has elapsed does not affect information recovery, since the protocol is not concerned with reconstructing the state of the black hole [8].

A PROTOCOL FOR RETRIEVING INDIVIDUAL QUBITS

Suppose that Alice sits outside a black hole and has in her possession a spin-1/2 particle in some state $|\phi\rangle_A = \alpha\,|\uparrow\rangle_A + \beta\,|\downarrow\rangle_A$ that is unknown to her. Here, the basis states $|\uparrow\rangle_A$ and $|\downarrow\rangle_A$ represent true spin states as measured along some axis, not merely abstract labels. Before dropping her qubit into the black hole, Alice measures the hole's angular momentum and finds it in the state $|j,m\rangle_B$. (We suppress the state of the black hole's internal degrees of freedom, $\rho_B^{(\mathrm{int})}$, which will play no role in our analysis.) Such a measurement is technologically formidable, but one which Alice could in principle perform with the help of a sufficiently large Stern-Gerlach apparatus or by carefully measuring frame dragging. She then drops in the qubit, collects a single Hawking particle, and then measures the angular momentum of the hole again, determining it to be $|j',m'\rangle_B$.

We assume that the emitted Hawking particle is one half of a pair, the other one of which falls into the hole. Various conservation laws assure us that the pairs of particles should have opposite gauge and Poincaré quantum

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[1] Concretely, suppose that there was some conditional interaction between the black hole's internal degrees of freedom and its spin which would take a state $|BH\rangle \otimes (\alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle)$ to a state $\alpha\,|BH^\uparrow\rangle \otimes |\uparrow\rangle + \beta\,|BH^\downarrow\rangle \otimes |\downarrow\rangle$, where $\langle BH^\uparrow|BH^\downarrow\rangle = 0$. If, for example, $\alpha = \beta = 1/\sqrt{2}$, then angular momentum in the x direction would not be conserved by the interaction.

numbers. Let us focus on spin. For simplicity, suppose that these Hawking particles have spin-1/2. The ingoing and outgoing particles must be created in the singlet state so that total angular momentum does not change:

$$|-\rangle_{io} \equiv \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_i |\downarrow\rangle_o - |\downarrow\rangle_i |\uparrow\rangle_o \right) = |0,0\rangle_{io} \,. \tag{1}$$

(Further justification for this model and its relation to Hawking photons, which have spin-1, is provided in the next section.) The subscripts i and o label the particles that remain inside and outside the black hole respectively.

After Alice drops her spin into the black hole and a Hawking pair forms, the total state of the black hole and the three spins is therefore

$$|\Psi\rangle = |j,m\rangle_B \otimes |\phi\rangle_A \otimes \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_i |\downarrow\rangle_o - |\downarrow\rangle_i |\uparrow\rangle_o \right) . \tag{2}$$

Alice is ignorant of what happens inside the black hole. What Alice can know, however, is the total angular momentum of the black hole and the projection of its angular momentum vector along some axis. As such, let us rewrite the AiB subsystem in the total angular momentum basis:

$$\begin{aligned} |\Psi\rangle = \frac{\alpha}{\sqrt{2}} \Big(& \langle {}^{j}_{m}\, {}^{1}_{1} | {}^{j+1}_{m+1} \rangle \, |j+1, m+1\rangle + \langle {}^{j}_{m}\, {}^{1}_{1} | {}^{j}_{m+1} \rangle \, |j, m+1\rangle + \langle {}^{j}_{m}\, {}^{1}_{1} | {}^{j-1}_{m+1} \rangle \, |j-1, m+1\rangle \Big) \otimes |\uparrow\rangle_o \\ & + \frac{1}{2} \left(\langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j+1}_{m} \rangle \, |j+1, m\rangle + \langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j}_{m} \rangle \, |j, m\rangle + \langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j-1}_{m} \rangle \, |j-1, m\rangle \right) \otimes \left(-\alpha |\uparrow\rangle_o + \beta |\downarrow\rangle_o \right) \\ & - \frac{1}{2} |j,m\rangle^{\perp} \otimes \left(\alpha |\uparrow\rangle_o + \beta |\downarrow\rangle_o \right) \\ & - \frac{\beta}{\sqrt{2}} \Big(\langle {}^{j}_{m}\, {}^{1}_{-1} | {}^{j+1}_{m-1} \rangle \, |j+1, m-1\rangle + \langle {}^{j}_{m}\, {}^{1}_{-1} | {}^{j}_{m-1} \rangle \, |j, m-1\rangle + \langle {}^{j}_{m}\, {}^{1}_{-1} | {}^{j-1}_{m-1} \rangle \, |j-1, m-1\rangle \Big) \otimes |\downarrow\rangle_o \,. \end{aligned} \tag{3}$$

The symbols $\langle {}^{j_1}_{m_1}\, {}^{j_2}_{m_2} | {}^{j}_{m} \rangle \equiv \langle j_1, m_1; j_2, m_2 | j, m \rangle$ denote appropriate Clebsch-Gordan coefficients. We have also suppressed the label AiB on the total angular momentum kets. Note that there are three cases: $m \geq j-1$, $m \leq -j+1$, and $-j+1 < m < j-1$, or in other words, some of the $\langle {}^{j_1}_{m_1}\, {}^{j_2}_{m_2} | {}^{j}_{m} \rangle$ could be zero. For now, we will just assume that $-j+1 < m < j-1$. In particular note the following two states: $|j,m\rangle$, which comes from $j \otimes 1$, and $|j,m\rangle^{\perp} \equiv |j,m\rangle |0,0\rangle$, which comes from $j \otimes 0$. These states have the same angular quantum numbers, but $\langle j,m | j,m\rangle^{\perp} = 0$.

Next, Alice queries the black hole's total angular momentum by performing the following orthogonal measurement on AiB:

$$\begin{gathered} \hat{F}_1 = \sum_a |a,m\rangle \langle a,m| \,, \\ \hat{F}_2 = \sum_a |a,m+1\rangle \langle a,m+1| + |a,m-1\rangle \langle a,m-1| \,, \\ \hat{F}_3 = \hat{I}_{AiB} - \hat{F}_1 - \hat{F}_2. \end{gathered} \tag{4}$$

Note that by construction, only the results $\hat{F}_1$ and $\hat{F}_2$ may be obtained for black hole states which may emerge from this protocol. The protocol for retrieving the state $|\phi\rangle$ is then as follows:

Case 1: Alice obtains the result $\hat{F}_1$*.* In this case, the whole system collapses to a state that is proportional to the second and third lines of Eq. (3). Alice then measures the total angular momentum $\hat{J}^2$ of the black hole.

If Alice measures the result $J^2 = \hbar^2 (j \pm 1)(j \pm 1 + 1)$, then she knows that the spin that she holds is in the state $|\phi'\rangle_o = -\alpha |\uparrow\rangle_o + \beta |\downarrow\rangle_o$. Therefore, she can apply the Pauli operator $\hat{\sigma}_z$ to restore the state $|\phi\rangle_o$.

If Alice measures the result $J^2 = \hbar^2 j(j+1)$, then the total system is in the state

$$|\Psi'\rangle \propto \langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j}_{m} \rangle \, |j,m\rangle \, |\phi'\rangle_o - |j,m\rangle^{\perp} |\phi\rangle_o \,. \tag{5}$$

This state represents a mixed density matrix for the spin that Alice holds so long as $\langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j}_{m} \rangle$ is nonzero, and so we must arrange for this coefficient to vanish. In particular, some algebra reveals that

$$\langle {}^{j}_{m}\, {}^{1}_{0} | {}^{j}_{m} \rangle = \frac{m/j}{\sqrt{1+1/j}}. \tag{6}$$

At the beginning of the protocol, Alice may measure j and determine if it is an integer. If not, she may repeatedly throw spin-1/2 particles into the black hole and measure j until she measures an integral value. She may then repeatedly measure the black hole's angular momentum projection along different axes until she obtains $m = 0$, before tossing her qubit into the hole. In this way, the Clebsch-Gordan coefficient may be made to vanish, allowing Alice to recover the qubit.

Case 2: Alice obtains the result $\hat{F}_2$*.* In this case, the whole system collapses to a state that is proportional to the first and last lines of Eq. (3). Next, Alice measures the total angular momentum $\hat{J}^2$, obtaining the result $J^2 = \hbar^2(j+\sigma)(j+\sigma+1)$ for some $\sigma \in \{-1,0,1\}$. The total state is then

$$\begin{aligned} |\Psi''\rangle \propto\ & \alpha \left\langle {}^{j}_{m}\,{}^{1}_{1} \middle| {}^{j+\sigma}_{m+1} \right\rangle |j+\sigma, m+1\rangle \otimes |\uparrow\rangle_o \\ & - \beta \left\langle {}^{j}_{m}\,{}^{1}_{-1} \middle| {}^{j+\sigma}_{m-1} \right\rangle |j+\sigma, m-1\rangle \otimes |\downarrow\rangle_o . \end{aligned} \tag{7}$$

We are faced with the problem of disentangling the AiB part of the system from the o part which Alice holds. She may accomplish this task with the help of a spin-1 ancilla and a local entangling unitary. Suppose Alice holds a spin-1 ancilla, A', that she prepares in the state $|1,1\rangle_{A'}$. If she then implements a local entangling unitary operator $U_{oA'}$ such that

$$\begin{aligned} U_{oA'} |\uparrow\rangle_o |1,1\rangle_{A'} &= |\uparrow\rangle_o |1,-1\rangle_{A'} \\ U_{oA'} |\downarrow\rangle_o |1,1\rangle_{A'} &= |\downarrow\rangle_o |1,1\rangle_{A'} , \end{aligned} \tag{8}$$

upon acting with $U_{oA'}$ on the spins that she holds, the total state $I_{AiB} \otimes U_{oA'} (|\Psi''\rangle \otimes |1,1\rangle_{A'})$ is proportional to

$$\begin{aligned} & \alpha \left\langle {}^{j}_{m}\,{}^{1}_{1} \middle| {}^{j+\sigma}_{m+1} \right\rangle |j+\sigma, m+1\rangle_{AiB} |\uparrow\rangle_o |1,-1\rangle_{A'} \\ & - \beta \left\langle {}^{j}_{m}\,{}^{1}_{-1} \middle| {}^{j+\sigma}_{m-1} \right\rangle |j+\sigma, m-1\rangle_{AiB} |\downarrow\rangle_o |1,1\rangle_{A'} . \end{aligned} \tag{9}$$

Next, Alice tosses her ancilla into the black hole and then measures the black hole's total angular momentum. The $AiBA'$ terms will consist of linear combinations of $|j+\sigma+1, m\rangle$, $|j+\sigma, m\rangle$, and $|j+\sigma-1, m\rangle$ weighted by the appropriate Clebsch-Gordan coefficients. If Alice finds $AiBA'$ in a total angular momentum $j+\sigma+\tau$ state, where $\tau \in \{1,0,-1\}$, it is straightforward to show that the spin that she still holds collapses to the state

$$\begin{aligned} |\phi''\rangle_o \propto\ & \alpha \left\langle {}^{j}_{m}\,{}^{1}_{1} \middle| {}^{j+\sigma}_{m+1} \right\rangle \left\langle {}^{j+\sigma}_{m+1}\,{}^{1}_{-1} \middle| {}^{j+\sigma+\tau}_{m} \right\rangle |\uparrow\rangle_o \\ & - \beta \left\langle {}^{j}_{m}\,{}^{1}_{-1} \middle| {}^{j+\sigma}_{m-1} \right\rangle \left\langle {}^{j+\sigma}_{m-1}\,{}^{1}_{1} \middle| {}^{j+\sigma+\tau}_{m} \right\rangle |\downarrow\rangle_o . \end{aligned} \tag{10}$$

As long as Alice measured the black hole angular momentum at the beginning of the protocol and ensured that $|m| \ll j$, then none of these coefficients vanish. Alice then performs the appropriate unitary transformation on the spin that she holds to restore the state $|\phi\rangle_o$.

DISCUSSION

We now consider several aspects of the proposed algorithm, as well as its consequences for black hole information theory.

Singlet State of the Hawking Photons: To see why the Hawking particles must be created in a singlet state, note that spacetime is locally flat on the horizon and becomes increasingly flat as the black hole mass M increases. As a result, the only way for a Hawking pair to have non-zero angular momentum is for the pair to pick it up via interactions with the vacuum, *i.e.*, with another Hawking pair. This requires, roughly speaking, that two Hawking pairs be present within one wavelength λ of one another in the time t it takes for a pair to separate. The relevant scaling relations in general are $\lambda \propto T^{-1}$, $t \propto \lambda$, and $F \propto T^d$, where d is the number of spatial dimensions, T is the Hawking temperature, and F is the particle number flux across the horizon. The fraction f of Hawking pairs which interact with an additional Hawking pair scales at tree order as $f \propto |\mathcal{A}|^2 (F\lambda^{d-1}t)^2 \propto |\mathcal{A}|^2$, where the mass-dependence of the phase-space factors dropped out.[2]

For photons, which are the exponentially dominant form of Hawking radiation at large M, the matrix element $|\mathcal{A}|^2$ must depend on the probability of producing a virtual electron-positron pair to mediate the Hawking pair interaction. This scales as $e^{-m_e/E_\gamma} \sim e^{-m_e GM}$. Thus for large black holes, we expect these interactions to be exceedingly rare, and hence are justified in assuming that the Bell pair carries no net angular momentum. We note that the creation of Hawking pairs in the singlet state relies on the assumption that the local spacetime around the horizon of the black hole is a low-energy, quiescent environment. Were there instead an energetic firewall at the horizon, we could not expect outgoing quanta to come from a singlet state. We also note that the precise sequence of events (dropping a spin, collecting radiation, etc.) is not important, as must be the case for a covariant protocol. The only requirement is that Alice must perform her measurement before a second Hawking photon arrives.

For the sake of simplicity and familiarity, in our protocol we let the Hawking quanta consist of spin-1/2 particles. The protocol works equally well for quanta that consist of photons, provided that Alice is far enough away from the black hole that she can measure qubits encoded in a plane-wave circular polarization basis. In particular, it does not matter that photons are spin-1 particles, since their state space is that of a qubit, and Clebsch-Gordan coefficients characterize state space structure.

When performing this analysis for other quantum numbers the same arguments apply: for large black holes, the Hawking pair must be created with zero net quantum number. The algorithm we describe will work for any conserved quantum number which photons may carry. If the relevant number is not quantized, the information recovered is only up to a precision limit given by the number of bits recovered. For those quantum numbers which photons do not carry, superpositions of states cannot be recovered except by waiting exponentially long in

[2] This is not entirely unexpected. Consider, for instance, that the characteristic wavelength of Hawking photons is on the order of the Schwarzschild radius. Roughly speaking, since $t \propto \lambda$, any two photons at the black hole horizon will therefore overlap before they separate.

M for the relevant particles to be emitted. If, on the other hand, it is known that a quantum number eigenstate fell in, and hence that only classical information was encoded in this way, then direct measurement of the black hole allows for recovery. For example, in order to learn the mass of a particle that fell into the black hole, then one may of course measure the mass of the black hole afterwards, assuming that the initial mass of the black hole was known. Altogether, this allows for unique recovery of classical information about any particle that fell in. This is because each known fundamental particle has a unique set of gauge quantum numbers—mass, spin, charge, and color. This feature is not necessary—it would not hold in a theory with two unbroken $U(1)$ symmetries—but it does hold true in the Standard Model.

Resource Considerations: In its essence, our protocol amounts to a quantum teleportation scheme [9] between a transmitting party—the black hole—and a receiving party—Alice. Its perfect fidelity when $m = 0$ is due to the fact that setting $m = 0$ eliminates any degeneracy in the states that the transmitting party could find after measuring in the total angular momentum basis, as opposed to a (nondegenerate) maximally-entangled basis. Alice would not be able to use an analogous procedure to recover more that a single qubit at a time, since the degeneracy of total angular momentum states rapidly increases as more and more spins are added.

We can also understand the difficulty of the multiple qubit case from the point of view of resources. Suppose that Alice wishes to recover more than a single qubit at a time through a quantum number conservation protocol. As these protocols amount to quantum teleportation schemes, Alice is bound by the resource inequality [10]

$$2[c \to c] + [qq] \geq [q \to q], \tag{11}$$

which says that two classical bits, or cbits, of communication and one entangled qubit pair shared between the two parties is necessary to achieve one qubit of communication. If Alice drops N spins into the black hole and collects N Hawking qubits, she only obtains $\sim \log_2(N^2) = 2\log_2 N$ cbits since there are $N+1$ possible outcomes for the total angular momentum measurement and $\sim N$ possible outcomes for the measurement of the projection of the angular momentum along the axis of quantization. As such, she cannot hope to recover some general state of N qubits, which would require $2N$ cbits. On the other hand, she may be able to recover a state that is encoded in some subspace of $\mathcal{H}$. For instance, Alice could try encoding her data in the total angular momentum of a set of N spin-1/2 qubits with total angular momentum s. Thus she is encoding her data in a Hilbert space $\mathcal{H}_s$ with $\dim \mathcal{H}_s = 2s+1$. Resource considerations do not prohibit the recovery of a state in $\mathcal{H}_s$, which only requires the extraction of $\log_2 \dim \mathcal{H}_s \leq \log_2(N+1)$ qubits and hence $\sim 2\log_2 N$ cbits. We suspect that the general method for doing this is similar to the single qubit case.

Timescale Considerations: During the protocol, Alice must wait for the black hole to emit a quantum of Hawking radiation. We may estimate this timescale as follows. The wavelength density of blackbody radiation is given in d spatial dimensions by [11]

$$B_\lambda(T) = \frac{\rho(\lambda, T)c}{2\pi^{d/2}}\Gamma\left(\frac{d}{2}\right) = \frac{2hc^2}{\lambda^{d+2}\left(e^{hc/\lambda k_B T} - 1\right)}. \tag{12}$$

This may be converted to an energy density by noting that $d\lambda/dE = -\lambda/E$, and hence

$$B_E(T) = 2c\left(\frac{E}{hc}\right)^d \frac{1}{e^{E/k_BT} - 1}. \tag{13}$$

Consequently, the photon number flux across the event horizon is

$$F = \int_0^\infty dE \frac{2(E/hc)^{d-1}}{h\left(e^{E/k_BT} - 1\right)} \int \cos\theta \, d\Omega. \tag{14}$$

Per the arguments of Landsberg and De Vos [11], the solid angle integral just produces the volume of the unit sphere in $d-1$ dimensions. As a result, we may evaluate the integrals and compute the characteristic timescale for emission of Hawking photons for a black hole with mass M and horizon radius R:

$$t_h = \frac{1}{AF} = R\left(\frac{8\pi}{\xi\chi}\right)^d \frac{\sqrt{\pi}\Gamma(d/2)\Gamma(d/2 + 1/2)}{4\Gamma(d)\mathrm{PolyLog}(d, 1)}. \tag{15}$$

In the above, we have defined $\xi \equiv 8\pi GMT$, $\chi \equiv R/(2GM)$, and used units where $\hbar$, c, and k_B are 1. This estimate agrees well with more precise calculations of Hawking emission rates—for instance, Eq. (15) gives a photon emission rate of $t_h^{-1} = 1.9 \times 10^{-4}\ c^3/GM$ for a Schwarzschild black hole in $3+1$ dimensions, compared to the rate $1.5 \times 10^{-4}\ c^3/GM$ reported by Page [12]. We note, however, that emission rates are modified, in some cases quite significantly, if the black hole has very large angular momentum [13].

It is interesting to compare the emission time to the scrambling time [8], which may be thought of as the time it takes for Alice's infalling qubit to become incorporated into the (stretched horizon of) the black hole. For a black hole of mass M, the scrambling time is

$$t_s = R\ln(R/l_p), \tag{16}$$

where l_p denotes the Planck length. This increases faster than t_h, so there is a critical radius R_{crit} above which the scrambling time is greater than the time required for a Hawking particle to be emitted, although the numerical factors in (15) can make this radius very large. In light of our single-qubit protocol, $R > R_{\mathrm{crit}}$ means that the qubit which falls in is essentially bounced off of the black hole, rather than being incorporated into it. For a Kerr black hole in $3+1$ dimensions, we find that

$$R_{\rm crit}^{d=3} = e^{\frac{32\pi^4}{\mathrm{PolyLog}(3,1)(1-\alpha^2)^{3/2}}} l_p \gtrsim e^{2600} l_p, \qquad (17)$$

where $\alpha \equiv J/J_{\rm max}$ parametrizes how close the black hole is to being extremal. This is considerably larger than the current Hubble radius, so the bouncing qubit case is not relevant for realistic black holes.

In AdS spacetime, on the other hand, the critical radius is tunable and may be considerably smaller. For a BTZ black hole in $2+1$ dimensions [14, 15], $R_{\rm crit}$ is given by

$$R_{\rm crit}^{d=2} = e^{\frac{1}{4}W\left(12\pi\left[1+\frac{1}{\sqrt{1-\alpha^2}}\right]^2 (b/l_p)^4\right)} l_p, \qquad (18)$$

where b is the AdS radius and W is the Lambert W function. In the limit as $b \to 0$, $R_{\rm crit} \to l_p$, while in the limit as $b \to \infty$, $R_{\rm crit} \to \infty$. Thus we see that an enormous range of critical radii may be achieved by tuning the AdS radius. Similar calculations may be done for AdS black holes in higher dimensions using the appropriate expressions for temperature [16]. For AdS black holes, the critical radius beyond which qubits bounce off the black hole is a way to characterize whether or not a black hole is small. This characterization of size is an alternative to asking whether or not the black hole is eternal [17].

CONCLUSION

We have described a protocol, based on quantum teleportation, that allows an external observer to recover a single spin qubit that has been dropped into a black hole, if the spin of the hole is measured before and after the qubit is dropped. Our procedure relies on the fact that the angular momentum states of the black hole span the possible states of the qubit; for more than one qubit, this condition would not hold, and an analogous procedure would be unable to recover the information. Although very far from allowing a complete reconstruction of the state of the hole from its emitted radiation, this process represents a small step in the right direction.

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