

and Petersen³ from measurements of the specific heat at low temperatures. According to reference 3 the wavelength of the unstable spin waves in spherical samples of yttrium garnet is given by $2\pi/\lambda = 4.60 \times 10^5 \text{ cm}^{-1}$. The line in Fig. 3 represents the theoretical prediction for the dependence of the slope on the diameter of the sample based on this numerical value.

Stimulating discussions with Professor R. V.

Jones are gratefully acknowledged.

*The portion of the work performed by J. J. Green was supported by the Air Force Cambridge Research Center.

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MANY-PARTICLE COOPERATIVE SCATTERING OF X-RAYS IN THE TWO-CRYSTAL SPECTROMETER

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(Received June 24, 1959)

It has been emphasized by Guinier and Fournet¹ that low-angle x-ray scattering patterns should theoretically exhibit a relatively high, extremely narrow central maximum because the entire irradiated sample will scatter coherently in the same phase at scattering angles very close to zero. This effect is very small outside of a central region where $h = (4\pi/\lambda)\sin(\phi/2)$ is less than $2\pi/d$, where d is the average dimension of the sample perpendicular to the incoming beam of x-rays.² The magnitude of d is almost always so large that the above effect occurs at angles too small to be observed. Parratt *et al.*³ have called this and similar effects "many-atom cooperative scattering," or "ultra-small-angle" scattering. In this Letter, I shall adopt the term "many-particle cooperative scattering" and abbreviate it as "m.p.c. scattering."

This high central scattering due to the total size of the sample irradiated should be broad enough to be observed in some cases where the x-ray beam has been collimated by means of a plane, nearly perfect crystal. Such a case arises when the two-crystal spectrometer is used in parallel position for the study of low-angle x-ray diffraction patterns, with the sample placed between the two crystals.⁴ M.p.c. scattering should be observable here because in this case the equivalent sample size is very small since it is determined, not by the region of the sample actually irradiated by x-rays, but by the size of the region of the sample irradiated by a coherent beam of x-rays. This means that we must consider radiation of one wavelength coming from one point on the anode of the x-ray tube. Since the colli-

ating crystal is between the x-ray tube and the sample, the angular width of such a coherent beam of x-rays will be determined by the angular width of the reflection curve for the crystal (as, for example, the Darwin diffraction curve). The actual width of the sample irradiated by this coherent radiation will of course depend on the distance of the target of the x-ray tube from the sample. Therefore the width of the resulting m.p.c. scattering pattern will depend on both the angular width of the reflection curve of the crystal and the distance of the x-ray tube target from the sample, and will be largest for smallest values of both.

Using the half-width of the Darwin diffraction curve, and approximating this curve and also the resulting m.p.c. scattering curve by a Gaussian, the m.p.c. scattering angle at half maximum is calculated to be 1.6 seconds and 9.6 seconds, respectively, for the first and second orders of reflection of copper K_α radiation from the cleavage planes of calcite, with the sample located at 30 cm from the anode of the x-ray tube. If this distance were only 3 cm, then the above scattering angles should be multiplied by ten. The angle is much greater for the second order reflection because the Darwin pattern is much narrower in this case. The preceding estimate does not take into consideration two other causes of broadening of the pattern. The first of these broadening effects comes from the fact that the rocking curve, in the case of the two-crystal spectrometer, must be folded into the entire scattering pattern from the sample to obtain the pattern which will actually be observed,

and this of course broadens the pattern. The second effect is due to multiple scattering in the sample. We believe that both of these effects may contribute significantly to the apparent width of the m.p.c. scattering.

M.p.c. scattering appears to explain some results which have been observed in attempts to obtain low-angle x-ray scattering patterns by means of the two-crystal spectrometer in parallel position. Thus it would seem to explain the high central scattering obtained by Parratt *et al.*⁵, particularly in the 2, -2 case, although it does not explain the scattering which they obtained at larger angles. It seems also to explain a similar central scattering obtained by Beeman and Kaesberg,⁶ although the dimensions of their instrument were not given in this reference. Since the width of the m.p.c. scattering depends on the target-to-sample distance, this distance should henceforth be published in reports on work done with the two-crystal spectrometer.

The two-crystal spectrometer has also been used to define and determine x-ray absorption coefficients, as recently discussed by Parratt *et al.*³ It has also been used in a similar way by Warren and others⁷ to determine the total small-angle scattering of a sample. In each of the above applications the results may well prove to be dependent upon the target-to-sample distance, since this distance partially determines the width of the m.p.c. scattering discussed in this Letter,

and this width in turn determines the amount of this scattering to be reflected from the second of the two crystals. Similarly, the results may prove to be more dependent on the reflection curve of the crystal than has been thought.

The author is indebted to his colleagues for valuable discussions and especially to Dr. Jesse W. M. DuMond, who also was of great assistance in preparing this Letter.

¹Guinier, Fournet, Walker, and Yudowitch, *The Small-Angle Scattering of x-Rays* (John Wiley and Sons, Inc., New York, 1955), pp. 37 and 71.

²A. Guinier *et al.*, reference 1, p. 37.

³Parratt, Porteus, Schnopper, and Watanabe, *Rev. Sci. Instr.* **30**, 344 (1959).

⁴Some of the references to this use of the two-crystal spectrometer, in addition to others cited in this Letter, are I. Fankuchen and M. H. Jellinek, *Phys. Rev.* **67**, 201 (1945); J. W. M. DuMond, *Phys. Rev.* **72**, 83 (1947); Ritland, Kaesberg, and Beeman, *J. Appl. Phys.* **21**, 838 (1950); and L. G. Parratt, *Rev. Sci. Instr.* **30**, 297 (1959).

⁵Parratt, Porteus, Schnopper, and Watanabe, Technical Report No. 14, December 20, 1958, Document Nos. AFOSR TN-58-781 and ASTIA AD 202 002 (unpublished).

⁶W. W. Beeman and P. Kaesberg, *Phys. Rev.* **72**, 512 (1947).

⁷B. E. Warren, *J. Appl. Phys.* **20**, 96 (1949); H. M. Barton, Jr., and R. Brill, *J. Appl. Phys.* **21**, 783 (1950).

STORED ENERGY OF IRRADIATED COPPER

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(Received July 6, 1959)

It is well known that reactor irradiation substantially changes the properties of metals.¹ These changes have usually been attributed to interstitial atoms and vacant lattice sites arising from the atomic displacements which result from the collision with energetic neutrons.² The appearance of a large annealing peak at 30 to 50°K in the radiation-induced resistivity of samples bombarded below 20°K has led to the suggestion that interstitial atoms are mobile at this temperature and migrate to and are annihilated by vacant lattice sites. This conclusion was largely based on the fact that computations of the migration energy of interstitials indicate that they

should be mobile in the 30 to 50°K range.³ It should be noted, however, that other experiments indicate that a more complicated model than freely moving interstitials is required to explain the low-temperature annealing.⁴ The fact that the formation energy of an interstitial-vacancy pair is known within narrow limits makes the measurement of the energy released important in the analysis of this peak as it gives an accurate measure of the number of defects being annihilated. In the past three years several measurements of the stored energy have been made by the authors⁵⁻⁷ and it has been estimated from these measurements that the most reliable