One-body parity and time reversal violating potentials

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It is of great importance to relate the results of nuclear gamma-ray distribution measurements to the fundamental parity (P) and time (PT) violating meson-nucleon coupling constants. To facilitate this program we derive a one-body mean-field potential from the two-body PT violating pion-exchange potentials. This potential includes a suppression due to the finite meson range of about 0.7 in the rare-earth nuclei and 0.4 in the *s-d* shell nuclei. We compare exact numerical results to an approximate analytic formula and find good agreement.

We discuss the energy distribution of the nuclear PT violating impurity. This is overwhelmingly concentrated at relatively large excitation energies, around $1\hbar\omega$. Nevertheless, in heavier deformed nuclei there is a small PT violating strength at low energies due to the interaction of natural parity states with the unnatural parity intruder subshell. This impurity can be significantly enhanced due to the small energy denominators.

Following the discovery of CP violation in neutral kaon decay, numerous attempts have been made to probe the extent of similar symmetry violations in the nuclear environment.[1] To date, none of these have shown any significant symmetry noninvariance. The most constraining limit on such effects is set by the upper limit on the neutron electric dipole moment[2] ($d_n < 1.2\times10^{-25}e$ cm). In order to relate the values of d_n to other nuclear symmetry violating processes, one may follow Herczeg[3] and Crewther *et al.*[4] and find a relationship between the neutron electric dipole moment and the coupling constant of time reversal (PT) violating pion exchange nucleon-nucleon interactions with different isospin structures [see Eq. (3) below for an example]. The possible isoscalar ($I=0$) and isotensor ($I=2$) interaction constants are restricted to $\bar{g}_{\pi NN}^{(I=0,2)} \lesssim 3\times10^{-11}$. However, due to a reduced sensitivity of the neutron electric dipole moment to the isovector interaction, the limit on the associated coupling constant is an order of magnitude weaker. By contrast, measurements involving the atomic nucleus are more sensitive to the isovector interaction since the isoscalar and isotensor terms are hindered by a factor $(N-Z)/A$ (i.e., only the excess neutrons contribute). It is therefore of interest to ascertain the competitiveness of symmetry experiments on the atomic nucleus as compared to the neutron electric dipole moment, especially with regard to PT violating isovector pion exchange. As a preliminary step in this program, we examine in this work to what extent the two-body PT violating interaction can be replaced by an effective one-body potential.

As a means to study symmetry violation, we wish to consider experimental techniques involving the angular distributions of gamma radiations following the decay of polarized nuclei. These are sensitive to the interference between two competing gamma multipolarities, i.e., to the relative phase of the corresponding multipole matrix elements. Explicitly we consider the correlation $(\mathbf{J}\cdot\mathbf{k}_2)(\mathbf{J}\cdot\mathbf{k}_1\times\mathbf{k}_2)$, where the photon $\mathbf{k}_1$ is of "regular" multipolarity $M2+E3$, with mixing ratio δ, and connects the initial and final nuclear states $|a_0\rangle$ and $|b\rangle$. The "irregular" multipole is $E2$, and connects the state $|a_z\rangle$, which is admixed into $|a_0\rangle$ due to the $\mathbf{P}T$ violation, with the final state $|b\rangle$. The second photon $\mathbf{k}_2$ determines the polarization of the final state. The observable asymmetry is then given by

$$A \sim \frac{\delta}{1+\delta^2+\varepsilon^2}\,\mathrm{Im}\sum_{a_z}\frac{\langle b||M(E2)||a_z\rangle}{\langle b||M(M2)||a_0\rangle}\frac{1}{E_0-E_z}\times[i\langle a_z|V_{PT}|a_0\rangle+\langle a_z|V_P|a_0\rangle(\xi_{E2}-\xi_{M2})]\ , \quad (1)$$

where ε is the net "irregular" $M2+E2$ mixing ratio and the ξ's are the atomic final-state phases that mimick (but do not represent) T violation. [A generalization of (1) to other multipolarities is straightforward; our conclusions are independent of the particular choice of multipolarities.] Using Eq. (1), the experimental effect may be related to nuclear matrix elements of the V_P and V_{PT} potentials, and ultimately, via nuclear-structure calculation, to the fundamental PT violating coupling constants.

In many applications it is desirable and much simpler to replace the two-body interaction by an effective mean-field one-body potential. Examples of such potentials have already been given by Herczeg.[3] However, these are strictly applicable only in the case where the mediating mesons have infinitely short range. The effects of finite range in the case of P violation have been investigated by Adelberger and Haxton.[5] Using a Fermi-gas model they conclude that, while the contributions arising from the direct term of the corresponding two-body potential remain unaltered, the exchange term may be significantly suppressed. For example, the one-body pion contribution is reduced by 80%. As noted in Ref. 3, the two-body PT violating potentials considered here contribute to the one-body reduction solely through the direct term and so, by analogy with the parity case, the suppression factor due to finite-range effects was not included.

Nevertheless, it is of interest to verify the extent to which this approximation is true and also to determine the characteristic nuclear length scale which governs its behavior.

We wish to derive the mean-field potential $U(1)$ arising from a two-body potential $V(1,2)$. In order to perform this reduction, we follow the method of Michel[6] and consider antisymmetrized matrix elements of the two-body interaction summed over all occupied states χ. Hence,

$$\langle \psi_\alpha | U(1) | \psi_\beta \rangle = \sum_\chi [\langle \psi_\alpha \chi | V(1,2) | \psi_\beta \chi \rangle - \langle \psi_\alpha \chi | V(1,2) | \chi \psi_\beta \rangle] , \quad (2)$$

where the wave functions $\psi_{\alpha,\beta}$ correspond to some arbitrary nuclei states.

In what follows we take the case of the PT violating isovector pion exchange potential, although our conclusions apply to all three isospin channels. Thus, we have[3]

$$V_{PT}^{(I=1)} = -\frac{m_\pi^2}{16\pi M} \bar{g}_{\pi NN}^{(1)} g_{\pi NN} [(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\hat{\mathbf{r}}(\tau_{1z}+\tau_{2z}) + (\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{r}}(\tau_{1z}-\tau_{2z})] \times \frac{\exp(-m_\pi r)}{m_\pi r}\left[1+\frac{1}{m_\pi r}\right] , \quad (3)$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. The constant $g_{\pi NN}$ $(=13.45)$ is the strong pion-nucleon coupling, while $\bar{g}_{\pi NN}^{(1)}$ defines the PT violating strength via the coupling

$$L_{PT}^{(I=1)} = \bar{g}_{\pi NN}^{(1)} \bar{N} N \pi^0 . \quad (4)$$

We first attempt an analytic evaluation of Eq. (2) under the assumption of a spin symmetric core. The exchange terms [the second term in square brackets in (2)] then drop out and we are left only with a direct term

$$\langle \psi_\alpha | U(1) | \psi_\beta \rangle = \left\langle \psi_\alpha \left| \int \sum_\chi \chi^*(2) V(1,2) \chi(2) d\mathbf{r}_2 \right| \psi_\beta \right\rangle , \quad (5)$$

so that we may express the mean-field potential independently of $\psi_{\alpha,\beta}$ as

$$U(1) = \int \sum_\chi \chi^*(2) V(1,2) \chi(2) d\mathbf{r}_2 = -\int \frac{m_\pi^2}{8\pi M} \bar{g}_{\pi NN}^{(1)} g_{\pi NN} \boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\tau_{1z}\rho(r_2) \times \frac{\exp(-m_\pi r)}{m_\pi r}\left[1+\frac{1}{m_\pi r}\right] d\mathbf{r}_2 , \quad (6)$$

where we have made the substitution $\sum_\chi \chi(2)^* \chi(2) = \rho(r_2)$. Due to the Yukawa term, the important contributions to this integral are confined to the pion range $r \lesssim 1/m_\pi$. Thus, for the purpose of carrying through the integral, we temporarily assume that $m_\pi \to \infty$ and expand the core density as a power series in the small quantity r about the point r_1 to obtain

$$U_{PT,\mathrm{asympt}}^{(I=1)}(1) = \frac{1}{2Mm_\pi^2} \bar{g}_{\pi NN}^{(1)} g_{\pi NN} \boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}_1 \tau_{1z} \frac{\partial \rho(r_1)}{\partial r_1} . \quad (7)$$

This last result has already been obtained by Herczeg.[3] However, we find an extra factor of $\frac{1}{2}$ which we believe is due to a double counting of the spin degrees of freedom in that work.

In order to test the validity of the short-range assumption, we have performed an exact numerical evaluation of the matrix elements of the mean field using Eq. (2). We calculate the antisymmetrized two-body matrix elements of $V_{PT}^{(I=1)}$ for harmonic-oscillator states ψ_α and ψ_β lying in the fourth and fifth oscillator shells, respectively. For the core states χ, we take the proton and neutron Fermi levels to lie at the top of the fourth and fifth shells, respectively. These are then normalized to the corresponding one-body matrix elements of the asymptotic potential Eq. (7). In Fig. 1 we show the ratio

$$F = \frac{\sum_\chi [\langle \psi_\alpha \chi | V(1,2) | \psi_\beta \chi \rangle - \langle \psi_\alpha \chi | V(1,2) | \chi \psi_\beta \rangle]}{\langle \psi_\alpha | U_{\mathrm{asympt}}(1) | \psi_\beta \rangle} , \quad (8)$$

of true mean field to asymptotic mean field, plotted as a function of the exchanged meson range for the representative cases of $\psi_\alpha, \psi_\beta = 4g_{9/2}, 5h_{9/2}$ (squares) and $4g_{7/2}, 5f_{7/2}$ (circles). (The other possible $\Delta j = 0$ matrix elements lie between the displayed cases.) As expected in the limit of infinite mass (short range), this ratio is indeed unity, however, for the physical pion mass, the asymptotic mean field is reduced by a factor of 0.7.

An alternative analytic reduction of Eq. (6) was obtained earlier by Haxton[7] for the case of a nucleon Fermi gas extending up to the nuclear radius R. [Due to the density derivative appearing in Eq. (7), one obtains a null result for the case of infinite nuclear matter.] The resulting potential in the region $r_1 \le R$ is then given by

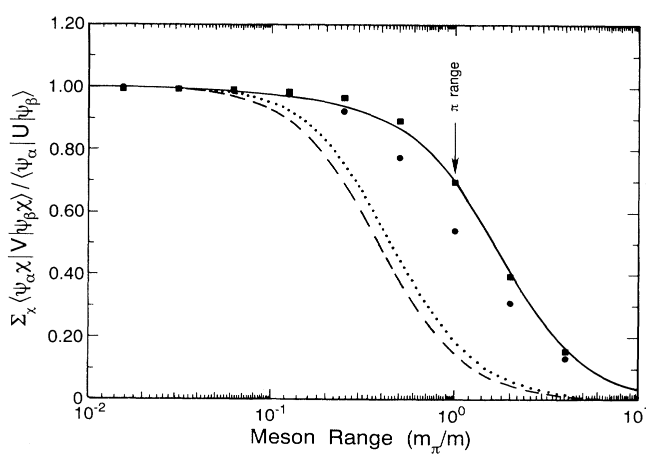

FIG. 1. Dependence on the exchanged meson range (mass) of the ratio F (8) between the matrix elements of the true and asymptotic one-body potential. The points are results of the numerical calculations described in the text. The solid line is the analytical expression $F(mR)$ (10). For comparison, the dotted (Z) and dashed (N) lines are the analogous suppression factors $W_{Z,N}$ for the P violating potentials (Ref. 5).

$$U^{(I=1)}_{PT,\text{Fermi gas}}(1)=-\frac{3}{16\pi M m_\pi^2}\tilde{g}^{(1)}_{\pi NN}g_{\pi NN}\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}_1\tau_{1z}\frac{\rho}{R}\times\left[\frac{8\pi}{3}\exp(-m_\pi R)(1+m_\pi R)\frac{m_\pi R}{(m_\pi r)^2}[m_\pi r_1\cosh(m_\pi r_1)-\sinh(m_\pi r_1)]\right] . \quad (9)$$

Here the nuclear density ρ is to be considered constant, as distinct from that appearing in Eq. (7). Owing to the rather extreme approximation used in its derivation, this potential is not really suitable for realistic calculations involving the matrix elements of general wave functions $\psi_{\alpha,\beta}$. Nevertheless, for the present purposes we only require the dependence on m_π. With this aim, and in the spirit of the original assumptions, we calculate matrix elements between states $\psi_{\alpha,\beta}$ which are themselves step functions. Equation (9) has been written in such a way that all the pion mass dependence over and above that already expressed by Eq. (7) is contained in the braced term. Focusing on this part of the matrix element, we then obtain from the integration over the radial coordinate

$$F(mR)=\int_0^R\left[\frac{8\pi}{3V}\exp(-mR)(1+mR)\frac{mR}{(mr)^2}\times[mr\cosh(mr)-\sinh(mr)]\right]r^2dr$$

$$=2\exp(-mR)\frac{(1+mR)}{(mR)^2}\times[mR\sinh(mR)-2\cosh(mR)+2] . \quad (10)$$

The nuclear volume V arises from the normalization of the wave functions $\psi^*_\alpha\psi_\beta$. Inasmuch as the step-function approximation is valid, this quantity represents that part of the exchanged meson mass (or range) dependence which would have been neglected in the matrix elements of the asymptotic potential (7). As such, it must constitute an analytic parametrization of the ratio (8); $F(mR)$ is therefore chosen to be normalized to unity in the infinite-mass limit. The function $F(mR)$ is plotted as the solid line in Fig. 1. As can be seen, it provides an excellent reproduction of the numerical calculations. We have also performed an explicit calculation of Eq. (8) for the symmetric nuclei at mass 16 and 40 with qualitatively similar results. Thus, we consider that a reasonable analytic approximation to the true pion exchange mean field is given by the potential

$$U^{(I=1)}_{PT}(1)=F(m_\pi R)U^{(I=1)}_{PT,\text{asympt}}(1) . \quad (11)$$

We stress again that this same suppression factor will apply to the isoscalar and isotensor asymptotic mean potentials given by Herczeg[3] (with an additional factor of $\frac{1}{2}$). It also generalizes to the exchange of heavier mesons.

Also in Fig. 1 we plot the P violating meson-exchange suppression factors $W_{Z,N}$ (Ref. 5) for comparison. From our previous discussion regarding the relative merits of direct and exchange terms, we expect that, at any given range, the P violating potential will be reduced more than the PT violating potential and, indeed, this is seen to be true. Nevertheless, the analogy between the two cases is limited since we have demonstrated that the direct PT violating potential does exhibit some suppression, a characteristic not shared by the corresponding direct terms in the parity case.

In Fig. 2 we show the dependence of the pion-exchange suppression on the atomic mass number through the relation $R=1.2A^{1/3}$. We see that the suppression increases rapidly below $A=50$. This is to be contrasted with P violating exchange where the suppression, which is referred to the nucleon density, is virtually independent of the nuclear mass. Therefore, when searching for PT violating effects in nuclei, there is some advantage to be had by choosing heavier masses.

In addition to the finite range of the exchanged meson, there is an entirely separate effect to be considered which arises from the short-range nucleon-nucleon correlations. These correlations may be included by calculating matrix elements of the modified interaction $\Gamma V(1,2)\Gamma$ in the numerical derivation of the one-body meson field (2), where

$$\Gamma=1-\exp(-\gamma_1 r^2)(1-\gamma_2 r^2) . \quad (12)$$

With the parameters $\gamma_1=1.1\ \text{fm}^{-2}$ and $\gamma_2=0.68\ \text{fm}^{-2}$ of Ref. 8, we find that the matrix elements of the one-body PT violating mean-field potential are reduced by only

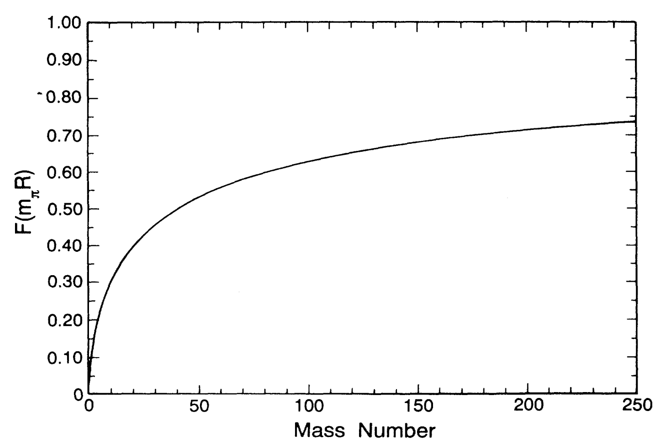

FIG. 2. Dependence of the function $F(m_\pi R)$ (10) on the nuclear mass number A.

$\sim 5\%$ for pion exchange (and $\sim 60\%$ for ρ-meson exchange). These are to be contrasted with the corresponding P violating potential where one finds suppressions of $\sim 30\%$ and $\sim 70\%$ (Ref. 5) for pion- and ρ-meson exchanges, respectively. In terms of the effective one-body mean field (11), we may approximate the suppression of the short-range correlations by the inclusion of a factor $G(m_\pi,\gamma_1,\gamma_2)$, where

$$G(m,\gamma_1,\gamma_2)=\int_0^\infty \Gamma r\frac{\exp(-mr)}{mr}\left[1+\frac{1}{mr}\right]\Gamma r^2 dr \Big/ \int_0^\infty r\frac{\exp(-mr)}{mr}\left[1+\frac{1}{mr}\right]r^2 dr \ . \tag{13}$$

The origin of this expression should be clear if we point out that the denominator is just the radial part of the integral performed in reducing Eq. (6) to the asymptotic mean field of Eq. (7). A comparison with the exact suppression of the true one-body mean field obtained from Eq. (2) as a function of exchanged meson range (mass) shows that $G(m,\gamma_1,\gamma_2)$ closely simulates the effects of the short-range correlations.

We wish to make some general comments concerning the best strategy when selecting cases to study PT noninvariance. First, we would like to point out that Eq. (1) does not imply that cases with multipole mixing ratios $\delta\sim 1$ lead to optimal sensitivity to the nuclear PT admixtures. In fact, since the considered combination of vectors does not measure the interference between electric and magnetic transitions of the same multipolarity, in the present example one can only observe the $E2$, $E3$ mixing. Thus, ideally we require a "regular" transition with a minimal $M2$ content for which we have $\delta \gg 1$.

It follows from Eq. (1) that a favorable case to study PT violation in nuclear gamma decay should simultaneously exhibit the following features: (i) There should exist a pair of close-lying states $|a_0\rangle$ and $|a_z\rangle$ (so that the energy denominator is small) of opposite parity and of the same angular momentum, (ii) the matrix element of PT violating interaction connecting $|a_0\rangle$ and $|a_z\rangle$ should be as large as possible, and (iii) the admixed "irregular" multipole [$E2$ in the case of the situation considered in Eq. (1)] should be enhanced, i.e., it should involve a collective transition (rotational or vibrational), while at the same time the "regular" multipole should be hindered.

To illustrate the difficulties encountered when trying to fulfill the above criteria, we plot in Fig. 3 the distribution of the one-body PT violating strength as a function of excitation energy. This has been constructed by considering all possible quasiparticle states which can be admixed into some two-quasiparticle (or one-quasiparticle) reference state via the operator

$$\sum_{s,t}(u_s u_t + c v_s v_t)\langle s|U|t\rangle \alpha_s^\dagger \alpha_t - \sum_{s>t}(u_s v_t - c v_s u_t)(\langle s|U|\tilde{t}\rangle \alpha_s^\dagger \alpha_t^\dagger + \text{H.c.}) \ ,$$

where $\alpha_s^\dagger$ is the quasiparticle creation operator and u_s, v_s are the BCS pairing amplitudes. The dichotomous quantum number c describes the properties of the potential under time reversal and takes the values $+1$ and -1 for PT and P violating potentials, respectively. We then average over all possible choices of reference states to obtain a typical set of matrix elements of the potential (11). These are then squared and plotted in Fig. 3(a) as a function of the corresponding excitation energy in histogram form. We confine the single-particle basis to Nilsson states from the $N=4,5$ shells, calculated at deformations $\delta=0$ (dashed lines) and 0.20 (solid lines), using modified

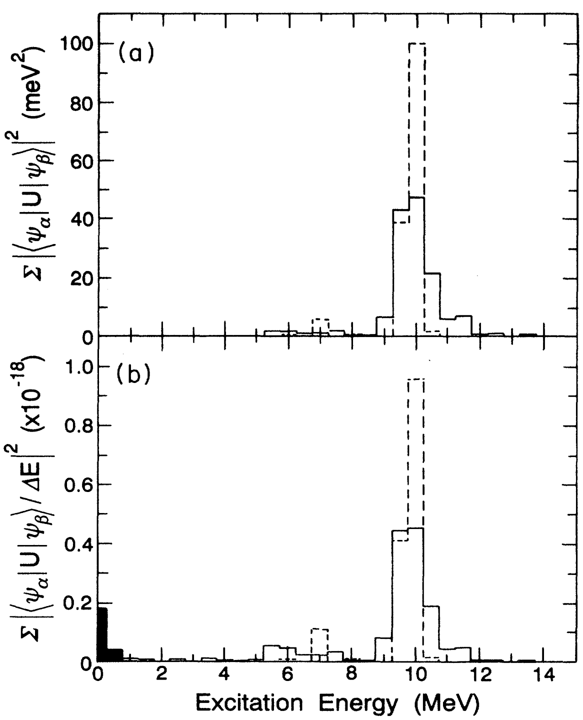

FIG. 3. Matrix elements of the one-body PT violating potential (11) as function of the excitation energy. The coupling constant $\bar{g}^{(1)}_{\pi NN}$ was set to 3×10^{-10}, the approximate upper limit set by the neutron electric dipole moment. Calculations were performed for the nucleus with $Z=74$, $A=182$, for the spherical case (dashed histogram) and deformation $\delta=0.20$ (solid histogram). In the upper part, (a), the sum of squared matrix elements within each energy channel is shown. In the lower part, (b), the matrix elements are divided by the excitation energy before squaring and adding.

oscillator parameters appropriate to $Z=74$, $A=182$.

It can be seen that essentially all the *PT* (and also *P* only) violating single-particle strength is concentrated at rather high excitation energy. That can be easily understood when one remembers the selection rules for the potential *U*, Eq. (11), namely, $\Delta j=0$, $\Delta\pi=$yes. In the spherical extreme, shell-model states that obey this selection rule are separated by $1\hbar\omega+\Delta_{\text{spin orbit}}$. These selection rules are weakened in deformed nuclei, but only a tiny piece of the strength appears at low excitation energy.

The actual admixture is determined by the ratio of the *PT* matrix element to the energy denominator, which we plot in Fig. 3(b) in a similar manner. One can see that, for a deformed nucleus, the energy denominator weighting significantly enhances the admixtures of close-lying states [shown solid in Fig. 3(b)]. Nevertheless, taken at face value, Fig. 3 would still suggest that one is generally not justified in restricting the summation in Eq. (1) to a single close-lying admixed state. To reach a valid conclusion, however, one has to also consider the criterion (iii) above, namely, the size of the matrix element of the admixed multipole. We shall show later[9] that detailed calculations do, in fact, suggest that, in carefully selected cases, the nearest state does dominate the sum in Eq. (1).

Another lesson of the plot in Fig. 3 is that, since the one-body potential only rarely connects two opposite parity states close in energy, in a detailed analysis one has to consider the parts of the two-body interaction that are not contained in the mean-field approximation. Again, we shall consider terms of this kind later.[9] Finally, since the strength is so small in the independent particle (or quasiparticle) model, it is essential to include configuration mixing in the analysis.

In conclusion, we have found an effective *PT* violating one-body potential that includes the suppression due to the finite meson range. We compared the exact numerical results with an approximate analytic formula and found a good agreement between them. Further, we have discussed various criteria that should help in selecting optimal cases for the experimental study of the *PT* violation in nuclear gamma transitions. While we have not found an ideal case, we argued that the heavy deformed nuclei offer the best opportunity for the enhancement of the *PT* violating effects.

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