Enhanced Heat Capacity and a New Temperature Instability in Superfluid ^{4}He in the Presence of a Constant Heat Flux Near T_λ

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We present the first experimental evidence that the heat capacity of superfluid ^{4}He, at temperatures very close to the lambda point T_λ, is enhanced by a constant heat flux Q. The heat capacity at constant Q, C_Q, is predicted to diverge at a temperature $T_c(Q) < T_\lambda$ at which superflow becomes unstable. In agreement with previous measurements, we find that dissipation enters our cell at a temperature, $T_{\rm DAS}(Q)$, below the theoretical value, $T_c(Q)$. We argue that $T_{\rm DAS}(Q)$ can be accounted for by a temperature instability at the cell wall, and is therefore distinct from $T_c(Q)$. The excess heat capacity we measure has the predicted scaling behavior as a function of T and Q, but it is much larger than predicted by current theory.

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There has been intense recent interest in the superfluid transition of liquid ^{4}He in the presence of a constant heat flux Q because it provides an ideal testing ground for the study of phase transitions under dynamical conditions [1–11]. A heat flux induces a counterflow between the superfluid and the normal fluid, which gives the system an extra degree of thermodynamic freedom and depresses the superfluid density ρ_s. These effects are predicted to lower the temperature at which superfluidity vanishes, and to have a significant influence on the thermodynamic properties.

In particular, superfluidity is destroyed at a temperature $T_c(Q)$ [12] where superflow becomes unstable because $\partial Q/\partial v_s = 0$, where v_s is the superfluid velocity. It is easy to show [1,2] that the heat capacity C_Q also diverges at this temperature. The instability occurs due to the depression in ρ_s, which limits the ability of the superfluid to conduct heat since, according to the two fluid model, $Q \simeq -\rho_s v_s S T$, near the lambda point, where T is the temperature and S the entropy. The instability occurs along a curve in the T-Q plane of the form

$$t_c(Q) = \frac{T_\lambda - T_c}{T_\lambda} = \left(\frac{Q}{Q_0}\right)^x. \qquad (1)$$

Theories [9,10,13] give $x = 1/2\nu = 0.746$ where ν is the correlation length exponent. Haussman and Dohm (HD) applied renormalization-group theory to the problem and found $Q_0 = 7395$ W/cm^2 [11], although Haussmann recently modified this prediction to be 6571 W/cm^2 [3].

We have measured C_Q in the range $1 \le Q \le 4$ μW/cm^2. A typical result, for $Q = 3.5$ μW/cm^2, is shown in Fig. 1. C_Q agrees well with the $Q = 0$ heat capacity data for all temperatures lower than about 1 μK below T_λ. Between 1 and 0.5 μK below T_λ, a significant increase of the heat capacity is observed. This is the first direct experimental evidence for the increase in C_Q near $T_c(Q)$. The increase is much larger than predicted [1,2], but a recent theory by Haussmann [3], shown in the figure, comes closer to the experimental result than earlier theories. Just after the point marked α in Fig. 1, a sudden increase in thermometer noise signals the breakdown of superfluidity in our cell. In a thermal conductivity experiment, Duncan, Alhers, and Steinberg (DAS) [4] observed that the onset of thermal resistance occurs at a temperature we call $T_{\rm DAS}(Q)$. They found that $T_{\rm DAS}(Q)$ obeyed the same power law as $T_c(Q)$, but with $x = 0.813 \pm 0.012$ and $Q_0 = 568 \pm 200$ W/cm^2. We identify the temperature at which dissipation enters our cell to be equal to $T_{\rm DAS}(Q)$, which is always below $T_c(Q)$. To understand our result, one must understand why the DAS phenomenon occurs.

There have been several explanations for the discrepancy between the theoretical prediction of superfluid breakdown and the experimental results. Because the order parameter

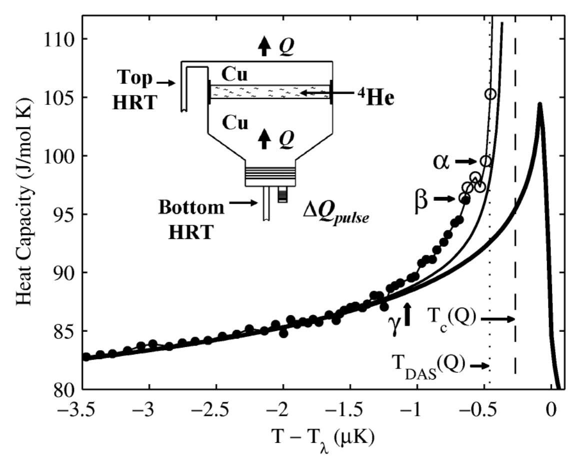

FIG. 1. $Q = 3.5$ μW/cm^2. Thick solid line: C_Q for $Q = 0$ (rounded for gravity); thin solid line: Haussmann's theory [3] (rounded for gravity); solid circles: data from the average of the top and bottom thermometers; open circles: data from the top thermometer only; α: the last point before dissipation enters the cell; β: the change in the boundary resistance affects the bottom thermometer; γ: the maximum temperature of the data of FBKA [17]. Inset: schematic diagram of the cell.

does not go to zero at $T_c(Q)$, HD [11] suggested that the transition is like a spinodal line of a first-order phase transition. This implies that, when approaching $T_c(Q)$ from the superfluid side, fluctuations will induce the transition to occur at a lower temperature. Liu and Ahlers (LA) [6] identify this lower temperature with $T_{\mathrm{DAS}}(Q)$. Furthermore, they report the observation of a region of small but finite resistivity that they believe lies between $T_{\mathrm{DAS}}(Q)$ and $T_c(Q)$. An experiment by Murphy and Meyer [7] confirmed the existence of this anomalous dissipative region, but called LA's placement of the region into question. An alternative explanation is given in the theory by Haussmann [3]. He calculated the thermal conductivity of superfluid ^{4}He in the presence of both a heat current and gravity, and identifies T_{DAS} with a gravity dependent transition.

We propose a quite different explanation of T_{DAS}. A heat flux Q, flowing across a solid wall into superfluid helium, produces a thermal gradient in the fluid within a correlation length of the wall [14]. The resulting thermal resistance, $R_b = \Delta T_b/Q$, is known as the singular contribution to the Kapitza resistance. R_b is thought to diverge when the mean boundary temperature approaches T_λ. We will show that if one assumes only that ΔT_b is linear in Q, then there is a temperature instability that drives the superfluid-normal fluid interface into the cell when the bulk superfluid temperature is very close to $T_{\mathrm{DAS}}(Q)$.

A constant heat flux, Q, may be maintained in a bulk sample of superfluid helium at a fixed temperature, T_{SF}, by flowing heat into one wall of the cell and out of the other. At the wall where the heat flows into the cell, the temperature in the boundary layer is higher than T_{SF}. The opposite is true at the wall where the heat exits the cell. It is found that the data for R_b at both walls collapse onto a single curve if R_b is expressed as a function of the mean boundary temperature $T_b = (T_{\mathrm{SF}} + T_W)/2$, where T_W is the temperature of the superfluid next to the wall [15].

Measurements [15–17] indicate that a logarithmic plot of the singular boundary resistance R_b versus the reduced boundary temperature, $t_b = 1 - T_b/T_\lambda$, is nearly a straight line, so that

$$\begin{aligned} \Delta T_b = T_W - T_{\mathrm{SF}} &= QR_b = Q\beta(t_b)^{-z}, \\ T_W - T_{\mathrm{SF}} &= Q\beta\left(1 - \frac{(T_W + T_{\mathrm{SF}})}{2T_\lambda}\right)^{-z}, \end{aligned} \tag{2}$$

where β and z are fitting parameters to the experimental singular Kapitza resistance data.

Equation (2), which is merely an empirical fit to experimental data, has an instability built into it. When $d\Delta T_b/dT_{\mathrm{SF}} \to \infty$, any change in T_{SF} causes T_W to run away. This is precisely the behavior observed in the experiments. If we solve Eq. (2) for T_{SF} and apply this condition in the form $dT_{\mathrm{SF}}/d\Delta T_b = 0$, we find that the instability occurs when

$$T_W - T_{\mathrm{SF}} = \left(\frac{2T_\lambda}{z}\right)^{z/(z+1)} (\beta Q)^{1/(z+1)}. \tag{3}$$

Setting Eq. (2) equal to Eq. (3) and rearranging the terms, we find the reduced temperature of the bulk superfluid, t_i, when the boundary temperature goes unstable,

$$t_i = \frac{T_\lambda - T_i}{T_\lambda} = \left[\frac{\beta Q z(\frac{z+1}{z})^{z+1}}{2T_\lambda}\right]^{1/(z+1)} = \left(\frac{Q}{Q_0}\right)^x, \tag{4}$$

where, for typical values of β and z, $Q_0 \approx 700$ W/cm^2 and $x = (z + 1)^{-1} \approx 0.8$. Thus the instability temperature is close to T_{DAS}. To make a more accurate comparison, we note that a logarithmic plot of the singular boundary resistance data has some curvature, so it cannot be fitted adequately by single values of z and β. However, for any t_b, one can do a local linear fit to find accurate values of β and z. By once again setting Eq. (2) equal to Eq. (3), but this time rearranging the terms differently, we find that a given reduced boundary temperature, t_b, will go unstable at a value of $Q = Q_i$,

$$Q_i = \frac{2T_\lambda}{z\beta}(t_b)^{z+1}. \tag{5}$$

We can now proceed directly from the boundary resistance data of Fu, Baddar, Kuehn, and Ahlers (FBKA) [17] to a prediction of superfluid breakdown. We simply find a local fit for each of nine values of t_b, and then plot Eq. (4) versus Eq. (5) on the same graph as the DAS data. This is shown in Fig. 2. The results predicted from the singular Kapitza resistance data are at larger reduced temperatures than the DAS data, but the agreement with the DAS data is excellent nevertheless. A fit to our predicted points in the form $t_i = (Q/Q_0)^x$ gives $x = 0.8163 \pm 0.0023$ and $Q_0 = 813 \pm 9$ W/cm^2, which is consistent with the DAS fit.

From the above equations, one can show that $(T_\lambda - T_i)/(T_W - T_i) = (1 + z)/2$. Since $z < 1$ for all t_b this implies that, when the temperature of the cell becomes unstable at T_i, $T_W > T_\lambda$. One might expect that as $T_W \to T_\lambda$, ΔT_b will depend nonlinearly on Q, and Eq. (2) will no longer hold true. If this is the case, the instability will enter the cell at a lower temperature than if the temperature drop across the boundary were to remain linear in Q. We would therefore expect the predicted points shown on Fig. 2 to lie slightly below an extrapolation of the DAS fit to larger reduced temperatures. For comparison, we have also plotted the reduced temperature of the bulk superfluid when $T_W = T_\lambda$ for the same nine data ranges of FBKA. This shows an even closer match to the DAS results.

In summary, we conclude that the breakdown of superfluidity in our cell, previously observed by DAS and others,

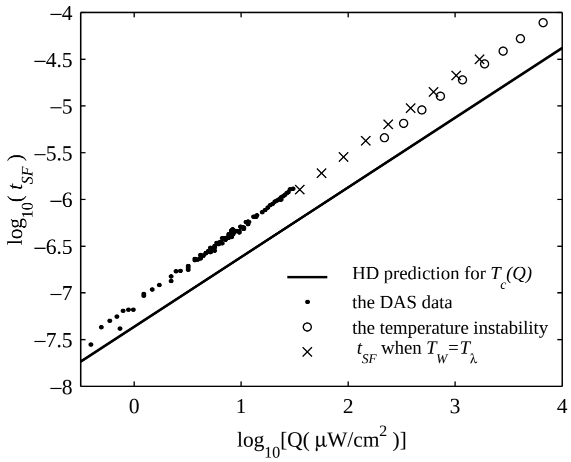

FIG. 2. The breakdown temperature as a function of Q.

is a boundary effect, quite distinct from the physics treated by recent theories [1–3,9–11,13]. It should be possible to devise experiments to test this hypothesis.

Our measurements of C_Q were taken in a cell constructed of two 6.985 cm diameter annealed oxygen-free high conductivity copper end plates connected by a 0.640 mm high stainless-steel sidewall. (See the inset of Fig. 1.) The small cell height was chosen so that it would minimize gravitational rounding of the heat capacity yet still be large enough to avoid finite-size effects. The cell was filled with ultrapure ^{4}He (^{3}He concentration < 0.07 ppb) and then sealed with a mechanical valve. A bubble filling 0.5% of the volume was trapped during this procedure, ensuring that heat capacity measurements would be taken at saturated vapor pressure. The helium temperature was measured with two high resolution paramagnetic salt thermometers (HRTs) [18], with a resolution of 5×10^{-11} K/$\sqrt{Hz}$, located on the top and bottom end plates of the calorimeter.

The adiabatically shielded calorimeter was connected to a 1.3 K continuously operated He-4 refrigerator via a three stage thermal isolation system. The third stage was connected to the calorimeter through a very large thermal impedance (867 K/W). The temperature of this stage was controlled to ± 0.2 μK with another HRT. A constant heat flux, Q, was applied by a wire heater at the bottom of the calorimeter, and extracted from the top through the large thermal impedance.

Measurements began when the sample temperature was approximately 4 μK below $T_c(Q)$. A series of heat pulses of ~ 0.7 μJ each were applied using a second wire heater also located at the bottom of the cell. Approximately two minutes before each heat pulse, the temperature of the third stage was automatically adjusted so that the drift rate of the sample was less than 1×10^{-11} K/sec. The third stage was then held at a constant temperature until approximately two minutes after the heat pulse, when it was then adjusted again in preparation for the next heat pulse. This procedure helped to compensate for the changing heat leak to the thermal network. The sample was pulsed, raising its temperature, until dissipation entered the cell, after which the third stage temperature was lowered so that the helium sample slowly cooled to its starting temperature. The third stage was then adjusted to null the drift rate, and the process began anew. This procedure was repeated consecutively 5 to 9 times per run so that the data could be averaged.

The temperature of the helium was inferred by averaging the measurements of the top and bottom thermometers, and then subtracting off a term to correct for the asymmetry between the top and bottom singular boundary resistances, using the data of FBKA [17]. However, the temperature range for these data comes only as close to T_λ as the point marked γ in Fig. 1. Between that point and the point marked β we, in effect, made our own measurements of the singular Kapitza resistance by choosing values of $R_b(t_b)$ such that the top and bottom HRTs both gave the same value for T_{SF}. These values of R_b turned out to be a smooth extrapolation of the data of FBKA.

The HRTs used in our experiment do not give a reading of the absolute temperature. It was therefore necessary to fix the temperature scale for each run by matching a feature in our data with a previously measured quantity. We did this by correlating the temperature at which dissipation left our cell on the downward temperature ramps with $T_{DAS}(Q)$. This point was determined by observing the temperature at which the noise of the bottom thermometer suddenly decreased. We then fixed the temperature scale of each thermometer by setting its temperature at this point equal to $T_{DAS}(Q) \pm QR_b$, where R_b is the singular boundary resistance. Thus, our thermometers were adjusted to read T_W rather than the temperature inside the copper, which would be affected by the regular Kapitza resistance. The correctness of our identification of $T_{DAS}(Q)$ is verified by the excellent agreement it produces between C_Q and the heat capacity at $Q = 0$, for temperatures lower than about 1μK below T_λ (see Fig. 1). The temperature scale for $Q = 0$ is established by identifying T_λ experimentally.

At the point marked β in Fig. 1, the temperature steps measured by the bottom thermometer suddenly became larger than those measured by the top thermometer. This phenomenon occurred at roughly the same value of T_W at the bottom surface for all values of Q used in our experiment. This value of T_W corresponds to a local correlation length, ξ, of a few micrometers, which we believe to be of the same order as the surface roughness of the wall. When ξ becomes larger than the surface roughness, the effective area through which heat passes from the interface into the bulk helium is reduced, thereby increasing the apparent R_b. The correlation length at the upper boundary never becomes large enough to produce this phenomenon there. Thus the points between β and α in Fig. 1 are measured using the upper thermometer only, corrected by values of

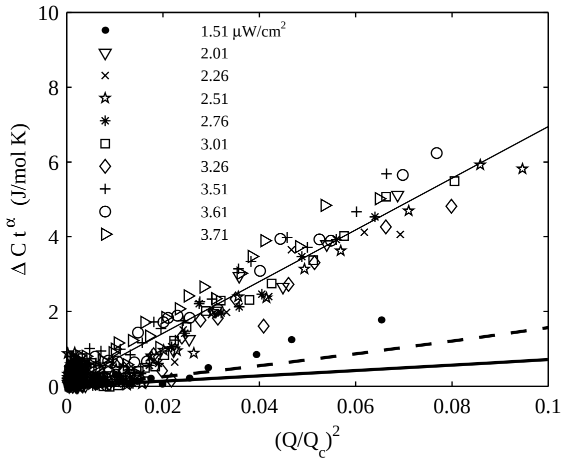

FIG. 3. Differential heat capacity measurements for various values of Q. The experimental data are terminated at β and scaled against the Q_c obtained by using $Q_0 = 6571\ \mathrm{W/cm^2}$. Thick solid line: original theoretical prediction [1,2] (not rounded for gravity); dashed line: Haussmann's prediction [3] (not rounded for gravity); thin solid line: fit to the $Q \geq 2\ \mu\mathrm{W/cm^2}$ data.

R_b that are extrapolated from our own data and those of FBKA.

Figure 3 shows the heat capacity enhancement, $\Delta C_Q = C_Q - C_{Q=0}$, for a number of values of Q, plotted versus $(Q/Q_c)^2$, where $Q_c(T)$ is obtained by inverting Eq. (1) with $Q_0 = 6571\ \mathrm{W/cm^2}$. The data are taken at constant Q, changing Q_c by changing T. They are measured with the average of the top and bottom thermometers, and are therefore terminated at point β of Fig. 1. The quantity $\Delta C_Q t^\alpha$, where $\alpha = 2 - 3\nu$, is expected to depend only on the ratio Q/Q_c. The data agree well with this prediction for all but the lowest heat currents. Theoretical predictions for $\Delta C_Q(Q/Q_c)t^\alpha$ are also shown. At small Q/Q_c, where the depression of ρ_s may be ignored, $\Delta C_Q t^\alpha$ should be proportional to Q^2. Thus, plotted as we have done in Fig. 3, all the data should fall on a single curve, which should be a straight line at small Q/Q_c. All of our data for $2\ \mu\mathrm{W/cm^2} \leq Q \leq 4\ \mu\mathrm{W/cm^2}$ can be represented by

$$\Delta C_Q t^\alpha = A(Q/Q_c)^2, \tag{6}$$

where $A = 69 \pm 4$ J/mol K. This value of A is much larger than predicted by any current theory.

We have shown that it is possible to measure the enhancement of the heat capacity of superfluid ^{4}He due to a heat current, and that the enhancement is larger than predicted by theory. Thus, there is new physics to be examined near $T_c(Q)$. We have also argued that the dissipation that limits the experimental approach to $T_c(Q)$ is due to a temperature instability at the cell wall arising from the singular boundary resistance. It should be possible to obtain better heat capacity data, in the same temperature range we have explored, using a penetrating sidewall thermometer to make direct temperature measurements in the bulk of the helium sample, unaffected by the singular Kapitza resistance. Ultimately, however, if our hypothesis is correct, only a microgravity experiment in a cell configured to avoid the wall instability will suffice to examine the new physics.

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