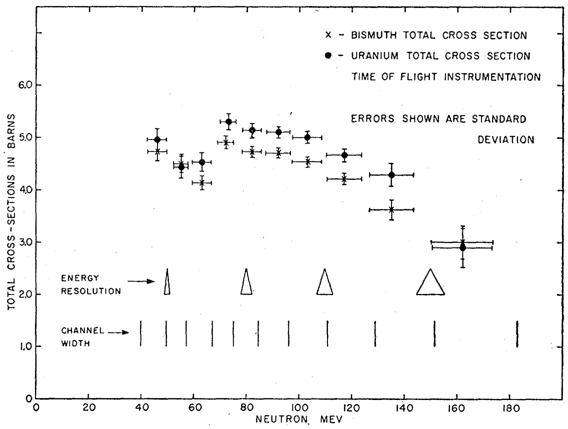

FIG. 1. The variation with energy of the neutron total cross section for bismuth and uranium, measured by time-of-flight instrumentation. The errors shown are standard deviations based on total counts and on energy channel width.

Absolute values of the cross section may be in error by a constant estimated to be ± 0.2 barn because of beam intensity variations. Inasmuch as such a constant would be added to all the points for an element, it would not affect the variation of cross section with energy.

The energy scale was calibrated by time-of-flight of gamma rays. The time-of-flight of a neutron or photon could be measured with a probable error of $\pm 0.2 \times 10^{-8}$ second, including the effect of neutron production time (this leads to a resolution of 5×10^{-11} second per meter at the flight distance of 43.7 meters). At 90 Mev the absolute value of the quoted energy seems to be in error by not more than ± 2 Mev.

We wish to thank Professor H. F. York for suggesting this technique and Professor L. W. Alverez, under whose guidance this work was carried out, for valuable suggestions; also James Vale, Lloyd Hauser, and the cyclotron crew for much cooperation. Thanks are due also Robert Silver and John Leahy whose help at times of runs was particularly welcome, and Vern Ogren and Don Paxson, both of whom rendered expert electronics assistance in many ways.

* This work was sponsored by the U. S. Atomic Energy Commission.

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The Giant Nuclear Dipole Resonance*

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(Received August 21, 1953)

WE have applied the collective description of nucleon interactions[1] to the investigation of the giant dipole resonance in heavy nuclei.[2] Our method follows closely that developed by Bohm and Pines[3] for the treatment of electron-electron interactions in metals. We start with a system of individual nucleons, interacting via short-range two-body forces, and investigate to what extent these forces lead to collective behavior. We find that the part of the nucleon-nucleon potential V_{ij} which is proportional to $\tau_z^i\tau_z^j$ can lead to an oscillation of the nucleus as a whole, similar to the dipole oscillation suggested by Goldhaber and Teller,[4] and investigated by Steinwedel and Jensen.[4] The excited level corresponding to this oscillation lies at an energy

$$\hbar\omega = W_0 A^{-\frac{1}{3}} \tag{1}$$

above the ground state, where a rough calculation indicates $W_0 \approx 80$ Mev, a result which is in agreement with the experimentally observed position of the giant resonance. We find the γ-ray width of this level to be proportional to $A^{\frac{1}{3}}$, and use the width to obtain the cross sections for absorption and scattering of γ rays in this region. Throughout we have neglected certain effects of the $(\tau_x^i\tau_x^j + \tau_y^i\tau_y^j)$ terms in the potential, which are known to give rise to an increase in the frequency of the oscillation, as can be seen from the sum rule calculations of Bethe and Levinger.[5] Preliminary results indicate that such corrections are not large. Surface effects are also neglected, an approximation which restricts the application of our method to heavy nuclei.

Our approach consists in concentrating our attention on the following portion of the nuclear Hamiltonian,

$$H_0 - \sum_i \frac{p_i^2}{2M} + \sum_{\mathbf{k}} \sum_{i \neq j} \frac{V_k}{2} \tau_z^i \tau_z^j e^{i\mathbf{k}\cdot(\mathbf{x}_i - \mathbf{x}_j)}, \tag{2}$$

where the V_k are the Fourier components in a box of nuclear dimensions of a nucleon-nucleon potential of range μ^{-1}. The potential V is formally repulsive since $\tau_z^i\tau_z^j$ appears in the complete Hamiltonian as part of an exchange operator, which we take to be that of Majorana. We perform a canonical transformation directly analogous to that applied by Bohm and Pines[3] in the electron case. We are then able to isolate a part of the Hamiltonian corresponding to harmonic oscillations of the quantities

$$\rho_{\mathbf{k}} = \Sigma_i \tau_z^i e^{-i\mathbf{k}\cdot\mathbf{x}_i} \quad (\mathbf{k} \neq 0), \tag{3}$$

which are the fluctuations in the difference between neutron and proton densities.

The lowest mode of oscillation $[k \approx \pi/R = (\pi/R_0)A^{-\frac{1}{3}}]$ is weakly coupled to the motions of individual nucleons, to the ordinary density fluctuations of the nucleus, and to the terms in the nuclear Hamiltonian that are omitted in (2). The higher modes are strongly coupled, appreciably damped, and thus unimportant. The frequency of the lowest mode is

$$\omega = \left(\frac{Ak^2V_k}{M}\right)^{\frac{1}{2}} = \left[\frac{3\pi^2 D_0}{MR_0^5\mu(\mu^2+k^2)}\right]^{\frac{1}{2}} A^{-\frac{1}{3}} = \frac{W_0}{\hbar} A^{-\frac{1}{3}}, \tag{4}$$

if V is chosen to be a Yukawa potential of depth D_0. The value of W_0 quoted above is obtained by fitting D_0 and μ to low-energy two-particle data.

We identify the giant dipole resonance with the first excited state of our lowest oscillator mode, and calculate the γ-ray width of this level by expressing the dipole moment operator in terms of the $\rho_{\mathbf{k}}$, (3). The matrix elements of the $\rho_{\mathbf{k}}$ are the well-known oscillator matrix elements. The result for the γ-ray width at resonance is

$$\Gamma_\gamma = \frac{1}{6}\frac{e^2}{\hbar c}\frac{\hbar^2\omega^2}{Mc^2}A = \frac{1}{6}\frac{e^2}{\hbar c}\frac{W_0^2}{Mc^2}A^{\frac{1}{3}} \tag{5}$$

for equal numbers of neutrons and protons. Γ_γ depends on V_k only through ω and is very much smaller than the total level width Γ. Inserting (5) in the Breit-Wigner formulae, one may obtain the γ-ray scattering and absorption cross sections in the neighborhood of resonance. Thus,

$$\sigma_{\text{scatt}}(\text{res.}) = \frac{\pi}{6}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{W_0}{\Gamma}\right)^2\left(\frac{\hbar}{Mc}\right)^2 A^{4/3}, \tag{6}$$

which, for example, ≈ 3 millibarns for Ta^{181}.

It is of interest to compare our results for the absorption cross section with the sum rules of Bethe and Levinger, (not including their exchange-force corrections). For the integrated absorption cross section, $\int \sigma dE$, we obtain $\frac{1}{2}\pi^2(e^2\hbar/Mc)A$, which is just their result, showing that our single dipole level "exhausts" the sum rule, just like the level of Goldhaber and Teller. On the other hand, we find for both the mean and harmonic mean energies for photon absorption simply $\hbar\omega$, since there is but a single level. These results are in disagreement with those of Bethe and Levinger only because they neglect correlations in the gound state wave function of the nucleus. Such correlations are of great importance in our model, and seem to be necessary in order to explain the experimentally observed variation of the resonance energy with atomic number.

Details will be published in a forthcoming paper.

* Research supported in part by the Office of Naval Research, Office of Ordnance Research, U. S. Army, and the U. S. Atomic Energy Commission.
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Nuclear Radii*

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(Received June 26, 1953; revised manuscript received September 8, 1953)

We would like to point out that nuclear radii as predicted from isotope shift and high-energy electron scattering are in excellent agreement. Recent experiments on μ-mesonic x-rays corroborate these results.[1,2] These radii are considerably smaller than those usually quoted. However, they are in excellent agreement with those obtained from the most recent semi-empirical mass formula and may be reconciled with radii as obtained from the Coulomb energy difference in light nuclei.

Electron scattering experiments have been performed by Lyman, Hanson, and Scott[3] with electrons at 15.7 Mev, by Hammer, Raka, and Pidd[4] at 33 and 43 Mev, and by Hofstadter *et al.*[5] at 116 Mev, for a variety of elements. We shall analyze the first set of experiments. The second set gives similar results as far as nuclear radii are concerned. The third set does not show Ramsauer minima, again indicating a small radius. For the lower-energy experiments, according to theory[6] only one phase shift, η_0' is required. We have, therefore, evaluated the phase shift required to match the experimental data at each scattering. The resulting values should be constant. There are, however, a number of difficulties. For very light elements, and for small angles for all elements, the effect of nuclear size is small and would require experiments of great accuracy. For this reason the aluminum data are not useful for the present purpose. For large angles and for heavy elements the scattering is very small, again making the experiments difficult. Moreover, the theoretical uncertainties are greatest at large angles. The most consistent results are obtained for copper and silver, less consistent results for gold. (See Table I.)

Table I. Values of the phase shift η_0'.

ϑ	Copper	Silver	Gold
90°	0.0054	0.0213	0.141
120°	0.0052	0.0228	0.112
150°	0.005	0.0213	0.120

Employing the theoretical results,[6,7] we find that for a homogeneous charge distribution the copper and silver radii are $1.0\times10^{-13}A^{\frac{1}{3}}$ cm and $1.1\times10^{-13}A^{\frac{1}{3}}$ cm, respectively. The nuclear radius for gold is not well determined; but if an average phase shift of $\eta_0'=0.120$ is taken, the nuclear radius is $1.2\times10^{-13}A^{\frac{1}{3}}$ cm. These results are in agreement with those of Raka *et al.*, who obtain a radius of $(1.1\pm0.075)\times10^{-13}A^{\frac{1}{3}}$ cm for Sn and $(1.03)\times10^{-13}A^{\frac{1}{3}}$ cm for W.

The isotope-shift data in Fig. 1 has been summarized by Brix and Kopfermann.[8] We have replotted the data taking the nuclear radius as $1.1\times10^{-13}A^{\frac{1}{3}}$ cm. The agreement with the data is very much better than that obtained with $1.5\times10^{-13}A^{\frac{1}{3}}$ cm. It clearly would be of interest to do electron scattering experiments with the rare earths Ce, Sm, Eu, as well as with Rb, Xe, Ba, which show large deviations from the average line. It should be noted that the tacit assumption is made here that the isotope shift is a pure volume effect.

It is of interest to show that these radii may not be reconciled with the larger Coulomb energy radii of $1.47\times10^{-13}A^{\frac{1}{3}}$ cm commonly quoted,[3] for we shall show that no positive definite charge distribution exists which will give a smaller nuclear radius for scattering and a larger one for the nuclear Coulomb energy. The effective nuclear Coulomb radius R_N is defined by

$$\frac{1}{R_N}=\frac{5}{6Z^2e^2}\int\int\frac{\rho(\mathbf{r})\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}\frac{d\mathbf{r}}{4\pi}\frac{d\mathbf{r}_0}{4\pi}. \tag{1}$$

The scattering and isotope shift depend primarily upon the volume integral of the perturbing potential,[6]

$$\int\frac{\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}d\mathbf{r}_0-\frac{Ze}{r}.$$

It may be shown that this difference is proportional to

$$\int\rho(\mathbf{r})r^2d\mathbf{r}.$$

Hence, the scattering radius R_s is defined by

$$R_s^2=\frac{5}{3Ze}\int\rho(\mathbf{r})r^2(d\mathbf{r}/4\pi). \tag{2}$$

We now ask for what charge distribution the ratio R_s/R_N is stationary:

$$\frac{\delta(R_s/R_N)}{\delta\rho}=0,\quad \delta\int\rho d\mathbf{r}=0;$$

or

$$\frac{1}{R_s}\frac{\delta R_s}{\delta\rho}=\frac{1}{R_N}\frac{\delta R_N}{\delta\rho};\quad \delta\int\rho d\mathbf{r}=0. \tag{3}$$

From Eq. (2) we find

$$\frac{\delta R_s}{\delta\rho}=\frac{5}{6Ze}\frac{r^2}{R_s}, \tag{4}$$

while from Eq. (1) it follows that

$$\frac{\delta R_N}{\delta\rho}=-\frac{5}{3}\frac{R_N^2}{Z^2e^2}V, \tag{5}$$

where

$$V(\mathbf{r})=\int\frac{\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}d\mathbf{r}_0.$$

Inserting (4) and (5) into (3), we find that $V(\mathbf{r})\sim A-Br^2$, where A and B are constants. For positive definite charge density the constants correspond to a homogeneous charge distribution. We determine that the homogeneous charge distribution corresponds to a minimum for the ratio (R_s/R_N) by evaluating the ratio for an actual example. It is, of course, possible to obtain $R_s<R_N$ by relaxing the positive definite charge density condition and thus permitting regions of negative charge within the nucleus, as might be possible in a meson theory of the nucleus.

We turn now to other evidence for nuclear radii. Here it is interesting to note that the most recent determination of the semi-empirical mass formula by Green and Engler[9] give the Coulomb energy term as $0.750(Z^2/A^{\frac{1}{3}})$mMU. This corresponds to the relation $R_N=(1.23\times10^{-13})A^{\frac{1}{3}}$ cm, in agreement with our determination. The second source of evidence is obtained from mirror nuclei. These are light and, as Wigner has pointed out, correlation effects are important. In particular, the exchange Coulomb energy has the effect of reducing the Coulomb energy and therefore increasing the effective radius. Both Elton[10] and Cooper and Henley[1] have pointed out that the nucleon involved in the β transition