

## Consequences of Charge Independence for Nuclear Reactions Involving Photons\*

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Some effects of the charge independence of nuclear forces on the emission and absorption of photons by light nuclei are investigated. It is found that the selection rules governing the change of isotopic spin  $T$  in such transitions are of practical importance in nuclei with  $T_z=0$ , particularly the rule that  $E1$  transitions without change of isotopic spin are forbidden. Two examples of forbidden  $E1$ -emission (in  $N^{14}$  and  $O^{16}$ ) are discussed, and detailed experimental reinvestigation is suggested in connection with each of them.

The main consequence for photonuclear reactions of the aforementioned selection rule is that for each  $T_z=0$  nucleus there is a threshold for (allowed)  $E1$  absorption, corresponding to the first level with  $J=1^-$ ,  $T=1$ . For  $(\gamma, \alpha)$ - and  $(\gamma, d)$ -reactions, where no isotopic spin can be carried off by the particle,  $E1$  absorption can become effective only when there is enough energy to leave the residual nucleus in a  $T=1$  state. The roles of  $E2$  and  $M1$  absorption, with and without change of isotopic spin, are discussed, and appropriate sum rules are derived. The rules governing  $(\gamma, \alpha)$  processes are applied particularly to reactions in  $C^{12}$  and  $O^{16}$ , where considerable experimental evidence is available and appears to support the theoretical conclusions.

### I. INTRODUCTION

CONSIDERABLE evidence<sup>1</sup> has accumulated during recent years in favor of the hypothesis of charge independence of nuclear forces. In light nuclei (say  $Z \leq 10$ ) the effects on nuclear wave functions of the Coulomb force between protons and of the neutron-proton mass difference are expected to be small,<sup>2</sup> and consequently the total isotopic spin  $T$  must be considered a good quantum number<sup>3</sup> in purely nuclear processes; some restrictions imposed on nuclear reactions by the conservation of isotopic spin have been discussed by Adair.<sup>4</sup>

When the emission or absorption of photons takes place, isotopic spin need not be conserved, since the electromagnetic interaction of nucleons is charge-dependent. However, as has been pointed out by Radicati<sup>5</sup> and others, there are definite selection rules governing the change of isotopic spin accompanying such processes. It is our purpose to show how these selection rules may be used in the interpretation of certain experimental observations on photonuclear reactions and  $\gamma$ -decay in light elements and to suggest further experiments for which the hypothesis of charge independence has interesting consequences. Kroll and Foldy<sup>6</sup> have shown that the selection rules follow equally well from the much weaker assumption of the

charge symmetry of nuclear forces. However, we shall adopt the point of view of charge independence since we shall be concerned frequently with the existence of charge triplets, which are sets of  $T=1$  states in neighboring isobars with equal (adjusted) energies. Such triplets are difficult to account for in the absence of charge independence.

In the discussion that follows we shall always assume that the specific nuclear forces are indeed charge independent, and we shall attribute to Coulomb and mass difference effects any failure of  $T$  to be a good quantum number for the nucleus.

### II. SELECTION RULES

Many of the results in this section have been given by Radicati<sup>5</sup> and are reproduced here only for completeness.

In order to derive the isotopic spin selection rules for electromagnetic transitions, let us consider the perturbation Hamiltonian  $H$  corresponding to the emission or absorption of a photon of wave number  $k$  by a system of  $A$  nucleons, neglecting certain magnetic exchange and interaction moments:

$$H = \sum_{i=1}^A \left\{ \frac{e}{c} \frac{d}{dt} \left[ \frac{1}{2} (1 + \tau_{iz}) \mathbf{r}_i \right] \cdot \mathbf{A}(\mathbf{r}_i) + \left[ \frac{\mu_P}{2} (1 + \tau_{iz}) + \frac{\mu_N}{2} (1 - \tau_{iz}) \right] \boldsymbol{\sigma}_i \cdot \nabla \times \mathbf{A}(\mathbf{r}_i) \right\}. \quad (1)$$

Here  $\tau_{iz}$ , the component along the charge axis of the isotopic spin operator for the  $i$ th nucleon, has the eigenvalue  $+1$  for a proton state and  $-1$  for a neutron state.  $\mathbf{r}_i$  is the position vector of the  $i$ th nucleon measured from the center of mass of the nucleus.  $(d/dt)$  applied to an operator means its time rate of change, or  $i\hbar$  times its commutator with the nuclear Hamiltonian.  $\mathbf{A}(\mathbf{r}_i)$  is the vector potential of the electromagnetic field, given

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<sup>1</sup> F. Ajzenberg and T. Lauritsen, *Revs. Modern Phys.* **24**, 321 (1952). All data concerning nuclear levels discussed by us are taken from this article, to which the reader is referred whenever no special references are given.

<sup>2</sup> L. A. Radicati, *Proc. Phys. Soc. (London)* **A66**, 139 (1953).

<sup>3</sup> The notation  $I$  has been used for isotopic spin in some recent papers on charge independence in phenomena involving nucleons and pions. However, for purely nuclear processes it is perhaps wise to retain the older symbol " $T$ " since in discussions of hyperfine structure  $I$  is often used to mean the total angular momentum of the nucleus.

<sup>4</sup> R. K. Adair, *Phys. Rev.* **87**, 1041 (1952).

<sup>5</sup> L. A. Radicati, *Phys. Rev.* **87**, 521 (1952).

<sup>6</sup> N. M. Kroll and L. L. Foldy, *Phys. Rev.* **88**, 1177 (1952).

by

$$\mathbf{A}(\mathbf{r}_i) = (2\pi c\hbar/kV)^{1/2} \mathbf{e} \exp[\pm i\mathbf{k}\mathbf{n} \cdot \mathbf{r}_i] \\ = A_0 \mathbf{e} \exp(\pm i\mathbf{k}\mathbf{n} \cdot \mathbf{r}_i) \quad (2)$$

in an obvious notation; the + sign applies to emission and the - sign to absorption.

For our purposes, it is convenient to separate  $H$  into two parts:

$$H_0 \equiv \sum_{i=1}^A \left\{ \frac{e}{2c} \mathbf{v}_i \cdot \mathbf{A}(\mathbf{r}_i) + \frac{(\mu_P + \mu_N)}{2} \boldsymbol{\sigma}_i \cdot \nabla \times \mathbf{A}(\mathbf{r}_i) \right\}, \quad (3)$$

$$H_1 \equiv \sum_{i=1}^A \left\{ \frac{e}{2c} \frac{d}{dt} (\mathbf{r}_i \tau_{iz}) \cdot \mathbf{A}(\mathbf{r}_i) \right. \\ \left. + \tau_{iz} \frac{(\mu_P - \mu_N)}{2} \boldsymbol{\sigma}_i \cdot \nabla \times \mathbf{A}(\mathbf{r}_i) \right\}. \quad (4)$$

Here  $\mathbf{v}_i$  is the velocity operator for the  $i$ th particle. It is clear that  $H_0$  is a scalar with respect to rotations in isotopic spin space, and hence transitions induced by  $H_0$  obey the selection rule

$$\Delta T = 0. \quad (5a)$$

On the other hand  $H_1$  is the  $z$ -component of a vector in isotopic spin space and hence gives the selection rules

$$\Delta T = 0, \pm 1 \text{ when } T_z \neq 0; \\ \Delta T = \pm 1 \text{ when } T_z = 0. \quad (5b)$$

Here  $T_z$  is equal, of course, to  $(-1/2)$  times the neutron excess in the nucleus.

The most significant fact contained in (5a, b) is that in nuclei with equal numbers of neutrons and protons ( $T_z = 0$ ) electromagnetic transitions without change of isotopic spin must arise from  $H_0$  alone. We will refer to such processes as  $H_0$  transitions.

A multipole expansion of  $H_0$  reveals certain important properties. We shall exhibit the parts of  $H_0$  corresponding to the first few multipoles as power series in the  $kr_i$ , keeping only the first term or two.

The electric dipole ( $E1$ ) part of  $H_0$  is

$$H_0(E1) = A_0 \mathbf{e} \cdot \left\{ \sum_{i=1}^A \left[ \frac{e}{2c} \mathbf{v}_i - \frac{e}{20c} k^2 (\mathbf{r}_i^2 \mathbf{v}_i + \mathbf{v}_i \mathbf{r}_i^2) + \frac{e}{40c} k^2 \right. \right. \\ \left. \left. + (\mathbf{r}_i \mathbf{r}_i \cdot \mathbf{v}_i + \mathbf{v}_i \cdot \mathbf{r}_i \mathbf{r}_i) + \frac{k^2}{4} (\mu_P + \mu_N) \mathbf{r}_i \times \boldsymbol{\sigma}_i + \dots \right] \right\}. \quad (6)$$

The term of lowest order *vanishes* since  $\sum_{i=1}^A \mathbf{r}_i = 0$  identically, and hence there is practically no  $E1$  contribution to  $H_0$  for photons having a wavelength large compared to the nuclear radius.

The electric quadrupole ( $E2$ ) part of  $H_0$  is

$$H_0(E2) = \pm i e \frac{k}{2c} A_0 \mathbf{e} \cdot \left\{ \sum_{i=1}^A \left[ \frac{1}{2} (\mathbf{v}_i \mathbf{r}_i \cdot \mathbf{n} + \mathbf{r}_i \mathbf{v}_i \cdot \mathbf{n}) + \dots \right] \right\}. \quad (7)$$

We shall estimate in the next section the contribution of (7) to the absorption of photons.

In connection with magnetic dipole ( $M1$ ) radiation, it is necessary to mention the magnetic exchange and interaction moments that have been omitted in (1). However the exchange and interaction moments that are usually assumed to be of importance in nuclei<sup>7</sup> and that are omitted in (1) are either of the form

$$\sum_{i,j} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z O_{ij}, \quad (8a)$$

or else of the form

$$\sum_{i,j} (\boldsymbol{\tau}_i - \boldsymbol{\tau}_j)_z O_{ij}, \quad (8b)$$

where  $O_{ij}$  is a two-particle operator on space and spin coordinates. Both expressions (8a, b) are  $z$ -components of vectors in isotopic spin space and thus will provide corrections to  $H_1$  and not to  $H_0$ . We are thus permitted to consider just the operator

$$H_0(M1) = \pm i A_0 \mathbf{n} \times \mathbf{e} \cdot \left\{ \sum_{i=1}^A \left[ \frac{ek}{4c} \right. \right. \\ \left. \left. + \frac{k}{2} (\mu_P + \mu_N) \boldsymbol{\sigma}_i + \dots \right] \right\}. \quad (9)$$

Corresponding expressions may, of course, be written for higher multipoles as well.

### III. SUM RULES

It is well known that sum rules can be obtained for the absorption of radiation of a given multipolarity by nuclei. In light nuclei with  $T_z = 0$ , it is useful to derive separate sum rules for the  $H_0$  transitions alone, using (7) and (8). If  $\sigma_0(E2, W)$  is the cross section for  $H_0$  absorption of  $E2$  radiation at energy  $W$ , then

$$\int \sigma_0(E2, W) \frac{dW}{W^2} = \frac{\pi^2}{137 \cdot 12} \frac{A \langle r^2 \rangle_{00}}{Mc^2}. \quad (10)$$

where  $M$  is the mass of a nucleon and  $\langle r^2 \rangle_{00}$  is the mean squared displacement of a nucleon from the center of mass in the ground state of the nucleus. For  $M1$  radiation, we have the sum rule

$$\int \sigma_0(M1, W) \frac{dW}{W} \\ = \frac{\pi^2}{137} \left( \frac{\hbar}{Mc} \right)^2 \frac{1}{4} \frac{\langle (\mathbf{L} + (\mu_P + \mu_N) 2\mathbf{S} / \mu_0)^2 \rangle_{00}}{\hbar^2}. \quad (11)$$

where  $\mu_0$  is the nuclear Bohr magneton. Equations (10) and (11) are derived on the assumption that the nuclear forces are not explicitly velocity-dependent.

<sup>7</sup> R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); R. K. Osborne and L. L. Foldy, Phys. Rev. **79**, 795 (1950).

However, unlike most nuclear sum rules, they require no correction on account of exchange forces or correlation. Moreover, the  $E2$  sum rule is correct even in the presence of interaction potentials that depend linearly on particle momentum, such as the spin-orbit interaction.

#### IV. EMISSION OF $\gamma$ -RAYS

The  $\gamma$ -ray spectra of nuclei with  $T_z=0$  should be affected in at least two ways by the forbiddenness of  $H_0$  ( $E1$ ) transitions:

(a)  $E1$  radiation will be very weak between two  $T=1$  states, while the  $E1$  transition between the two analogous levels in each of the neighboring isobars may be of normal strength. For example, a level at 6.1 Mev in  $C^{14}$  has been found in the reaction  $C^{13}(d,p)C^{14}$ ;<sup>1</sup> the level decays by  $\gamma$ -ray emission to the ground state ( $J=0^+$ ), and there is some evidence that the  $\gamma$ -ray is  $E1$ .<sup>8</sup> Let us assume for the purpose of illustration that such is the case. Now in  $N^{14}$  the level at 2.31 Mev is analogous to the ground state of  $C^{14}$ . At around 8 Mev, then, there must be a state with  $J=1^-$  analogous to the one at 6.1 Mev in  $C^{14}$ ; perhaps it is the level at 8.05 Mev observed in  $C^{13}(p,\gamma)N^{14}$ . At any rate, the  $\gamma$ -transition from that state to the one at 2.31 Mev, though fully allowed by conservation of angular momentum and parity, is forbidden by the isotopic spin selection rules, while the corresponding transitions in the isobars  $C^{14}$  and  $O^{14}$  are allowed. Of course the transition in  $N^{14}$  is not *totally* forbidden; there are two mechanisms by which it can occur—the intervention of the higher term in the expansion (6) of  $H_0(E1)$  and the impurity with respect to isotopic spin of the initial and final nuclear states.

The former mechanism should lead to a width for decay to the 2.31 Mev state of the order of  $(kR)^4$  times a normal  $E1$  width, where  $R$  is the nuclear radius—that is, about 0.01 percent. The isotopic spin impurity is certainly more effective; Radicati<sup>2</sup> estimates it as 1 percent in amplitude for the ground state, but for a highly excited  $T=1$  state it should be much greater. Still, an impurity of 10 percent in amplitude would yield a width about 1 percent of normal. It would be interesting to make a quantitative experimental comparison of the widths for the forbidden transition and for the allowed  $E1$  transition to the ground state.

(b)  $E1$  radiation will be very weak between two  $T=0$  states. For example, let us consider the level at 7.1 Mev in  $O^{16}$ , found in  $F^{19}(p,\alpha)O^{16}$ <sup>9</sup> and  $N^{16}(\beta^-)O^{16}$ .<sup>10,11</sup> Its spin and parity,  $J=1^-$ , have been determined by  $\alpha$ - $\gamma$  angular correlations.<sup>12</sup> Its isotopic spin is 0 since the lowest  $T=1$  state of  $O^{16}$  is analogous to the ground state

of  $N^{16}$  and lies at about 13.0 Mev. Since the ground state of  $O^{16}$  has  $J=0^+$  and  $T=0$ , the  $\gamma$ -ray transition from the 7.1-Mev level to ground must be  $E1$  and consequently forbidden by (5b). However, we must consider the effect of the two mechanisms of transition mentioned in (a). The first one alone would provide a transition rate of an order of magnitude corresponding to  $M2$ . But any competing mode of de-excitation should proceed even more slowly, according to simple order-of-magnitude estimates. The possibilities are:

(i)  $\gamma$ -decay to the 6.0-Mev level ( $J=0^+$ ) suffers from the same disability as the ground-state decay and there is much less energy available;

(ii)  $\gamma$ -decay to the 6.9-Mev level ( $J=2^+$ ) would proceed by the  $E1$  correction term (as above) and by genuine  $M2$  and is unfavored by the tiny energy difference;

(iii)  $\gamma$ -decay to the 6.1 Mev level ( $J=3^-$ ) by  $E2$  is favored by a factor of  $(c/v)^2 \simeq 100$  by comparison with the ground state transition but the ratio of energy differences enters to the fifth power, so that the ground state should be preferred by a factor of  $7^5/100 \simeq 200$ ;

(iv) internal pair emission to the ground state through the intervention of the scalar potential is unfavored by a factor of at least  $e^2/\hbar c \simeq 1/100$ .

The experimental evidence, however, seems to indicate that some process *does* compete favorably with the ground-state transition. The 7.1-Mev  $\gamma$ -ray is observed in both the  $(p,\alpha)$ -reaction and the  $\beta$ -decay; in the latter case certain information has been obtained about the intensity of the line. It has been reported<sup>10,11</sup> that the  $\beta$ -decay of  $N^{16}$  leads with equal probability (40 percent in each case) to the levels at 7.1 Mev and 6.1 Mev, while Millar *et al.*<sup>13</sup> have found 12 times as many 6.1-Mev  $\gamma$ -rays as 7.1-Mev  $\gamma$ -rays following the  $\beta$ -emission.

If both experimental results are accepted as correct, we must conclude that the matrix element for the  $E1$  correction term is much smaller than simple dimensional arguments indicate, and furthermore that the admixture of  $T=1$  in the wave functions of both the initial and final states is either astonishingly small ( $\simeq 0.1$  percent in amplitude) or else anomalously ineffective in inducing  $E1$  transitions. In short, the selection rule (5b) seems to be *oversatisfied*. This curious circumstance calls, in our opinion, for a re-investigation of the  $\beta$ -decay branching ratio and a search for the 1-Mev  $\gamma$ -ray (7.1 $\rightarrow$ 6.1).<sup>†</sup>

<sup>13</sup> Millar, Bartholomew, and Kinsey, Phys. Rev. **81**, 150 (1951).

<sup>†</sup> Note added in proof.—Jones and Wilkinson [Phys. Rev. **90**, 722 (1953)] report that the 7.1-Mev  $\gamma$ -ray is at least 120 times more intense than the 1-Mev  $\gamma$ -ray, in agreement with (iii). However, their claim that this result sets a lower limit on the isotopic spin impurity of the 7.1-Mev state appears to us unjustified. They have neglected the effect of the  $E1$  correction term, which alone should yield an intensity ratio comparable to that observed.

Boehm, Peaslee, and Perez-Mendez [Phys. Rev. (to be published)] have recently found that the 1-Mev  $\gamma$ -ray following the  $\beta$ -decay of  $N^{16}$  is less than one-twentieth as frequent as the

<sup>8</sup> R. G. Thomas and T. Lauritsen, Phys. Rev. **88**, 969 (1952).

<sup>9</sup> A. P. French and J. Seed (unpublished).

<sup>10</sup> Bleuler, Scherrer, Walter, and Zünti, Helv. Phys. Acta **20**, 96 (1947).

<sup>11</sup> H. S. Sommers, Jr., and R. Sherr, Phys. Rev. **69**, 21 (1946).

<sup>12</sup> Kraus, French, Fowler, and Lauritsen, Phys. Rev. **89**, 299 (1953).

### V. ABSORPTION OF $\gamma$ -RAYS

In nuclear reactions induced by photons, the observation of forbidden electromagnetic transitions may be expected to play no important role. We must rather be concerned with the kinds of particle emission that can follow  $\gamma$ -ray absorption; for  $W$  less than about 50 Mev, moreover, we may restrict ourselves to  $E1$ ,  $E2$ , and  $M1$  transitions. The effects of isotopic spin conservation are most striking in even-even nuclei with  $T_z=0$ ; for such nuclei, the lowest  $T=1$  state is at a high excitation energy  $W_1$  and the ground state always has  $J=0^+$ . The following rules can then be deduced from (5b) and the usual conservation laws *in the approximation in which charge independence is rigorously true*:

(1) For  $W < W_1$ , all absorption proceeds either by  $M1$  through a compound state with  $J=1^+$ ,  $T=0$ , or by  $E2$  through a compound state with  $J=2^+$ ,  $T=0$ .

(2) For  $W > W_1$ , absorption into  $T=1$  states is possible by  $M1$ ,  $E2$ , or  $E1$ ; but such  $H_1$  absorption can result in emission of deuterons or  $\alpha$ -particles only if there is sufficient energy to leave the residual nucleus in a  $T=1$  state.

(3) At any energy, a  $(\gamma, \alpha)$ - or  $(\gamma, d)$ -reaction that leaves the residual nucleus in a  $T=0$  state must proceed as in (1).

(4) In particular, a  $(\gamma, \alpha)$ -reaction to the ground state of the residual nucleus may proceed only by  $E2$  absorption through a state with  $T=0$ ,  $J=2^+$ .

(5)  $(\gamma, n)$  and  $(\gamma, p)$  cross sections must, for each residual state, be identical with each other as functions of energy and angle except to the extent that there is interference between  $H_0$  absorption and  $H_1$  absorption.

The existence of Coulomb forces and of the neutron-proton mass difference introduces important deviations from some of these rules:

(1') Following  $H_1$  absorption, there is in general some probability of  $\alpha$ - and  $d$ -emission to a  $T=0$  state on account of isotopic spin impurity in the initial and final states and, to a lesser extent, in the  $\alpha$ -particle. While re-emission of  $\gamma$ -rays may not, in general, compete strongly enough to suppress such a process, strong neutron or proton emission should be expected to do so. Consequently there may be a region of energy, between  $W_1$  and the effective threshold for  $n$ - or  $p$ -emission, in which  $\alpha$ -particles are produced with an appreciable probability in violation of rules (2), (3), or (4).

(2') For each state in the residual mirror nuclei, the thresholds and barrier penetrabilities for  $n$  and  $p$  are in reality different; only after correction for this effect should (5) be valid. It is not likely that isotopic spin impurity is of any importance here.

The charge-dependent perturbations contribute also to the fate of a residual  $T=1$  state produced in ac-

cordance with rule (2). Again, if  $\gamma$ -ray emission is the only process that competes effectively, further  $\alpha$ -decay to a  $T=0$  state may occur. It should be noted that such an  $\alpha$ - $\alpha$  cascade through a level with  $T=1$  will prevail not only over a corresponding cascade through a  $T=0$  level but also over the direct emission of two  $\alpha$ 's. For these "forbidden" modes of decay of the initial  $T=1$  state we expect the smallness of isotopic spin impurity not to be compensated by phase-space factors. In the allowed mode of decay of the initial  $T=1$  state the residual nucleus may find itself unable to do anything but violate the conservation law.

Let us apply the preceding considerations to  $C^{12}$ . The lowest  $T=1$  level, analogous to the ground states of  $B^{12}$  and  $N^{12}$ , should lie at about 15.2 Mev and have  $J=1^+$ . Probably it is a level found at 15.09 Mev in  $B^{11}(d, n)C^{12*}$ . The analog of the first excited state of  $B^{12}$  at 0.95 Mev is probably the level at 16.07 Mev with  $J=2^+$ . That the latter level does indeed have  $T=1$  is borne out by the fact that it emits  $\alpha$ 's to the lowest two states of  $Be^8$  while its total width is only about 5 kev, including the width for proton emission. (Since a typical width for allowed  $\alpha$ -emission at this energy should be of the order of 1 Mev, it is apparently an isotopic spin impurity of less than 10 percent in amplitude that is responsible for the  $(p, \alpha)$ -process.) We have thus located the thresholds for  $H_1$  absorption of  $M1$  and  $E2$  radiation, respectively. At some higher energy  $W_1$  ( $E1$ ), there is presumably a level with  $T=1$  and  $J=1^-$ , which is the lowest one accessible to  $E1$  radiation. Above this energy the  $E1$ -absorption should increase rapidly and soon predominate over other multipolarities. It should manifest itself principally in  $(\gamma, p)$ - and  $(\gamma, n)$ -processes as long as the energy is insufficient to meet condition (2). In this energy range,  $E1$ -induced  $(\gamma, \alpha)$ -reactions may proceed only through isotopic spin impurity. The threshold for the *allowed* reaction leading to the lowest  $T=1$  level of  $Be^8$  is about 26 Mev, a value obtained by adding the binding energy of an  $\alpha$ -particle in  $C^{12}$  and the Coulomb barrier to 16.8 Mev, the energy at which the level in  $Be^8$  is expected to lie. (It is the analog of the ground states of  $Li^8$  and  $B^8$  and should have  $J=2^+$ .) Thus above 26 Mev there is a fully allowed  $(\gamma, \alpha)$ -reaction induced by  $E1$  and it should overshadow all other  $(\gamma, \alpha)$  process. The  $(\gamma, d)$ -reaction should exhibit much less striking behavior. The threshold is at about 25 Mev while that for reactions leading to  $T=1$  states of  $B^{10}$ , allowed for  $E1$ , is at about 28 Mev.

The experimental evidence on the photodisintegration of carbon, which in fact prompted this investigation, appears to agree with the conclusions presented here. The data on  $(\gamma, n)$ -<sup>14</sup> and  $(\gamma, p)$ -reactions<sup>15</sup> are too conflicting to be compared with rule (5). However, they indicate a steep rise in the absorption cross section in the neighborhood of 20 Mev, which must correspond to the onset of  $E1$  absorption. The

6.1-Mev  $\gamma$ -ray. Their result, together with that of Millar *et al.*, seems to indicate that the  $\beta$ -decay branching ratio is indeed not 1:1, but heavily favors the 6.1-Mev state.

<sup>14</sup> Haslam, Johns, and Horsley, Phys. Rev. 82, 270 (1951).

<sup>15</sup> J. Halpern and K. Mann, Phys. Rev. 83, 370 (1951).

$(\gamma, \alpha)$  cross section<sup>16,17</sup> not resolved into "fine structure"<sup>16</sup> due to absorption into discrete levels, exhibits two principal peaks around 18 and 29 Mev, respectively, each of half-width about 4 Mev and height about 0.3 mb. Let us divide the entire energy range into three regions. The first peak lies in region "A" between threshold and 20 Mev in which we expect all absorption to proceed through  $H_0(E2)$  and  $H_0(M1)$ . In fact Telegdi<sup>18</sup> has found that at 17.6 Mev  $E2$  and  $M1$  contribute about equally and that  $E1$  does not participate noticeably. On the basis of Eq. (11) one sees that appreciable  $H_0(M1)$  absorption indicates that the ground state of  $C^{12}$  is by no means a pure  $^1S_0$  state, as the independent particle model in  $L$ - $S$  coupling would predict. In region A, the  $(\gamma, \alpha)$ -reaction leads predominantly to the 3 Mev state ( $J=2^+$ ,  $T=0$ ) of  $Be^8$ . While, in accordance with rule (4),  $M1$ -induced transitions cannot leave  $Be^8$  in its ground state, it is not clear why the  $E2$ -induced transitions show a marked preference for the 3 Mev state at energies high enough for the  $s$ -wave and  $d$ -wave barrier penetrabilities to be roughly equal.

Energy region "B," between 20 and 26 Mev, includes the giant  $E1$  resonance displayed by the  $(\gamma, n)$ - and  $(\gamma, p)$ -reactions. It is clear, however, from the theoretical discussion above, that the  $(\gamma, \alpha)$ -process should not exhibit such behavior. Rather, we expect in B only forbidden  $\alpha$ -emission induced by  $H_1(E1)$  and allowed  $\alpha$ -emission induced by  $H_0(M1)$  and  $H_0(E2)$ . In B the experimental cross section is indeed smaller than in A and C. The sum rules (10) and (11) for the allowed transitions appear to be exhausted by  $(\gamma, p)$  and  $(\gamma, \alpha)$ -processes in A, and the forbidden  $(\gamma, \alpha)$  transitions in B compete very unfavorably with  $(\gamma, n)$  and  $(\gamma, p)$  reactions. Forbidden transitions due to  $H_1(M1)$  and  $H_1(E2)$  are certainly negligibly rare.

The second  $(\gamma, \alpha)$  peak lies in region "C" above 26 Mev, in which we expect allowed  $E1$  processes to predominate. That this is the case is borne out by the experimental fact that in  $C^{12}$  more than 95 percent of the transitions leave  $Be^8$  in an excited state<sup>19,20</sup> at  $17.0 \pm 0.2$  Mev with  $J=2^+$ .<sup>20</sup> There seems to be no reason to doubt that this is the lowest  $T=1$  state of  $Be^8$ . Since the total  $\gamma$ -ray absorption in C, as indicated by the  $(\gamma, n)$  and  $(\gamma, p)$  cross sections, is only about 4 percent of the value at the giant resonance peak in B, it is understandable that the allowed  $E1$ -induced  $(\gamma, \alpha)$ -reactions in C do not produce a very great increase in the  $(\gamma, \alpha)$ -cross section over region B. The higher density of final states for the  $(\gamma, n)$ -process accounts for its predominating by a factor of 10 over  $(\gamma, \alpha)$  even in C.

It should be mentioned that one preliminary experi-

mental result conflicts with the point of view we have expressed. Goward and Wilkins<sup>21</sup> report that the angular distribution (with respect to the incident  $\gamma$ -ray) of  $\alpha$ 's to the ground state of  $Be^8$  displays strong forward-backward asymmetry in the energy regions 13–15.5 Mev and 17–19 Mev, indicating interference between  $E1$  and  $E2$ . While in the lower interval we can offer no explanation whatever for such an effect, it is conceivable that we could interpret the effect in the upper interval by supposing that the threshold for  $E1$  absorption is at 17 Mev rather than around 20 Mev. However, the experimental result<sup>18</sup> obtained at 17.6 Mev appears to be in disagreement with the hypothesis of a large  $E1$  contribution, unless we assume that  $E1$  absorption is followed with strong preference by transitions to the ground state of  $Be^8$ . It should perhaps be emphasized that a unique identification of events as ground-state transitions is extremely difficult, while misassignments may lead to spurious angular distributions.

The situation in  $O^{16}$  is roughly analogous to that in  $C^{12}$ , though the experimental evidence is less detailed. The lowest  $T=1$  state, analogous to the ground states of  $N^{16}$  and  $F^{16}$ , must lie at about 13 Mev and have  $J=2^-$ . It may be either of two levels with  $J=2^-$  at 12.51 Mev and 12.95 Mev found in  $N^{15}(p, \alpha)C^{12*}$ . The location of the  $E1$  absorption threshold, which defines the boundary between energy regions "A" and "B," cannot be deduced from the known level structure above 13 Mev and it is difficult to obtain information about it from the energy dependence of  $\sigma(\gamma, n)$ . This cross section rises sharply from threshold at 16 Mev to a plateau of 0.5 mb extending from 17 to 20 Mev and then rises again to the giant resonance peak value of 11 mb at 24 Mev.<sup>22</sup> The plateau has been attributed<sup>22</sup> to  $E2$  and  $M1$  absorption following a suggestion of Blatt and Weisskopf.<sup>23</sup> In the same energy region we have the results of Millar and Cameron<sup>24</sup> and Nabholz *et al.*<sup>25</sup> on the reaction  $O^{16}(\gamma, \alpha)C^{12}$  and a single measurement<sup>26</sup> (0.7 mb at 17.6 Mev) of  $\sigma(\gamma, p)$ . Now it does not appear possible that all of the absorption below 20 Mev can be due to  $H_0(E2)$  and  $H_0(M1)$ , in view of the sum rules (10) and (11). Both the  $L$ - $S$  and  $j$ - $j$  independent particle models would make (11) vanish for  $O^{16}$ , and it seems unlikely that corrections due to interaction moments would yield  $M1$  absorption of normal strength. The sum rule (10) for  $H_0(E2)$  alone is overexhausted by the observed cross sections below 20 Mev. It is possible that absorption due to  $H_1(E2)$  and  $H_1(M1)$  may account for the discrepancy, or it may be that  $E1$  absorption is responsible for an appreciable part of the plateau, as

<sup>21</sup> F. K. Goward and J. J. Wilkins (private communication, December 1952).

<sup>22</sup> Johns, Horsley, Haslam, and Quinton, *Phys. Rev.* **84**, 856 (1951).

<sup>23</sup> J. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 656.

<sup>24</sup> Quoted by M. A. Preston, *Phys. Rev.* **80**, 307 (1950).

<sup>25</sup> Nabholz, Stoll, and Waffler, *Phys. Rev.* **86**, 1043 (1952).

<sup>26</sup> H. Waffler and S. Younis, *Helv. Phys. Acta* **22**, 614 (1949).

<sup>16</sup> F. K. Goward and J. J. Wilkins, Atomic Energy Research Establishment Memo G/M 127, March, 1952 (unpublished) and private communication (December, 1952).

<sup>17</sup> V. L. Telegdi, *Phys. Rev.* **87**, 196 (1952); and to be published.

<sup>18</sup> V. L. Telegdi, *Phys. Rev.* **84**, 600 (1951).

<sup>19</sup> J. J. Wilkins and F. K. Goward, *Proc. Phys. Soc. (London)* **A64**, 1056 (1951).

<sup>20</sup> V. L. Telegdi (to be published).

suggested for different reasons by Peaslee.<sup>27</sup> In any case, the  $(\gamma, \alpha)$  reaction to the ground state should be due almost entirely to  $H_0(E2)$  and seems to exhaust about half of the sum rule (10).

In the region of the giant  $E1$  absorption resonance, Goward and Wilkins<sup>28</sup> have observed several peaks in the cross section for  $O^{16}(\gamma, \alpha)C^{12} \rightarrow 3\alpha$ , of heights roughly equal to that of the ground state peak of Millar and Cameron,<sup>24</sup> as in  $C^{12}$ , the absence of a spectacular rise in the  $(\gamma, \alpha)$  cross section is in agreement with our theory.

The threshold for the allowed  $E1$ -induced  $(\gamma, \alpha)$ -process is  $7+15+3=25$  Mev. Peaks in the cross section observed by Goward and Wilkins<sup>28</sup> above this energy (up to 30 Mev) are of roughly the same magnitude as in region "B," though the absorption cross section has presumably fallen considerably; again the theory appears to be borne out. It is, of course, extremely important to check whether an overwhelming proportion of the events in region "C" really proceeds via  $T=1$  levels in  $C^{12}$ . It may be significant that Livesey and Smith<sup>29</sup> have reported a change in mechanism around 25 Mev.

At sufficiently high energies ( $>35$  Mev), the most allowed transition should lead through  $T=1$  states in  $O^{16}$ ,  $C^{12}$ , and  $Be^8$ ; the violation of isotopic spin conservation would be postponed as long as possible. It

would be interesting to know whether such cascades do occur.

Experimental effects of the isotopic spin selection rules for  $\gamma$ -ray absorption should, on the whole, be less striking in odd-odd nuclei with  $T_z=0$  than in the even-even ones. Perhaps the most noteworthy prediction that can be made is one concerning the competition between  $(\gamma, d)$ - and  $(\gamma, np)$ -reactions in the region of  $E1$  absorption. Below the energy at which the residual even-even nucleus can be left in a  $T=1$  state, the  $(\gamma, d)$  process is forbidden, while a corresponding reaction in which a neutron and a proton are emitted together in a  $T=1$  state (particularly in the virtual  $^1S_0$  state near zero energy) is fully allowed. Goward and Wilkins<sup>21</sup> appear to have observed an excess of  $(\gamma, np)$  over  $(\gamma, d)$  reactions. It would be useful to search for the allowed  $(\gamma, d)$  process as well; for example, above 25 Mev the reaction  $B^{10}(\gamma, d)Be^{8*}$  should lead predominantly to the state at 17 Mev or higher  $T=1$  states.<sup>‡</sup>

For a more detailed discussion of  $(\gamma, \alpha)$ - and  $(\gamma, d)$ -reactions in light nuclei, the reader is referred to a forthcoming review article by one of us (V.L.T.) to appear in the *Reviews of Modern Physics*.

<sup>‡</sup> *Note added in proof.*—Hsiao and Telegdi [Phys. Rev. **90**, 494 (1953)] have recently presented strong evidence that the reaction  $O^{16}(\gamma, \alpha)C^{12*} \rightarrow 3\alpha$  does lead to  $T=1$  states of  $C^{12}$  in energy region C. They have also reported cascades of the type predicted in the text. Our discussion of these cascades ought to be completed by the remark that their probability is greatly reduced through the fact that the emission of the second  $\alpha$  suffers competition from nucleon emission.

The same authors, in unpublished work, have found that the reaction  $N^{14}(\gamma, d)C^{12} \rightarrow 3\alpha$  proceeds predominantly through a 16-Mev level in  $C^{12}$ , presumably with  $T=1$ , as predicted.

<sup>27</sup> D. C. Peaslee, Phys. Rev. **88**, 812 (1952).

<sup>28</sup> F. K. Goward and J. J. Wilkins, Proc. Phys. Soc. (London) **A65**, 671 (1952).

<sup>29</sup> D. L. Livesey and C. L. Smith, Proc. Phys. Soc. (London) **A65**, 758 (1952).

## A WKB-Type Approximation to the Schrödinger Equation

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A method of approximating solutions of the one-dimensional Schrödinger equation is presented in this paper. The method closely resembles the usual WKB approximation. Whereas in the ordinary WKB method the exponential function is used as the basis of the approximation, in this paper the solutions of an arbitrary Schrödinger equation are used. The general advantage is that by proper choice of the arbitrary equation an improved approximation can be obtained. The method is illustrated by treating the potential well and potential barrier problems when there are two turning points. The approximations to the wave functions are continuous even across the turning points. The barrier transmission problem is treated uniformly for energies above and below the peak of the barrier.

### I. INTRODUCTION

THE WKB method, as well as showing the correspondence between classical and quantum mechanics, provides useful approximations to the solutions of the one-dimensional Schrödinger equation. A limitation on its usefulness as an approximation is that it becomes infinite at the classical turning points of the motion. Langer<sup>1</sup> introduced an approximation based on

<sup>1</sup> R. E. Langer, Phys. Rev. **51**, 669 (1937).

Bessel functions which remains finite at any one turning point and, far from the turning point, becomes identical with the WKB approximation. However, at a second turning point his result is infinite and, to obtain approximate solutions which are everywhere finite, one must join it to a similar approximation finite at the second turning point.

The ordinary WKB method is based on the exponential function and Langer's approximation on Bessel