Determination of Source Parameters by Amplitude Equalization of Seismic Surface Waves

2. Release of Tectonic Strain by Underground Nuclear Explosions and Mechanisms of Earthquakes[1]

M. Nafi Toksöz, David G. Harkrider, and Ari Ben-Menahem

Seismological Laboratory
California Institute of Technology, Pasadena

Abstract. The radiation patterns of Love and Rayleigh waves from three nuclear explosions (Hardhat, Haymaker, and Shoal) are studied to determine the nature of the asymmetry of radiation and the mechanism of Love wave generation. From a comparative study of different explosions it is reasoned that the Love waves are generated at the source of the explosion. The source function, represented as the superimposition of an isotropic dilatational component due to the explosion and a multipolar component due to the release of tectonic strain energy, is consistent with the observed radiation patterns and the amplitude spectrums. The amount of seismic energy due to the strain release is computed. In some cases (Haymaker and Shoal) it is found that this energy may be due to the relaxation of the pre-stressed medium by the explosion- formed cavity. In the case of Hardhat it is concluded that the explosion must have triggered some other strain release mechanism, such as an earthquake. The amplitude equalization method is applied to surface waves from an earthquake to determine the source parameters. From only the amplitude spectrums and radiation patterns of Love and Rayleigh waves, the source functions, source depth, strike and dip of the fault plane, and the rake of displacement are determined for the July 20, 1964, Fallon earthquake.

Introduction

Many of the underground nuclear explosions have generated horizontally polarized seismic shear waves (*SH* and Love) along with *P*, *SV*, and Rayleigh waves. Theoretically, an explosive source in a horizontally layered, homogeneous, isotropic medium should not generate *SH* or Love waves. One purpose of this study is to determine the mechanism of generation of these horizontally polarized transverse waves by nuclear explosions. The second part of the paper is devoted to a study of applicability of the method of amplitude equalization to earthquakes for determining their source parameters.

Nuclear Explosions and Tectonic Strain Release

The significance of the magnitude of *SH* waves radiated from explosions has been demonstrated by many investigators. *Press and Archambeau* [1962] gave an example showing that the short-period *SH* waves from the Rainier blast were of comparable amplitude with those of an earthquake located in the vicinity of the explosion site. *Brune and Pomeroy* [1963] illustrated clear examples of relatively long period transverse waves from Hardhat and Mississippi explosions. Other examples can be found in the *Long Range Seismic Measurements* project reports on underground explosions prepared for the Air Force Technical Applications Center.

The important question is not the verification of the existence of horizontally polarized waves from the explosion but the understanding of the mechanism of the generation of these waves. To that end, several mechanisms need to be considered.

1. Wave conversion: *P*, *SV*, and Rayleigh waves may be converted to *SH* and Love waves because of inhomogeneities along the path of propagation. However, *SH* and Love waves are not always observed from explosions fired in the same general area. Examples of these would be the Sedan and the Mississippi explosions which

[1] Contribution 1290, Division of Geological Sciences, California Institute of Technology.

were located less than 5 km apart. Although there were almost no horizontal transverse waves from Sedan, strong Love waves were recorded from Mississippi. Most of the noncratering explosions in alluvium were followed by the collapse of the cavities. In some cases, such as Haymaker, the Rayleigh waves from the collapse were as large as those from the explosions. Although Love waves were recorded from the Haymaker and Mississippi explosions, no horizontally polarized waves were observed from the collapses that followed [*Brune and Pomeroy,* 1963]. Since the paths were identical for both the explosion and the collapse, any conversion during propagation could not affect the waves from one event without affecting those from the other. In other words, we can rule out wave conversion as a possible source of *SH* waves observed from these nuclear explosions.

2. Near-source effects: Although an explosion would not generate any *SH* waves in an isotropic, homogeneous, horizontally layered medium, most of the explosion sites did not satisfy these conditions. The question, then, was whether the *SH* waves could have been due to near-source irregularities. This possibility can also be ruled out with the argument that, if this were the case, the collapse events would also generate *SH* waves.

3. Tectonic strain release: A nuclear explosion is not an ideal point source. The shock wave generated vaporizes, melts, crushes, and cracks the rocks out to some distance, depending on the yield of the explosion and the strength of the medium. The cracks caused by the explosion would not generate any *SH* waves unless there were differential movement along them. In a medium which is not pre-stressed the cracks and the movement along cracks would be distributed uniformly, with radial symmetry around the point of explosion. Therefore, one would not expect an *SH* wave radiation pattern which could be represented by a dipole or a quadrupole source.

In a pre-stressed medium, an explosion may release a part of the existing strain energy by one or more of three mechanisms. First, intense stress waves near the explosion may trigger a small earthquake. The radiation pattern of Love and Rayleigh waves from the Hardhat explosion which fits a double-couple source function [*Brune and Pomeroy,* 1963; *Aki,* 1964] was explained by such a mechanism.

A second mechanism of releasing the strain energy is directional cracking. When the shock wave propagates from an explosion in a pre-stressed medium, the cracking will occur in preferred directions in such a way that the existing strain energy is minimized. The radiation pattern of seismic waves due to an induced rupture in a pre-strained medium was studied by *Archambeau* [1964]. He found that all pre-stress conditions give symmetric quadrupole radiation patterns.

Another source of strain energy that may be released by an explosion is provided by the cavity. This problem was investigated by *Press and Archambeau* [1962]. Not only would the strain energy stored in the volume of material that was molten and crushed be released, the strain energy outside the linear zone would also be reduced. The latter release would primarily be as elastic radiation.

If there is tectonic strain release, *P*, *SV*, *SH*, Love, and Rayleigh waves radiated from the release will be superimposed on the explosion-generated waves—*P*, *SV*, and Rayleigh. Although the *SH* and Love waves will be due to the released strain energy only, the *P*, *SV*, and Rayleigh waves will contain fractions due to both sources. It is not possible to separate the fraction of the energy due to the explosion alone unless the space and time functions of both sources are known exactly. Since this information is not available, we resort to some indirect techniques and make certain assumptions. These are listed for each case.

In this study we investigate three nuclear explosions, the Hardhat and Haymaker explosions fired at the Nevada test site and the Shoal explosion detonated in the Sand Springs Range, Nevada. In addition, two small earthquakes whose epicenters were in the general vicinity of the Shoal explosion site are included for comparison. Seismic surface wave recordings from these events at some or all of four CIT stations (Pasadena, California; Ruth, Nevada; Jamestown, California; and Albuquerque, New Mexico) equipped with narrow-band, high-sensitivity Benioff seismographs and from LRSM program mobile stations are used in the study (see Table 1 and Figure 1).

Shoal explosion. The method of amplitude

TABLE 1*a*. Epicenter Data

Event	Origin Time, UT h	m	s	Location: Latitude, N	Longitude, W	Shot or Focal Depth, km	Yield or Magnitude
Explosion							
Hardhat (Feb. 15, 1962)	18	00	00	37°13′35″	116°03′34″	0.290	6 kt
Haymaker (June 27, 1962)	18	00	00	37°02′30″	116°02′07″	0.410	56 kt
Shoal (Oct. 26, 1963)	17	00	00	39°12′01″	118°22′49″	0.367	12.5 kt
Earthquake							
April 13, 1962	15	38	47	38.4°	119.2°	25	$m = 4.8$
June 20, 1962	09	02	08	39.5°	118.3°	25	$m = 5.2$

TABLE 1*b*. Station Data

Station	Location: Latitude, N	Longitude, W	Distance km: Haymaker	Shoal
Albuquerque, New Mexico	34°57′00″	106°27′29″	894	
Jamestown, California	37°56′53″	120°27′17″	402	
Pasadena, California	34°08′53″	118°10′16″	375	358
Ruth, Nevada	39°15′00″	115°00′00″	261	

equalization as described in our earlier paper [*Toksöz et al.*, 1964] will be utilized to determine the source function of the explosion and the nature of the multipolar component of the source responsible for Love waves. Since the isotropic explosive source does not contribute to the radiated Love waves, the properties of the multipolar source function can be deduced from the transverse waves alone. The time function of the explosive source, however, cannot be readily determined from the Rayleigh waves because of contamination with waves from the multipolar source.

We make two assumptions concerning the superimposed multipolar source: (1) The space function of the source can be represented as a horizontal (strike-slip) couple or orthogonal double couple; (2) the orientation of the axis of the couple or the double couple is parallel to those of earthquakes occurring in the region. The validity of these assumptions will be justified by showing the consistency of theoretical models with observations at all stations surrounding the explosion site.

A horizontal quadrupole of the type described above will have an azimuthal radiation pattern for Rayleigh waves given by $\sin 2\theta$, and for Love waves by $\cos 2\theta$. Thus in the case of an explosion with a superimposed quadrupole component, the Rayleigh waves recorded at stations set up along $\theta = n\pi/2$, where $n = 0, 1, 2, 3, 4$, would be due to the explosive source only. The ideal method of studying the explosion-generated waves, then, would be to find a station or stations situated along these azimuths and use their recorded waves.

The Shoal explosion was in a region of moderate seismic activity where numerous shallow earthquakes have occurred. The movements associated with the earthquakes that have been investigated were along faults whose strikes are, in general, in the N-S direction [*Slemmons*, 1956, 1957; *Benioff*, 1962]. The recordings of these earthquakes in Pasadena, which is due south of the epicentral areas, show that the Rayleigh wave amplitudes are much smaller than the Love wave amplitudes. The examples shown in Figure 2 clearly illustrate the passage

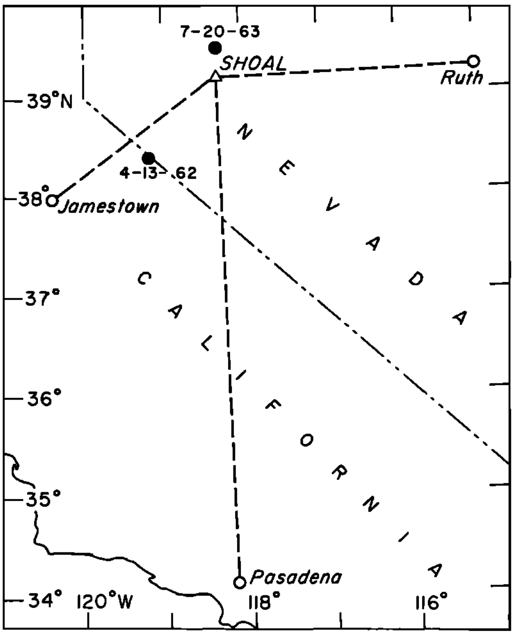

Fig. 1. Locations of the Shoal explosion, Fallon earthquake epicenters, and CIT stations.

of the Rayleigh wave nodal line through or near Pasadena. The records of Rayleigh and Love waves from the Shoal explosion show that their amplitudes are approximately equal. Invoking our first assumption—that the orientation of the quadrupole component of the field is the same as the tectonic axis as determined by earthquakes—we can state that the Rayleigh waves recorded in Pasadena were primarily due to the explosive source only. Using these waves, we can determine the source-time function of the explosion.

The method used in determining the source function is the amplitude equalization technique which was previously used in the case of some other nuclear explosions [*Toksöz et al.*, 1964]. To determine the amplitude response of the layered medium for a source, we employ the available information on the structure of the crust and the upper mantle. For propagation paths from Fallon to Pasadena, Ruth, and Jamestown, the structures are complicated and the thicknesses of the layers change along a given path [*Eaton*, 1963; *Healy*, 1963]. In each case, however, we represented the structure as a horizontally layered homogeneous medium where the layer thicknesses and the velocities were computed as the weighted average of those along the paths (Table 2).

To determine the source-time function of the Shoal explosion we corrected the amplitude spectrum of the Rayleigh wave recorded at Pasadena for the response of the instrument [*Toksöz and Ben-Menahem*, 1964] and the response of the layered medium for an explosive source buried at a depth of 370 m (i.e., shot depth). The observed vertical displacement of the ground and the theoretical response of the medium are shown in Figure 3.

The ground displacement was corrected for the response of the layered medium by pointwise division to obtain the amplitude spectrum of the source function at the boundary of the linear zone. The time function of the source was then assumed to be of the form $p(t) = P_0 te^{-\eta t}$, where $p(t)$ is the pressure, P_0 is a constant, t is time, and η is the source parameter [*Toksöz et al.*, 1964]. For a value of $\eta = 1.0$ sec^{-1} the amplitude spectrum of the theoretical pressure pulse, which is shown in Figure 4, agrees with the spectrum of the experimental pulse better than other values of η.

Three points should be clarified in regard to the shape of the pressure pulse: (1) The observed spectrum is not a true experimental quantity, since we computed the layering response theoretically using an assumed structure. The complexities of the structure, such as the lateral inhomogeneities, could not be taken into account. The oscillations of the observed spectrum are probably due to such complications. (2) The uniqueness of the particular pressure function is not proved. There are other pulses with similar amplitude spectrums. Some of these might even have a negative phase, where a dilatation phase follows the initial compression. This particular pulse was chosen because of our earlier study, in which pulses of this nature were justified by physical reasoning and by the consistency with initial phase studies. We should also mention that the $p(t) = P_0 te^{-t}$ represents the shape of the pressure function at the boundary of the region where the motion is elastic. (3) The long-period waves from which the source function is determined would not yield information on the sharper features of the pulse. Therefore, the pressure pulse shown in Figure 4 should be regarded as a

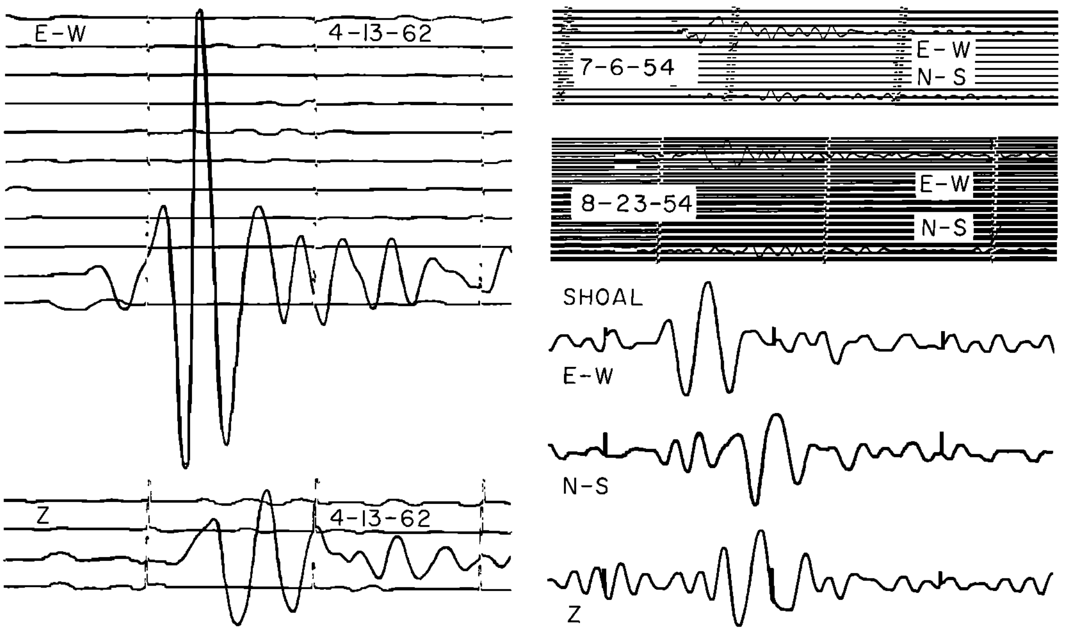

Fig. 2. Pasadena seismograms of the July 6, 1954, August 23, 1954, April 13, 1962, earthquakes and the Shoal explosion. July 6 and August 23 events are recorded with strong-motion instrument, others with narrow-band Benioff seismograph. The N–S component is almost purely radial, and the E–W component is purely transverse.

smoothed approximation rather than as the exact pulse.

The source of the Love waves generated by the Shoal explosion can be investigated with the same amplitude equalization technique. Other difficulties, however, result from the uncertainty of the depth and the spatial distribution of the source. Let us assume that the force configuration at the source can be represented as a double couple similar to that of the majority of earthquakes. For further simplification we take this to be an orthogonal double couple in the horizontal plane. Since the medium is going from a stressed to a relaxed state in releasing the strain energy, we would also expect the source-time function to be similar to a step function. It may seem that we have assumed everything about the mechanism of the generation of Love waves. However, we will show later that these assumptions are consistent with the observations.

Comparison of the Love waves generated by the explosion and the two small earthquakes provides clues to the nature of the explosion-generated waves. The Pasadena records from one set of instruments for three events with almost identical source-station paths were used in computing the graphs shown in Figure 5.

TABLE 2. Structures Used in Computing the Medium Layering Responses

Region	Thickness, km	Compressional Velocity, km/sec	Shear Velocity, km/sec	Density, g/cm³
Fallon to Pasadena	1	5.00	2.75	2.60
	21	6.00	3.40	2.75
	15	6.60	3.70	3.00
	50	7.82	4.35	3.35
	∞	8.00	4.40	3.40
Fallon to Ruth	1	5.00	2.75	2.60
	21	6.00	3.40	3.75
	9	6.60	3.70	3.00
	50	7.82	4.35	3.35
	∞	8.00	4.40	3.40
Fallon to Jamestown	1	5.00	2.75	2.60
	19	6.00	3.40	2.75
	15	6.70	3.75	3.05
	50	7.90	4.37	3.35
	∞	8.00	4.40	3.40

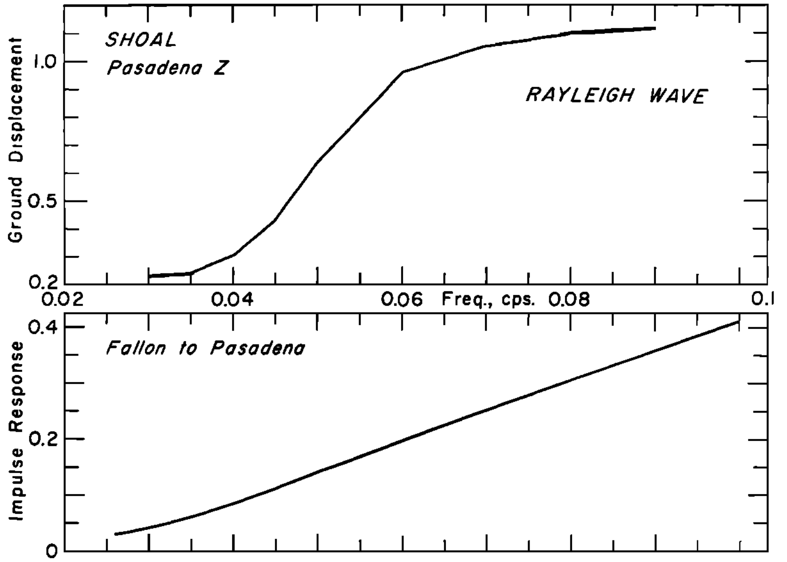

Fig. 3. Observed ground displacement at Pasadena and amplitude response of layered medium for an explosive surface source with impulsive time function.

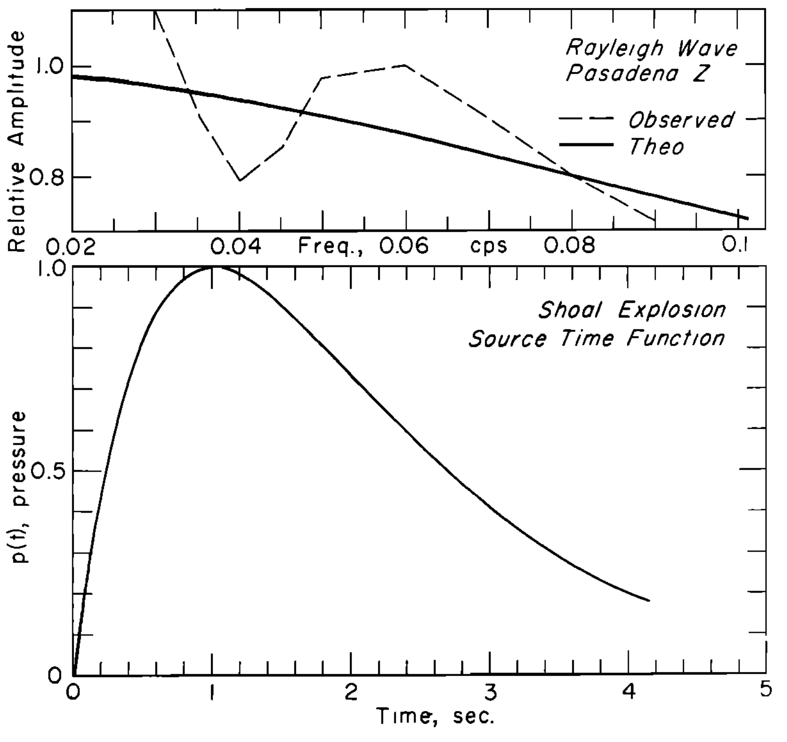

Fig. 4. Comparison of the theoretical and experimental amplitude spectrums of the Shoal pressure pulse and the shape of the pressure pulse at the boundary of the linear zone.

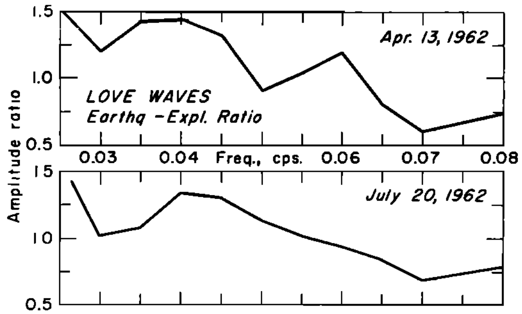

Fig. 5. The ratios of the amplitude spectrums of the Love waves from earthquakes to the amplitude spectrums of the Love waves generated by the Shoal explosion.

The outstanding feature of these curves is that the earthquake-explosion amplitude ratio decreases with increasing frequency. The relative enhancement of the long-period Love waves in the case of earthquakes cannot be explained simply by the volume of the source, since the equivalent magnitude of the explosion and those of the earthquakes were fairly close. The depth of the source, however, could affect the spectrums of the seismic surface waves generated, the deeper events producing relatively less amplitude of the short-period surface waves. These curves, shown in Figure 6, were computed by using the idealized structure from Fallon to Pasadena that is listed in Table 2 and medium response programs based on the matrix solutions in *Harkrider* [1964]. From the figure it is obvious that at $f = 0.03$ cps the amplitudes are nearly equal for sources at depths of 0, 11.5, and 25 km, while at $f = 0.09$ cps a surface source would generate Love waves whose amplitudes are nearly three times as large as those of the source at the depth of 25 km. Comparing Figures 5 and 6, we can state that the source of the Love waves generated by the Shoal explosion was near the surface and those of the earthquakes were deeper, provided that all the other source parameters were the same.

With the source near the surface, we can investigate whether the theoretical double-couple force configuration and the step-function source-time variation of the explosion-generated Love waves are consistent with the observations. Correcting the amplitude spectrum of the Love waves recorded at Pasadena for the response of the seismograph, we obtain the ground displacement spectrum. The theoretical spectrum is simply the product of the transform of the source-time function (i.e., ω^{-1} for a step function) and the amplitude response of the medium shown in Figure 6 ($h = 0$ km). The ratio of the observed ground displacement spectrum to the theoretical is shown in Figure 7. If the theoretical model exactly represented the actual source, this ratio would be a constant independent of frequency. The ratios plotted in Figure 7 show a slight increase with frequency. This increase can be due either to experimental errors or to the inexact representation of the structure, as well as some variation of the theoretical source from the actual. However, the effect shown in Figure 7 is so small that it can be neglected, and the consistency of the observed Love wave amplitudes with theoretical amplitudes can be accepted.

So far we have shown that our assumed source is an adequate model at this azimuth. There is

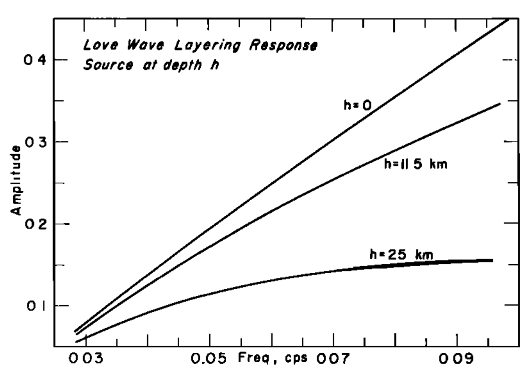

Fig. 6. The Love wave amplitude response of the medium to a double-couple, impulsive source-time function buried at depth h.

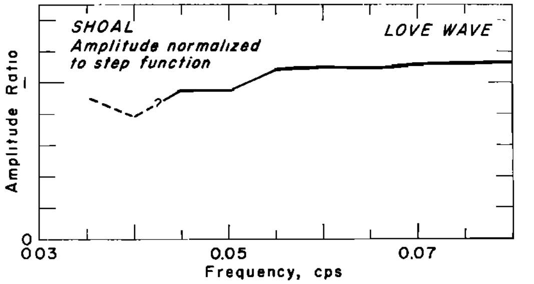

Fig. 7. The ratio of the observed displacement spectrum of Love waves from the Shoal explosion to the theoretical spectrum for a double-couple source with step-function time variation.

a problem of lack of uniqueness in this procedure, and various combinations of source space configurations and time functions may produce the same results. For example, in amplitude studies at one station it makes no difference whether the source is a couple or double couple [*Ben-Menahem and Toksöz,* 1963]. Furthermore, a single horizontal force with impulsive time function would give approximately the same results as a double-couple source with step-function time dependence. The double-couple (or couple) nature of the source, rather than a single force, will be shown in the next section.

Radiation patterns from three nuclear explosions. When tectonic strain energy is released in the form of seismic energy, the isotropic radiation pattern of an explosion is altered. It was shown earlier that, at least in the case of the Shoal explosion, this strain energy was released from a shallow depth. It will be assumed that the same condition will hold for the two other explosions, Haymaker and Hardhat, which are included in this study. The geometry of the strain-release source function is assumed to be an orthogonal, horizontal double couple. This double-couple condition was observed for many earthquakes, and it is a reasonable assumption. The horizontal orientation is the simplest geometry for an initial assumption. We will now determine two parameters of the source function, the orientation of the principal axis and the relative strength of the multipolar source compared to the explosive source.

The displacements observed at distant stations are the vectorial sum of displacements due to the explosion and the tectonic strain release. In a cylindrical coordinate system, the far-field expressions for the approximate ground displacements from a near-surface explosive point source are [*Harkrider,* 1964; *Toksöz et al.,* 1964]

$$w_e(\omega) = \frac{C_1}{(2\pi r)^{1/2}} \cdot \exp(-\gamma_R r) k_R^{1/2} \left(\frac{\dot{u}_0^*}{\dot{w}_0}\right) A_R(\omega) T(\omega) \cdot \exp\left[i\left(\omega t - k_R r - \phi_t + \frac{3\pi}{4}\right)\right]$$

$$q_e(\omega) = \frac{C_1}{(2\pi r)^{1/2}} \cdot \exp(-\gamma_R r) k_R^{1/2} \left(\frac{\dot{u}_0^*}{\dot{w}_0}\right)^2 A_R(\omega) T(\omega) \cdot \exp\left[i\left(\omega t - k_R r - \phi_t - \frac{3\pi}{4}\right)\right]$$

$$v_e(\omega) = 0 \tag{1}$$

where w_e, q_e, and v_e are the vertical, radial, and tangential components of the displacement, k_R is the wave number, r is the radial distance from the source, γ_R is the Rayleigh wave attenuation coefficient, $A_R(\omega)$ is the medium response for Rayleigh waves from a vertical force at the surface point, ω is the angular frequency, $\dot{u}_0$ and $\dot{w}_0$ are components of particle velocity at the surface, and $T(\omega)$ and $\phi_t(\omega)$ are the amplitude and phase response of the source-time function. The approximate ground displacement due to a near-surface orthogonal, horizontal double couple is [*Ben-Menahem and Toksöz,* 1963, *Ben-Menahem and Harkrider,* 1964]

$$w_{dc}(\omega) = \frac{C'}{(2\pi r)^{1/2}} \cdot \exp(-\gamma_R r) k_R^{1/2} \sin 2\theta \left(\frac{\dot{u}_0^*}{\dot{w}_0}\right) A_R(\omega) T'(\omega) \cdot \exp\left[i\left(\omega t - k_R r - \phi_t' + \frac{3\pi}{4}\right)\right]$$

$$q_{dc}(\omega) = \frac{C'}{(2\pi r)^{1/2}} \cdot \exp(-\gamma_R r) k_R^{1/2} \sin 2\theta \left(\frac{\dot{u}_0^*}{\dot{w}_0}\right)^2 A_R(\omega) T'(\omega) \cdot \exp\left[i\left(\omega t - k_R r - \phi_t' - \frac{3\pi}{4}\right)\right]$$

$$v_{dc}(\omega) = \frac{C'}{(2\pi r)^{1/2}} \cdot \exp(-\gamma_L r) k_L^{1/2} \cos 2\theta A_L(\omega) T'(\omega) \cdot \exp\left[i\left(\omega t - k_L r - \phi_t' - \frac{3\pi}{4}\right)\right] \tag{2}$$

The subscripts R and L refer to Rayleigh and Love waves, respectively; azimuthal angle θ is measured counterclockwise from the fault plane. If the time delay between the explosion and the strain release is negligible, the observed Rayleigh and Love wave displacements can be written as

$$U_{R_z}(\omega) = w_e(\omega) + w_{dc}(\omega)$$
$$= w_e(\omega)\left[1 + F\frac{T'(\omega)}{T(\omega)}\sin 2\theta\cdot e^{i\delta\varphi}\right]$$
$$U_{L_\theta}(\omega) = v_{dc}(\omega) \qquad (3)$$

The term $\delta\varphi$ in (3) is the phase difference between source-time functions and the source-space functions. The constant F is the strength of the multipolar source relative to that of the explosion.

Assuming the difference between time functions $T(\omega)$ and $T'(\omega)$ to be negligible, we computed theoretical radiation patterns for Haymaker and Hardhat explosions to fit the observed Rayleigh wave amplitudes of the long-period records from the LRSM stations. We can ignore the effects of geometric spreading, instrument magnification, and the layering in the case of the Haymaker explosion by normalizing the Rayleigh wave amplitudes to those of the Haymaker collapse event. It was concluded earlier [*Brune and Pomeroy,* 1963; *Toksöz et al.,* 1964] that the Haymaker collapse was a symmetric event. The experimental amplitude ratios are taken from the report *Haymaker* (prepared for AFTAC by the Geotechnical Corporation, 1962) and are the ratios of the maximum amplitudes in the period range of 13 to 16 seconds. We have also computed four ratios from CIT stations. The radiation pattern shown in Figure 8 is well defined, and the amplitude along the 20° azimuth is approximately one-half of that along azimuth 110°. To explain this theoretically

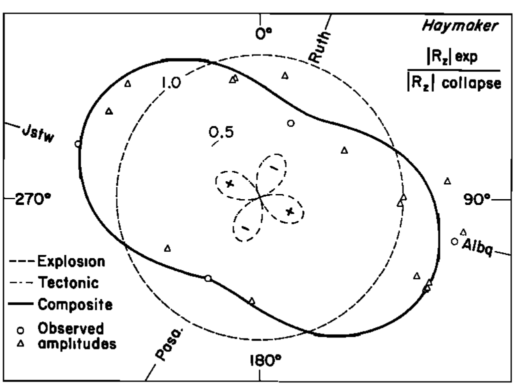

Fig. 8. The observed and the theoretical Rayleigh wave radiation pattern for the Haymaker explosion and the theoretical source components. Observed amplitudes are normalized to those of the Haymaker collapse.

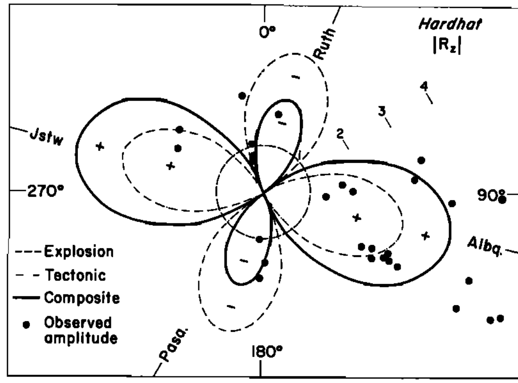

Fig. 9. The observed and the theoretical Rayleigh wave radiation pattern for the Hardhat explosion. The data is from LRSM stations.

we superimposed a double-couple source over the isotropic explosion and determined the orientation and the relative strength as given in (3) with $\delta\varphi = 0$. The best-fit theoretical double-couple pattern and the composite pattern are also shown in Figure 8. The double-couple orientation is such that it represents either a right-hand strike-slip fault striking at 340° azimuth or a left-hand fault striking at 70° azimuth. The relative strength of the double couple is $F = 0.333$.

For the Hardhat explosion we corrected the maximum amplitudes of the Rayleigh waves given in the AFTAC report *Hardhat* (prepared for ARPA, 1962) for the geometric spreading. We assumed that the response of the medium A_R remained constant for all azimuths, and we neglected the effect of attenuation due to absorption. From the observed radiation pattern (Figure 9) we determined the composite theoretical source with a procedure similar to that followed for Haymaker. In this case, the data scattered more than with Haymaker, but the intensity of the source in the southeasterly direction is seen to be definitely larger than it is in the northeasterly direction. For this event, the double-couple source can be the model of a right-hand strike-slip fault with the azimuth of the strike at 330°, and the conjugate of this (left-hand strike-slip fault at azimuth 60°) is also a possible solution. The amplitude of the double couple is $F = 3.0$ relative to the explosion. The polarity of the composite source changes from one quadrant to the other. The change of polarity of the Rayleigh waves recorded at Ruth and Pasadena relative to that recorded in Albuquerque was observed in our earlier study [*Toksöz et al.*, 1964]. From a study of the Hardhat, Love and Rayleigh wave phases and amplitudes, *Aki* [1964] obtained exactly the same double couple for the source. The contribution from the explosion in his case was one-half of that from the double couple, as compared with our value of one-third. It is, however, very encouraging to obtain almost the same source functions from two independent techniques.

The general agreement between the orientation and the sense of the double-couple components for both Haymaker and Hardhat explosions suggests that the strain energy released was part of the regional strain field.

To determine the mechanism at the source of the Shoal explosion we use the ratios of Love wave to Rayleigh wave amplitudes. Neglecting the difference between the source time functions of the explosion and the double couple and using (1), (2), and (3), we can write the ratio of the observed Love wave amplitude to the amplitude of the vertical component of the Rayleigh wave.

$$\frac{|U_{L\theta}|}{|U_{R_z}|} = \frac{F k_L^{1/2} A_L \cos 2\theta}{(1 + F \sin 2\theta) k_R^{1/2} A_R \left(\dfrac{\dot{u}_0^*}{\dot{w}_0}\right)} \tag{4}$$

The quantities k_L, A_L, k_R, and $[A_R(\dot{u}_0^*/\dot{w}_0)]$ were computed theoretically for the structure described earlier.

The amplitude ratios of the observed Love and Rayleigh waves were determined from the recordings of the LRSM stations as reported in the report *Shoal* (prepared for AFTAC by the United Electrodynamics, Inc., 1963). The average period of the maximum amplitude waves was about 14 seconds.

The value of F and the orientation of the double couple were determined so that the theoretical values from (4) and the observed values were in agreement (Figure 10). The amplitude of the double couple relative to the explosion is $F = 0.9$. The azimuth of the strike of the fault plane is 346°. In this case, the choice between the two possible conjugate orientations was made from the initial phase of the Love waves observed in Pasadena. This orientation is in agreement with the general tectonic axis of the region.

The tectonic strain energy. The strength of the multipolar component relative to the isotropic component demonstrates the significance of the tectonic strain release in each case. In determining the source of the strain energy, it is of great importance to know the total amount of seismic energy released by the multipolar component.

In general, the determination of the absolute value of the seismic energy released from a disturbance involves many corrections. In this case, however, we can compute the seismic energy due to the multipolar source relative to the explosion, and we can then compute the seismic energy of the explosion from the magnitude assigned to it and the energy-magnitude relation. Since the magnitude is measured from amplitudes of the waves from a number of stations along various azimuths, the effect of the multipolar components should cancel out.

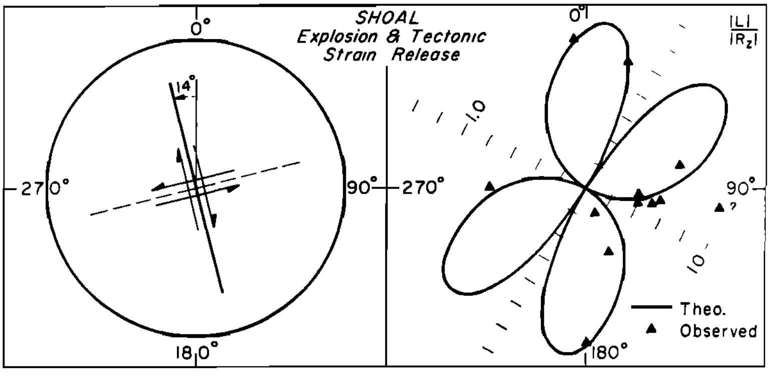

Fig. 10. The theoretical and observed $|L|/|R_z|$ (Love to vertical component of Rayleigh) amplitude ratios for the Shoal explosion. The theoretical model of the explosion and tectonic strain release are shown on the left-hand side.

The energy density of Rayleigh or Love waves recorded at a station is given by

$$E = \rho_0 \int_0^T V_i(\dot{u}_i)^2 \, dt \tag{5}$$

where V is the group velocity, ρ is density, and $\dot{u}_i$ is the particle velocity. The time integration is taken over the duration of the surface wave arrival. To compute the total energy of surface waves, we need to correct for the effect of geometric spreading and then to integrate over the azimuth and over the depth. The geometric spreading factor and the depth factor are independent of the source type. Since we are interested in the energy of one event, relative to the other, we can exclude the common factors and integrate over the azimuthal variation to determine an energy factor, E_i^f, which is the surface energy density at a unit distance.

$$\begin{aligned} E_i^f &= \rho \int_0^{2\pi} \int_0^T V_i[f(\theta)\dot{U}_i(t)]^2 \, d\theta \, dt \\ &\approx 2\pi\rho \int_0^{2\pi} \int_0^T V_i(\omega)[f(\theta)]^2 \, |\dot{U}_i(\omega)|^2 \, d\theta \, d\omega \end{aligned} \tag{6}$$

where $f(\theta)$ is the azimuthal factor of the source and $|\dot{U}(\omega)|^2$ is the power spectrum density, $f(\theta) = 1$ for explosions, and $f(\theta) = F \sin 2\theta$, $F \cos 2\theta$ for the double-couple type source functions where F is the source strength. From (6), (1), and (2), and after the azimuthal integration, the ratio of the energy of the tectonic strain release to the energy of the explosion can be derived

$$\frac{E_{\text{tect}}}{E_{\text{exp}}} = \frac{F^2}{2} \cdot \left\{ 1 + \frac{1}{\left(\frac{\dot{u}_0^*}{\dot{w}_0}\right)^2 \left[1 + \left(\frac{\dot{u}_0^*}{\dot{w}_0}\right)^2\right]} \left(\frac{A_L^2}{A_R^2}\right)\left(\frac{k_L}{k_R}\right)\left(\frac{V_L}{V_R}\right) \right\} \tag{7}$$

In the above derivation the source-time functions are assumed to be the same for both events. Since $\dot{u}_0^*/\dot{w}_0$, A_L, A_R, k_L, k_R, V_L, and V_R are frequency-dependent quantities, the ratio of energy densities would also depend on the frequency. However, since the ratios $\dot{u}_0^*/\dot{w}_0$, A_L/A_R, k_L/k_R, and V_L/V_R generally vary gradually with frequency $E_{\text{tect}}/E_{\text{exp}}$ also varies very gradually with frequency.

We computed the ratios of the strain energy released (i.e., energy of the multipolar component of the source) and the explosion energy for the three explosions at successive periods between 10 and 40 seconds. We should point out that the values shown in Table 3 are the ratios of the surface wave energies. They also represent the ratios of the total energies, since the partition of the energy between the body and surface waves is not likely to change significantly from one source to another if the source depth and type are not changed.

The absolute values of the strain energy released in each case were computed from the seismic energy of the explosion. The equivalent magnitude of each explosion is listed in the explosion reports. Using the formula given by *Gutenberg and Richter* [1956],

$$\log E = 5.8 + 2.4m \tag{8}$$

where m is the unified magnitude, we computed the seismic energy radiated from the Haymaker and Shoal explosions. In the case of Hardhat, it is very difficult to justify the assumption that the measured magnitude would represent the magnitude of the explosion and that the effect of the multipolar components would cancel out.

The source of strain energy. The source of the strain energy and the mechanism of its release by the explosion is of great interest. An explosion detonated in a strained medium would release some strain energy in the form of seismic waves because of the insertion of the cavity

TABLE 3. Seismic Energies from Explosions and the Associated Tectonic Strain Release

Explosion	Assigned Magnitude (m)	F	Seismic Energy from Explosion, ergs	Ratio of Strain Release to Explosion Energy	Seismic Energy from Tectonic Strain Release, ergs
Hardhat	4.9(?)	3.00	3.63×10^{17}(?)	12.00	4.3×10^{18}(?)
Haymaker	4.9	0.33	3.63×10^{17}	0.14	5.1×10^{16}
Shoal	4.9	0.9	3.63×10^{17}	1.05	3.8×10^{17}

in the medium and the relaxation of the stress field in the elastic zone around the cavity. This problem was studied by *Press and Archambeau* [1962] for a spherical cavity in a pre-strained elastic space. The expression they derive for the reduction of the strain energy due to the cavity is

$$\Delta W = \frac{\mu s^2}{2}\left(\frac{4}{3}\pi a^3\right) f\left(\frac{\mu}{\lambda}\right) + \frac{\mu s^2}{2}\left(\frac{4}{3}\pi a^3\right) \qquad (9)$$

where $\mu = \beta^2 \rho$ is the modulus of rigidity of the medium, S is the strain, and a is the radius of the cavity. The function $f(\mu/\lambda)$ is approximately equal to 1. The first term in (9) represents the reduction in strain energy in the elastic zone outside the cavity, and the second term the reduction of strain energy within the cavity. The maximum value of seismic energy radiated from stress relaxation must be equal to or less than ΔW.

An underground nuclear explosion crushes and cracks rocks far beyond the cavity. So, in applying (9), we can assume that the effective cavity radius is equal to the radius of the nonlinear zone around the explosion. These radii can be computed from the yields and strengths of the mediums as described in our earlier paper [*Toksöz et al.*, 1964]. The value of strain S is a difficult quantity to estimate. $s = 10^{-4}$ was chosen to be the upper limit for Nevada test site by *Press and Archambeau* [1962]. For the region where Shoal was located, $s = 10^{-4}$ is probably a good average value, judging from the seismicity of the Fallon area. With these values we computed the strain energy reduction ΔW by each of the three explosions. These results, listed in Table 4, represent the upper limit of the seismic energy that can be attributed to the strain release, since not all the strain energy is likely to be radiated in the form of seismic energy.

TABLE 4. Theoretical Values of Strain Energy That May Be Released by Explosions

Explosion	Nonlinear Zone Radius, *m*	Rigidity μ, dynes/cm²	Strain *s*,	Strain Energy Reduction ΔW, ergs
Hardhat	270	3×10^{11}	10^{-4}	2.4×10^{17}
Haymaker	400	2×10^{10}	10^{-4}	5.4×10^{16}
Shoal	350	3×10^{11}	10^{-4}	5.4×10^{17}

Let us compare these values with the seismic energy due to the multipolar component of the sources listed in Table 3. For Haymaker the maximum available strain energy was 5.4×10^{16} ergs, and the observed seismic energy was 5.1×10^{16} ergs. For Shoal, the values were 5.4×10^{17} ergs and 3.8×10^{17} ergs, respectively. In both of these cases the seismic energy due to the multipolar component of the source is less than the maximum value of strain energy released because of the explosion-generated 'cavity.' All of the observed strain energy could therefore be due to the relaxation of the stress field around the 'cavity,' and it is not necessary to have movements along the faults to radiate the strain energy in these cases.

The Hardhat explosion poses a completely different problem. Here our estimate of the seismic energy due to release of tectonic strain is not very accurate; it is probably higher than the true value. However, this figure (4.3×10^{18} ergs) is about 18 times higher than maximum available strain energy released due to the 'cavity.' Therefore, tectonic strain energy must be released from a volume that is much larger than the volume of the cavity and relaxed zone. This would mean that the shock wave from the explosion triggered a strain release mechanism outside the nonlinear zone. An earthquake, triggered in this manner near the explosion, would be an explanation of the magnitude of the observed strain energy. This was also concluded by *Brune and Pomeroy* [1963] from the after shocks that followed the explosion.

Source Parameters of Earthquakes

The radiation patterns of seismic surface waves from buried dipolar sources in a stratified earth were studied theoretically by *Ben-Menahem and Harkrider* [1964]. The necessary source-depth theory was given by *Harkrider* [1964]. We will determine the source characteristics of the July 20, 1962, Fallon earthquake using the amplitude equalization method and the radiation patterns from buried sources. The earthquake was a relatively small event ($m = 5.2$), and the finiteness of the source can be neglected when seismic surface waves of 40 km or longer wavelengths are used. The parameters that will be determined are the source-

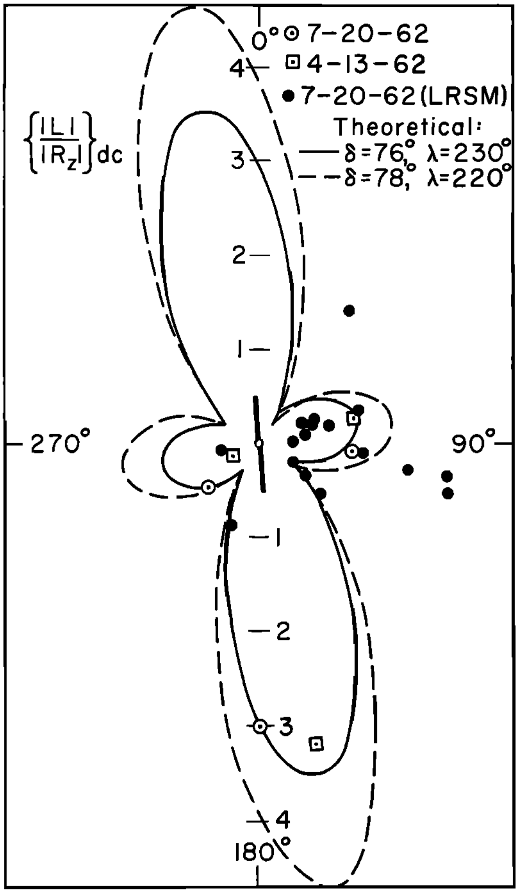

Fig. 11. The plot of $|L|/|R_z|$ amplitude ratios for the July 20, 1962, earthquake. Open symbols are data from CIT stations. δ is the dip angle and λ is the slip angle.

space function (i.e., singlet, couple, or double couple), strike and dip of fault plane, source depth, and the orientation of the force vector (i.e., strike-slip and the dip-slip components of the motion).

The radiation patterns and the ground displacement spectrums of both and Love and the Rayleigh waves were used in determining all the parameters. Unfortunately, when this work was done, we had only the seismograms from three Caltech stations from which to determine the radiation patterns, and we were not aware of the LRSM data. Instead of using absolute amplitudes at each station, we used the ratio of the Love to Rayleigh wave peak amplitudes (T = 16 sec) to determine the points that are shown in Figure 11.

A set of source parameters can be determined by fitting a theoretical radiation pattern curve to the experimental data, and the solid curve shown in Figure 11 is our 'best' fit. It is for a fault plane with a strike at azimuth 355° and an easterly dip of 76°. The slip angle λ is 230° (i.e., the direction if displacement is 50° from horizontal, rake is 50°S) and the source depth is 20 km. Although the agreement between the theoretical and observed radiation patterns is good, it is essentially based on three experimental points. The LRSM data that were added to the figure later do not strengthen or contradict this picture.

In obtaining a best fit, it was found that the source depth had relatively little effect. From Figure 11 it is clear that the strike direction cannot be changed appreciably. The dip of the fault plane affects the (Love/Rayleigh) amplitude ratios very strongly, and the effect of the slip angle is significant on the small lobes, especially in the directions of θ = 90° and θ = 270°. The dashed curve in Figure 11 demonstrates the results of varying the dip angle by 2° and the slip angle by 10°. The difference between the two theoretical curves is outstanding.

To check the validity of the source parameters, we computed the theoretical ground displacements for the Love and Rayleigh waves from the above source at three stations and compared the results with the observed amplitudes. The time variation of the source was assumed to be a step function. At Jamestown and Ruth, the Rayleigh waves were recorded with much greater amplitudes than at Pasadena, since the latter is very close to a nodal point. The Love waves, however, were very clear, with no interference on the transverse component at Pasadena. The Fourier amplitude spectrums of Rayleigh wave pulses at Ruth and Jamestown were corrected for the instrument response to determine the ground displacement. These observed spectrums are shown in Figure 12 along with the theoretical spectrums. In each part of the figure an arbitrary amplitude scaling factor is included.

In comparing the observed and the theoretical curves, we see that at Jamestown the Rayleigh wave spectrums are almost identical over the entire frequency band (f = 0.025 to 0.10 cps). At Ruth and Pasadena, however, the agreement between the experimental and the theoretical

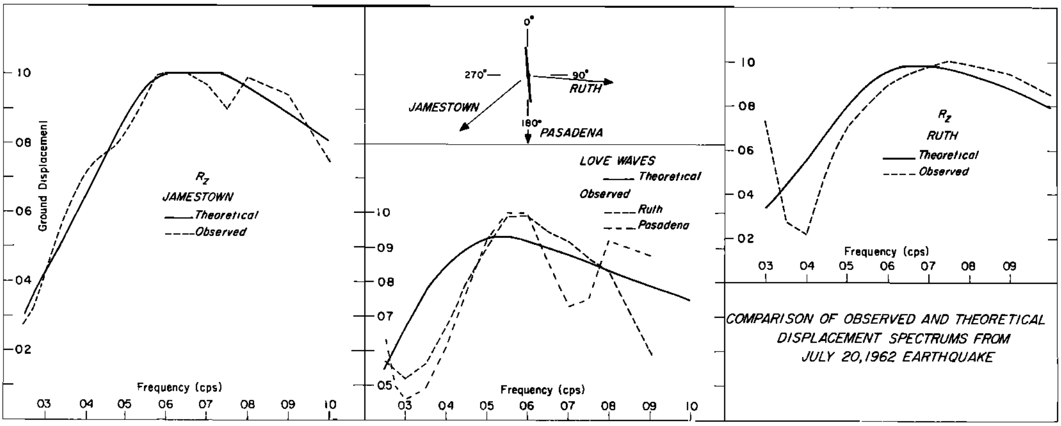

Fig. 12. Observed and theoretical ground displacement spectrums of Rayleigh and Love waves. The theoretical source parameters are the same as in Figure 11.

curves is not as good. There is a 'dip' in the observed spectrums centered around $f = 0.035$ cps. This is present in Love waves recorded at Pasadena and in Love and Rayleigh waves recorded at Ruth. A similar dip was also observed in the spectrums of Love and Rayleigh waves recorded at Pasadena from the Shoal explosion. The structures from the epicentral area to Pasadena and to Ruth are far from being horizontally layered, as assumed in our theoretical computations. The dips in the spectrums are probably due either to the selective filtering of some geologic structure in the path or to the interference of waves traveling along slightly different paths.

At Ruth the shape of the theoretical Rayleigh wave spectrum is different from that at Jamestown. This is primarily because the azimuths of the stations relative to the strike of the asymmetric source are different. The agreement between the theoretical and observed spectrums is fair, considering the big 'dip' in the observed spectrum.

The theoretical curves for Love waves at Pasadena and Ruth are almost identical except for a scale factor. For this reason we plotted the experimental Love wave spectrums of both stations on the same graph (Figure 12). Again, the agreement between the theoretical and observed curves is excellent for frequencies higher than 0.05 cps. At lower frequencies, however, the curves diverge because of the 'dip.'

It may appear that the theoretical spectrums should be shifted toward higher frequencies for a better fit with the records for Pasadena and Ruth. This can be done by moving the source to a shallower depth (see Figures 6 and 13). Comparing each of these theoretical curves with the observed curve (Figure 12), we can immediately rule out the depth of 5.5 km. The $h = 14$ km case gives a fair fit at frequencies lower than $f = 0.06$ cps, but at higher frequencies it diverges from the experimental curve, which reaches a maximum and then declines. The depth of 20 km gives the best over-all fit. We also computed the theoretical curves for the listed focal depth of 25 km and deeper source depths, but these did not fit the observed spectrums and were ruled out.

It should be noted that the source is not a

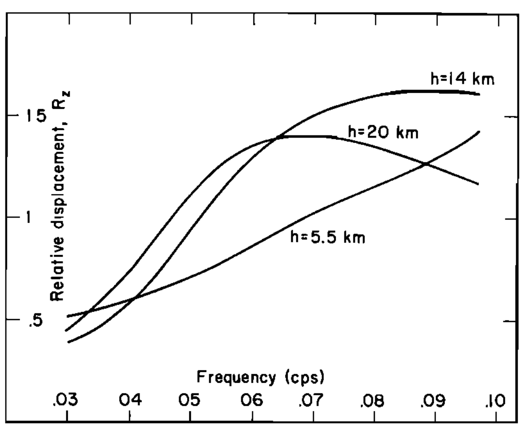

Fig. 13. The frequency response of the medium to Rayleigh waves generated by the source of Figure 11 placed at three different depths.

single point and that it has a finite volume. The depth of the source in this case is the center of the volume, and it may differ from the focal depth. In general, the focal depth refers to the depth of the point where the first motion occurred at the source. If the rupture along the fault propagated upward or downward, the source depth would refer to the depth of a central point and the focal depth would be the depth of the initial point.

Let us summarize the source parameters of the Fallon earthquake of July 20, 1962. The azimuth of the strike is 355° and the dip is 76°E. The force system is double couple. The projection of the displacement vector on the footwall makes an angle of 50° from the horizontal (i.e., rake is 50°S), which means that the dip- and strike-slip components of the motion were approximately equal. The sense of the strike-slip motion is dextral. The source depth is 20 km. Although we make no claim of uniqueness, the result is in agreement with the data and general tectonic features of the region.

Discussion and Conclusions

In the first part of this paper we utilized some assumptions and qualitative physical arguments to reach to the conclusions about the nature of the complicated source of explosion-generated, transverse seismic waves. In comparing the observations and theoretical results we did not always find perfect agreement. Some discrepancies are expected in this type of study because of the errors and scatter in the data and because of the simplified theoretical models chosen to represent complicated cases. There are also some uncertainties about the parameters we chose, such as values of strain in energy computations. In spite of these limitations, we found models for complicated sources that are consistent with the observations for different explosions. The conclusions stated below about the source mechanism of these explosions must be considered along with the limiting assumptions.

1. The source-time function of the Shoal explosion at the boundary of the linear zone as determined by amplitude equalization of surface waves is of the form $p(t) = P_0te^{-t}$.

2. The asymmetry of the radiation patterns of the Rayleigh waves and the generation of the Love waves from nuclear explosions can be explained by superimposing a quadrupole force system over the isotropic explosion source. The quadrupole source represents the radiation of seismic energy from tectonic strain release.

3. The strength of the quadrupole source relative to the explosion, and its orientation, can be determined from the radiation patterns of the Love and/or Rayleigh waves. The orientation seems to agree with the general tectonic features of the region.

4. The seismic energy released by the quadrupole component of the source varies from a small fraction of the explosion energy to many times the value of it. In the Haymaker and Shoal explosions, the relaxation of the tectonic stress field due to the explosion-generated 'cavity' can account for all of the seismic energy from the multipolar source. The Hardhat explosion, however, must have triggered a small earthquake or some other strain release mechanism besides the cavity.

5. No attempt was made to determine the time delay, if any, between the explosion and the tectonic strain release. This can be done by checking the radiation pattern of the first cycle of the P waves. In the Hardhat explosion the polarity of the first motion around the explosion would yield an answer. If time delay was much less than about 0.2 sec the polarity of the first motion (compression or tension) would have a quadrant distribution similar to the Rayleigh wave polarities. All of the reported first motions from LRSM stations were compression (TWG 2), although many stations did not specify the polarity.

6. The application of the amplitude equalization method with the theoretical radiation patterns from dipolar sources yields much useful information about the source mechanism of earthquakes. The procedure to be followed in determining the source parameters is a step by step process, as demonstrated for the Fallon earthquake. The strike, dip, and displacement direction are determined from the radiation pattern of the Love and Rayleigh waves or from the azimuthal variation of Love to Rayleigh amplitude ratios. In such a ratio, the time and space variation of the source, as well as the effect of its depth, nearly cancel out. The sense of the motion must be determined from the initial phase of the Love or the Rayleigh waves. The spatial source function and the depth are

determined from the ground displacement spectrums. Thus, the source is completely specified using only seismic surface waves. The uniqueness, however, remains a problem when the data are scattered.

Acknowledgment. This research was supported by contract AF-49(638)-1337 of the Air Force Office of Scientific Research as part of the Advanced Research Projects Agency project Vela.

References

Aki, K., A note on surface wave generation from the Hardhat nuclear explosion, *J. Geophys. Res.*, *69*, 1131–1134, 1964.

Archambeau, C. B., Elastodynamic source theory, Ph.D. thesis, California Institute of Technology, Pasadena, 1964.

Benioff, H., Movements on major transcurrent faults, in *Continental Drift*, edited by S. K. Runcorn, pp. 103–134, Academic Press, New York, 1962.

Ben-Menahem, A., and M. N. Toksöz, Source mechanism from spectrums of long-period surface waves, 2, Kamchatka earthquake of November 4, 1952, *J. Geophys. Res.*, *68*, 5207–5222, 1963.

Ben-Menahem, A., and D. G. Harkrider, Radiation patterns of seismic surface waves from buried dipolar point sources in a flat stratified earth, *J. Geophys. Res.*, *69*, 2605–2620, 1964.

Brune, J. N., and P. W. Pomeroy, Surface wave radiation patterns for underground nuclear explosions and small-magnitude earthquakes, *J. Geophys. Res.*, *68*, 5005–5028, 1963.

Eaton, J. P., Crustal structure from San Francisco, California, to Eureka, Nevada, from seismic-refraction measurements, *J. Geophys. Res.*, *68*, 5789–5806, 1963.

Gutenberg, B., and C. F. Richter, Magnitude and energy of earthquakes, *Ann. Geofis, Rome*, *9*, 1–15, 1956.

Harkrider, D. G., Surface waves in multilayered elastic media, 1, Rayleigh and Love waves from buried sources in a multilayered elastic half-space, *Bull. Seismol. Soc. Am.*, *54*, 627–680, 1964.

Healy, J. H., Crustal structure along the coast of California from seismic-refraction measurements, *J. Geophys. Res.*, *68*, 5777–5787, 1963.

Press, F., and C. Archambeau, Release of tectonic strain by underground nuclear explosions, *J. Geophys. Res.*, *67*, 337–343, 1962.

Slemmons, D. B., Geologic setting for the Fallon-Stillwater earthquakes of 1954, *Bull. Seismol. Soc. Am.*, *46*, 4–9, 1956.

Slemmons, D. B., Geologic effects of the Dixie Valley–Fairview Peak, Nevada, earthquakes of December 16, 1954, *Bull. Seismol. Soc. Am.*, *47*, 353–375, 1957.

Toksöz, M. N., and A. Ben-Menahem, Excitation of seismic surface waves by atmospheric nuclear explosions, *J. Geophys. Res.*, *69*, 1639–1648, 1964.

Toksöz, M. N., A. Ben-Menahem, and D. G. Harkrider, Determination of source parameters by amplitude equalization of seismic surface waves, 1, Underground nuclear explosions, *J. Geophys. Res.*, *69*, 4355–4366, 1964.

(Manuscript received September 18, 1964; revised November 11, 1964.)