again sets in with $\sigma_L/\sigma_T \to 0$. In a model where the conventional Gell-Mann–Zweig quarks are "bound states" of Han-Nambu quarks and gluons, the vector gluon carries charge so above the gluon production threshold scaling may be changed and the ratio σ_L/σ_T most probably will not approach zero. In any case, we could be assured of a new regime of physics.

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Breakdown of scaling in neutrino and electron scattering*

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Observation of deviations from scaling in the structure functions for deep-inelastic inclusive lepton-hadron scattering may provide a test of the hypothesis that the strong interactions are described by an asymptotically free field theory. Tests not involving additional assumptions are obtained for the combinations of structure functions $F_2(ep)-F_2(en)$, $F_2(\nu)-F_2(\bar{\nu})$, and $xF_3(\nu \text{ or } \bar{\nu})$. Neutrino and electron scattering experiments are compared as possible tests of asymptotic freedom.

I. INTRODUCTION

Approximate scaling, as observed in deep-inelastic lepton-hadron scattering, can be understood qualitatively if the strong interactions are described by an asymptotically free non-Abelian gauge field theory.[1] Such theories predict small deviations from exact scaling.[2] A real test of asymptotic freedom must involve the observation and measurement of this scaling breakdown. In this paper we discuss possible experimental tests of the quantitative predictions of asymptotic freedom in deep-inelastic inclusive electroproduction (elaborating on earlier work[3] by Parisi) and in the corresponding neutrino scattering process. In Sec. II we review the consequences of asymptotic freedom for the structure functions in lepton-hadron scattering. We explore possible explicit applications of these ideas in Sec. III. In Sec. IV we compare neutrino and electron scattering experiments as tests of asymptotic freedom. Section V contains conclusions.

II. STRUCTURE FUNCTIONS IN ASYMPTOTICALLY FREE FIELD THEORIES

An asymptotically free non-Abelian gauge theory is characterized by an effective coupling constant which for large spacelike momentum transfer $Q^2 = -q_\mu q^\mu$ is

$$\bar{g}^2(Q^2) = \frac{1}{B\ln(Q^2/\Lambda^2)} . \tag{1}$$

In some sense, $\bar{g}$ describes the strength of the interaction for a momentum transfer q. The constant B depends on the gauge group and the kinds of fundamental fields in the theory. For definiteness, we assume that the strong gauge group is a color SU(3) group with four triplets of quarks ($\mathcal{P}$, $\mathfrak{N}$, λ, and $\mathcal{P}'$ each in three colors), in which case $16\pi^2 B = \frac{25}{3}$.[1] The phenomenological parameter Λ measures the Euclidean momentum at which $\bar{g}$ gets large and the approximation Eq. (1) breaks down. The effective coupling $\bar{g}$ goes to zero as Q^2 goes to ∞, so the theory is "asymptotically free."

In such a theory, the Q^2 dependence of the moments of deep-inelastic structure functions is given by[4,5]

$$\int_0^1 F(x,Q^2)\,x^{n-2}\,dx = \sum_\alpha A_\alpha(n)\exp[-sa_\alpha(n)] \times[1+O(\bar{g}^2/4\pi)]+O(m_p^2/Q^2), \tag{2a}$$

$$s \equiv \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda)}\right), \tag{2b}$$

where F is xF_1, F_2, or xF_3 and Q_0 is an arbitrary reference momentum chosen such that $Q_0 > \Lambda$. The quantity x is the scaling variable introduced by Bjorken, $x = Q^2/2m_p\nu$, where ν is the difference between the lab energies of the incoming and the outgoing lepton. It is sometimes suggested that better scaling is obtained when the structure functions are expressed in terms of some other variable which approaches x in the Bjorken limit (for example, the Bloom-Gilman variable $x' = x/[1+xm_p^2/Q^2]$). All these distinctions are buried in the terms of order m_p^2/Q^2 about which we can say very little. In the context of asymptotically free field theories, we have no way of determining which scaling variable is "best." We can only extract useful information from Eq. (2) for high Q^2 (say, 20 GeV), where terms of order m_p^2/Q^2 are hopefully negligible and the choice of scaling variable is immaterial.

The $A_\alpha(n)$ are unknown constants, but the $a_\alpha(n)$ have been calculated.[4,5] If the terms of order m_p^2/Q^2 can be neglected *and* if $\bar{g}^2/4\pi$ is small so that $\bar{g}$ has the form (1), then the moments have only a logarithmic dependence on Q^2. This is still not very useful because in general Eq. (2) involves several terms with different Q^2 dependences. The sum over α in Eq. (2) refers to the different operators appearing in the operator-product expansion of a product of two currents (to lowest order in $\bar{g}$). In general, there will be operators bilinear in the quark fields and also gluon operators [involving the gauge fields of the color SU(3) group]. But there are at least three structure functions now or soon to be experimentally accessible which have the property that only a single operator contributes to each moment:

(1) The difference of electroproduction structure functions on proton and neutron targets,[3] $F_2(ep) - F_2(en)$. Proton-neutron differences are not isosinglet, so only fermion operators contribute (xF_1 structure functions will not be considered separately because these models imply the Callan-Gross relation $2xF_1 \simeq F_2$).

(2) The structure function xF_3 for neutrino or antineutrino scattering off any target. G-parity conservation implies that gluon operators do not contribute[6] to xF_3, which is a vector-axial-vector interference and is therefore G-odd (we assume the currents are first class).

(3) The neutrino-antineutrino difference $F_2(\nu p) - F_2(\bar{\nu} p)$. This is related by charge symmetry to $F_2(\nu p) - F_2(\nu n)$, so only fermion operators contribute,[6] as in case (1). Indeed, only fermion operators contribute to $F_2(\nu) - F_2(\bar{\nu})$ for any target. Targets with the same number of protons and neutrons are not useful because charge symmetry implies $F_2(\nu) \simeq F_2(\bar{\nu})$, but heavy nuclear targets may be interesting.

For these structure functions, in the large-Q^2, small-$\bar{g}$ limit, Eq. (2) becomes simply

$$\int_0^1 F(x,Q^2)\,x^{n-2}\,dx = A(n)\exp[-sa(n)] , \tag{3a}$$

where

$$a(n) = G\left(-3 - \frac{2}{n} + \frac{2}{n+1} + 4\sum_{k=1}^{n}\frac{1}{k}\right) . \tag{3b}$$

The constant G is a group-theory number. For the the four-triplet model, $G = \frac{4}{25}$. This may still be difficult to test because it involves information on $F(x)$ for all x. But Parisi has shown[3] that Eq. (3) is equivalent to the integro-differential equation

$$\frac{1}{G}\frac{\partial}{\partial s}F(x,s) = [3+4\ln(1-x)]\,F(x,s) + \int_x^1 dz(2-2z)\,F(x/z,s) + 4\int_x^1 \frac{dz}{1-z}\,[zF(x/z,s) - F(x,s)] . \tag{4}$$

If F is known for some value $Q^2=Q_0^2$ at which Eq. (4) is applicable, we can predict $F(x,Q^2)$ for all higher Q^2 in terms of the phenomenological parameter Λ^2 and the model-dependent group-theory number G. We can also predict F at any lower Q^2 for which Eq. (4) is a good approximation. In fact, it is not even necessary to know $F(x,Q_0^2)$ for all x. If it is known for $x \geqslant x_0$, we can predict it for $x \geqslant x_0$ at different Q^2.

III. EXPLICIT CALCULATIONS

To study the consequences of Eq. (4), we consider fitting the data at some value of $s=s_0$ by an expression of the form

$$F(x,s_0)=\sum_{n=n_0}^{N} d_n x^{1/2}(1-x)^n \ . \tag{5}$$

Then Eq. (4) implies

$$\frac{1}{G}\frac{\partial}{\partial s}F(x,s_0)=\sum_{n=n_0}^{N} d_n p_n(x)\,, \tag{6a}$$

where

$$p_n(x)=\sum_{m=0}^{n}(-1)^m\binom{n}{m}l_m(x), \tag{6b}$$

with

$$l_m(x)=x^{m+1/2}\left[-\tfrac{23}{3}+4\ln(1-x)-8\ln(1+x^{1/2})+8\ln2+\frac{8}{(2m-1)(2m-3)}-8\sum_{j=0}^{m}\frac{1}{2j-3}\right]$$
$$+\frac{4x}{2m-1}-\frac{4x^2}{2m-3}+8x^{m+1}+\tfrac{8}{3}x^{m+2}+8\sum_{j=0}^{m}\frac{x^{m-j+2}}{2j-3}\ . \tag{6c}$$

In Eq. (5), the $x^{1/2}$ behavior at small x is suggested by Regge-pole-dominance arguments[7] for the generalized forward Compton amplitude at fixed large Q^2 and $\nu\to\infty$. The leading trajectories that contribute to the processes under consideration all have intercept approximately equal to $\frac{1}{2}$. The $x^{1/2}$ "Regge behavior" is stable under changes in Q^2. The lowest value of $n=n_0$ may be related to the behavior of elastic form factors in a given Q^2 range.[8,2]

The relevant experimental data now available are at low Q^2 for $F_2(ep)-F_2(en)$ (Ref. 9) and $xF_3(\nu)-xF_3(\text{Freon})$ (Ref. 10). The gross features of these data can be described by the function $x'^{1/2}(1-x')^3$ of the Bloom-Gilman variable. To illustrate the main features of the Q^2 dependence of the structure functions, we will assume that this form (but as a function of x rather than x') describes the structure functions for some large $Q^2=Q_0^2$. In other words, we assume that only the $n=3$ term is present in Eq. (5). Then Eq. (6) becomes

$$\frac{1}{G}\frac{\partial}{\partial s}F(x,s_0)=\{x^{1/2}(1-x)^3[-\tfrac{7}{3}+4\ln(1-x)-8\ln(1+x^{1/2})+8\ln2]-\tfrac{24}{5}x(1-x^{5/2})+8x^{3/2}(1-x^{3/2})\}\,. \tag{7}$$

This function is negative for $x \gtrsim 0.08$ and positive for very small x. We can use it to calculate the change in F for a small change in s. To extract more information, we have numerically integrated Eq. (4) with $F(x,s_0)=d_3x^{1/2}(1-x)^3$ for a large ange in s, which corresponds to an enormous range of Q^2. We have used $G=\frac{4}{25}$. The results are summarized in a series of figures. In the figures, $d_3=4$.

Figure 1 shows the function $F(x,s)$ for various values of s. The curves are labeled by their s values ($s=0$ corresponds to the input function). Negative (positive) s corresponds to $Q^2<Q_0^2$ ($Q^2>Q_0^2$). The general features of the evolution in s are easy to understand. All the moments decrease as s increases and the decrease is faster for the higher moments, so the area under the curve decreases while the curve becomes more sharply peaked at low x.

We cannot label the curves with values of Q^2 until we know the value of the parameter Λ^2. This parameter can be determined experimentally by fitting the data at two different large values of Q^2 to two curves in Fig. 1 [or their generalizations from Eq. (6)].[11] For definiteness, take $\Lambda^2=1$ GeV^2 and $Q_0^2=100$ GeV^2. The curves labeled $s=-0.69$, -0.43, 0, 0.55, and 2.2 then correspond to $Q^2=10$, 20, 100, 2860, and 7×10^{18} GeV^2.

Figure 2 is a redrawing of Fig. 1 in terms of the variable $\omega=1/x$. This emphasizes the behavior of the structure functions at large ω (small x).

Figure 3 shows the Q^2 evolution of the structure function at fixed x, normalized to its value at Q_0^2. The input is again $F(x,Q_0^2)=x^{1/2}(1-x)^3$ and $G=\frac{4}{25}$

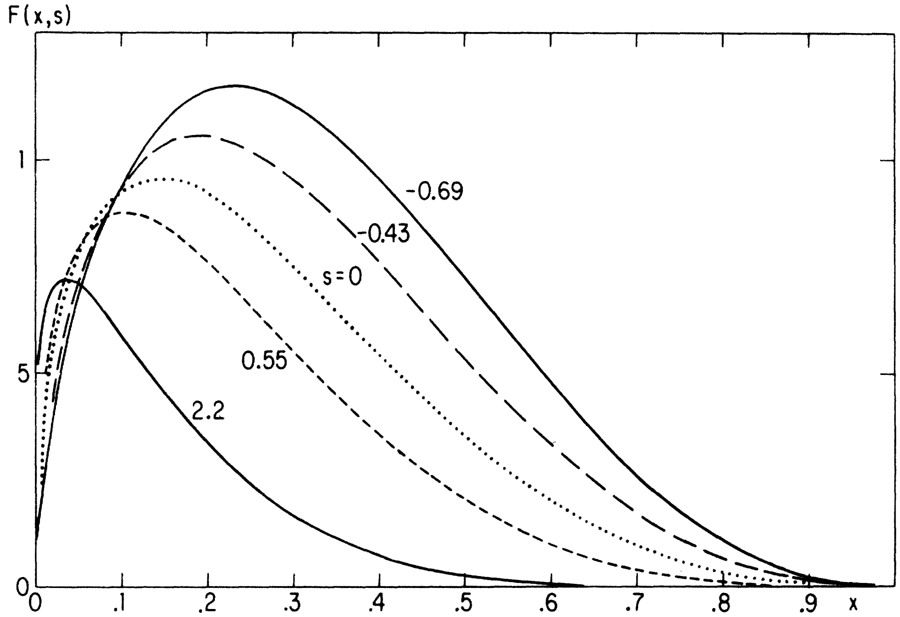

FIG. 1. $F(x, s)$ versus x at various values of s.

as in Figs. 1 and 2. Once more, it is not possible to show the results as a function of Q^2, but only as a function of s. The structure function varies most rapidly at large x and decreases for increasing s (or Q^2). Only for very small x does it increase with s. For reasonable values of Λ^2 and thinkable ranges of Q^2, it will be very difficult to observe nonlinear effects (except possibly at very large x). With good data at large Q^2 the experimentalist should try a fit of the form

$$F(x, s) = \sum_{n=n_0}^{N} d_n x^{1/2}(1-x)^n + Gs \sum_{n=n_0}^{N} d_n p_n(x), \quad (8)$$

with $p_n(x)$ given by Eq. (6). With the d_n determined by a fit at some $Q^2 = Q_0^2$, Eq. (8) involves two additional parameters: the group-theory number G and the scale parameter Λ^2.

In Fig. 4 we present the results of a theoretical experiment on the effects of terms of order m_p^2/Q^2:

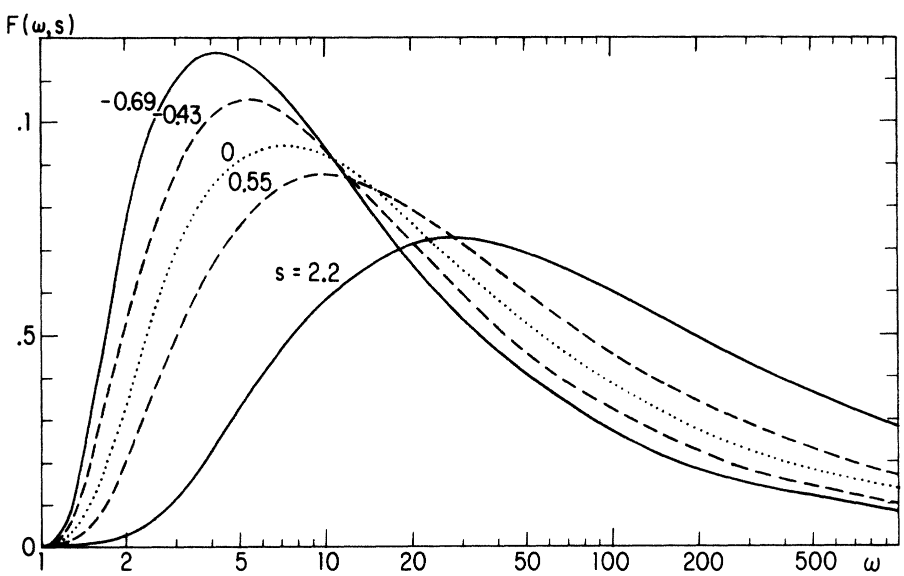

FIG. 2. $F(\omega, s)$ versus ω at various values of s.

We plot the function $F(x,Q^2)=x'^{1/2}(1-x')^3$ for $Q^2=2$, 4, 20, and ∞ GeV^2. In other words, we show the Q^2 dependence of the structure function if scaling is exact in the Bloom-Gilman variable x'. Except at very small x or large Q^2 ($Q^2>10$ GeV^2, say) the behavior is qualitatively similar to the asymptotic-freedom prediction shown in Fig. 1, for some suitably chosen Λ^2. We include this figure to emphasize again the need for data at large Q^2.

IV. NEUTRINO VERSUS ELECTRON

The neutrino structure functions $F_2(\nu)-F_2(\bar{\nu})$ and $xF_3(\nu$ or $\bar{\nu})$ may have some practical advantages over $F_2(ep)-F_2(en)$ as tests of asymptotic freedom. Neutrino experiments do not require the comparison of two different targets. Nuclear physics corrections are unnecessary. Furthermore, although the neutrino rates are much smaller, there is no suppression of the high-Q^2 region, where electroproduction is damped by the photon propagator.

There are also several difficulties peculiar to the neutrino tests besides the obvious rate limitation. To observe deviations from scaling, we need good knowledge of the flux. Other problems are radiative corrections, W-boson propagator effects, and the presence of unconventional terms in the weak current.

Precise neutrino-scattering experiments will raise the specter of radiative corrections. For charged-current events, the effects of radiation from the charged lepton cannot be separated in a gauge-invariant fashion from radiation off the hadron. Thus, an estimate of radiative corrections requires a model of the hadron. In general, the

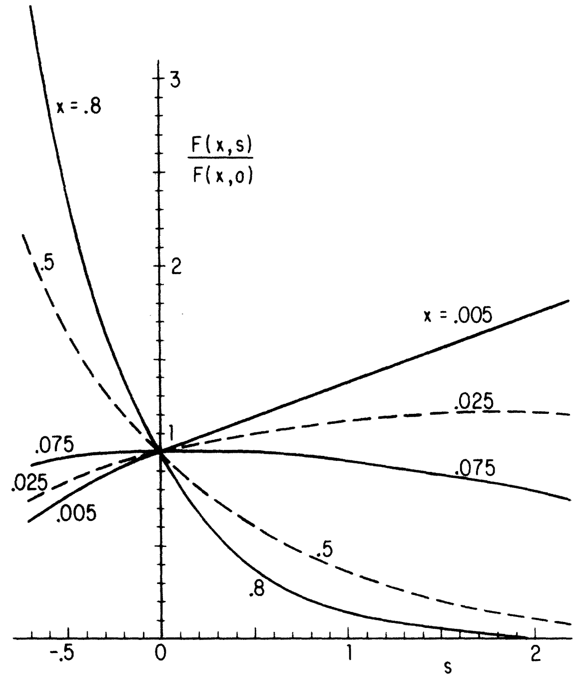

FIG. 3. $F(x,s)/F(x,0)$ versus s at various values of x.

corrections will change both the x and Q^2 dependences of the differential cross section. The correction to the x dependence does not affect the general features of our predictions. If scaling were exact, the extra Q^2 dependence introduced in the raw data by the radiative corrections could be embarrassing since it might be confused with the Q^2 behavior at fixed x shown in Fig. 3. However, in model calculations[12] the radiative corrections increase the raw differential cross section at fixed x as Q^2 increases. The effect is

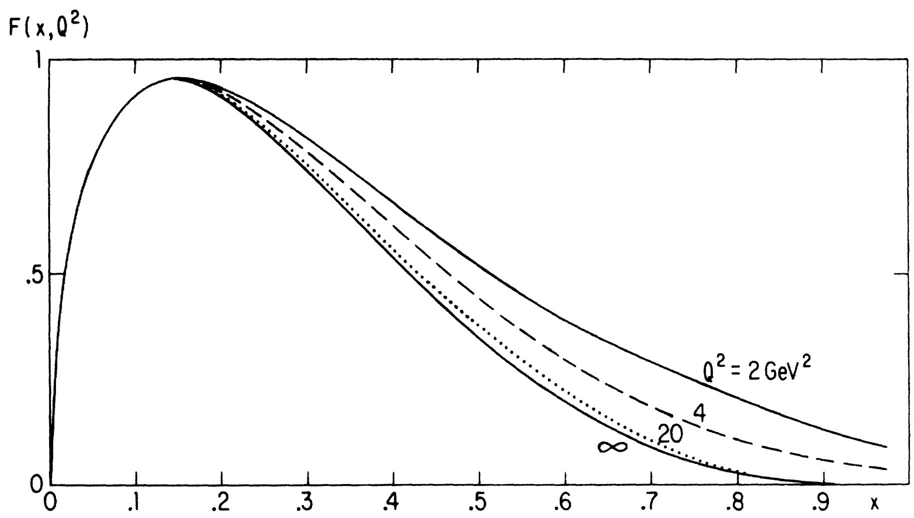

FIG. 4. $F(x,Q^2)=f(x')$ versus x at various values of Q^2.

$\lesssim 5\%$ from $Q^2 = 5$ to 40 GeV^2. This has the opposite sign to what we expect from asymptotic freedom at large x. Therefore, if the behavior shown in Fig. 3 is observed in the raw data, it is probably not due to the effect of radiative corrections.

If the weak interactions are also described by a gauge theory, one expects other surprises in high-energy neutrino scattering: the opening of charm thresholds and the existence of intermediate vector bosons. The W-boson propagator suppresses high-Q^2 events by an over-all factor of $(Q^2 + M_W{}^2)^{-2}$ Since the suppression is x independent it does not lead to the behavior shown in Fig. 3.

Above charm threshold there could be an increase in xF_3 at small x. Such an effect should be easy to distinguish from the effects we discuss because it sets in at a definite value of W^2, the mass of the hadron system.[13] Nevertheless, it would probably be impossible to correct for such an effect, so the small-x behavior of xF_3 would be useless as a test of asymptotic freedom.

V. CONCLUSIONS

Asymptotically free non-Abelian gauge field theories of the strong interactions make specific predictions of deviations from exact scaling in deep-inelastic inclusive lepton-hadron scattering experiments at large Q^2. It may be possible to test asymptotic freedom by observing deviations from scaling in xF_3 for neutrino (antineutrino) scattering, $F_2(\nu) - F_2(\bar{\nu})$, and $F_2(ep) - F_2(en)$. The observability of these effects depends on the value of the parameter Λ^2. If Λ^2 is too small, deviations from exact scaling will be very small at large Q^2. If Λ^2 is too big, asymptotic freedom is useless as an explanation of scaling in contemporary experiments; what is observed at SLAC energies would in this context be an incomprehensible epiphenomenon. The tests we have discussed only are interesting if Λ^2 is of order 1 GeV^2, in which case Λ^2 and the group-theory number G can in principle be measured experimentally. Unfortunately, the techniques described above cannot be simply applied to $F_2(ep)$. To check asymptotic freedom using $F_2(ep)$ we can either devise very complicated (and probably useless) tests involving four different values of Q^2 or we can use information from very model-dependent extra-field-theoretic sources.

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