Probing the Dark Sector with Dark Matter Bound States

Haipeng An,[1] Bertrand Echenard,[2] Maxim Pospelov,[3,4] and Yue Zhang[1]

[1]Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA
[2]California Institute of Technology, Pasadena, California 91125, USA
[3]Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, V8P 1A1 Canada
[4]Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

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A model of the dark sector where $O(\text{few GeV})$ mass dark matter particles χ couple to a lighter dark force mediator V, $m_V \ll m_\chi$, is motivated by the recently discovered mismatch between simulated and observed shapes of galactic halos. Such models, in general, provide a challenge for direct detection efforts and collider searches. We show that for a large range of coupling constants and masses, the production and decay of the bound states of χ, such as 0^{-+} and 1^{--} states, η_D and Υ_D, is an important search channel. We show that $e^+e^- \to \eta_D + V$ or $\Upsilon_D + \gamma$ production at B factories for $\alpha_D > 0.1$ is sufficiently strong to result in multiple pairs of charged leptons and pions via $\eta_D \to 2V \to 2(l^+l^-)$ and $\Upsilon_D \to 3V \to 3(l^+l^-)$ $(l = e, \mu, \pi)$. The absence of such final states in the existing searches performed at *BABAR* and Belle sets new constraints on the parameter space of the model. We also show that a search for multiple bremsstrahlung of dark force mediators, $e^+e^- \to \chi\bar{\chi} + nV$, resulting in missing energy and multiple leptons, will further improve the sensitivity to self-interacting dark matter.

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Introduction.—Identifying dark matter is an open question of central importance in particle physics and cosmology. In recent years, the paradigm of weakly interacting dark matter supplied by a new force in the dark sector came to prominence [1,2], motivated by a variety of unexplained astrophysical signatures. It was later shown [3,4] that this model provides a straightforward realization of self-interaction dark matter [5], which helps to alleviate tensions between observed and simulated shapes of dark matter halos (see, e.g., Ref. [6]).

It is of great phenomenological interest to check whether such a dark force could be probed in laboratories. The simplest way for dark matter to interact with the standard model (SM) sector is through a vector or scalar mediators coupled to the SM fields via the kinetic mixing or the Higgs portals. For dark matter heavier than 4–5 GeV, direct detection experiments provide the strongest constraints on such models. High-energy collider probes typically require more effective production channels [7–11]. For dark matter lighter than 4–5 GeV, the limits from direct detection experiments arise from electron recoil [12] and are much weaker. In this mass range, strong CMB constraints on dark matter annihilation [13,14] naturally point to particle-antiparticle asymmetry in the dark sector. Constituents of such a dark sector, light dark matter, and a light mediator, can be searched for in meson decays [15], fixed target experiments [16], mono-photon events at colliders [17], or via the production or scattering sequence in proton [18] and electron [19] beam dump experiments, or perhaps via new galactic substructures and minihalos [20]. Most of the existing searches of light particles [21] are insensitive to dark matter with $m_\chi > m_{\text{mediator}}$, and, therefore, would not be able to establish any candidate signal as coming specifically from the dark force carrier.

In this Letter, we show that the presence of self-interacting dark matter within the kinematic reach of existing colliders provides opportunities for the new search channels. We outline such possibilities in the minimal setup where the dark force carrier also mediates the interaction between dark matter and the SM particles. A light mediator gives an attractive force between χ and $\bar{\chi}$ particles, leading to the formation of bound states, which can be produced on-shell at colliders (weakly coupled dark matter bound states have been studied in various contexts [22–28]). In addition, the production of continuum $\chi\bar{\chi}$ leads to final state radiation (FSR) of light mediators. Both channels typically result in a striking multilepton final state, which can be searched for at B factories and fixed target experiments. It is well known that heavy flavor mesons and heavy quarkonia were instrumental for uncovering a wealth of information about the SM. Similarly, should a dark force exist, the aforementioned channels may allow for genuine tests of the detailed content of the dark sector.

Dark matter bound states production.—We illustrate these ideas in the well-studied example of the vector mediator model. The Lagrangian for dark matter and dark photons is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\chi} i\gamma^\mu(\partial_\mu - i g_D V_\mu)\chi - m_\chi \bar{\chi}\chi - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu, \qquad (1)$$

where κ is the kinetic mixing between the photon and the vector field V. The dark matter particle χ is a Dirac fermion, neutral under the SM gauge group, but charged under the dark $U(1)_D$ interaction that has a new vector particle V_μ (sometimes called a“dark photon”) as a force carrier. It is assumed that the correct cosmological abundance of dark matter is controlled by particle-antiparticle asymmetry in the dark sector. (Other well-motivated realizations of self-interacting dark matter based on a new strongly interacting sector would also typically require the existence of dark photons [29].)

As discussed in the introduction, sufficiently strong dark interaction strength and light dark photons will result in the formation of dark matter particles $(\chi\bar{\chi})$. The two lowest $(1S)$ bound states, ${}^1S_0$ ($J^{PC} = 0^{-+}$) and ${}^3S_1$ ($J^{PC} = 1^{--}$), will be called η_D and Υ_D, respectively. The condition for their existence has been determined numerically [30] (It is known that too large α_D would run to the Landau pole very quickly at higher scale [31]. Hereafter, we focus on $\alpha_D \leq 0.5$, and work with leading-order results in α_D.), $1.68 m_V < \alpha_D m_\chi$, with $\alpha_D = g_D^2/(4\pi)$. Their quantum numbers suggest the following production mechanisms at colliders:

$$\begin{aligned} &e^+e^- \to \eta_D + V; \qquad e^+e^- \to \Upsilon_D + \gamma; \\ &p + p \to \Upsilon_D + X. \end{aligned} \tag{2}$$

The last process represents the direct production of Υ_D from $q\bar{q}$ fusion. All production processes are mediated by a mixed $\gamma - V$ propagator, as shown in Fig. 1.

In order to obtain the rate for the first process in Eq. (2), we calculate the amplitude of $e^+e^- \to \chi\bar{\chi}V$ with $\chi, \bar{\chi}$ having the same four momentum p (with $p^2 = m_\chi^2$), and apply the projection operator,

$$\Pi_\eta = \sqrt{\frac{1}{32\pi m_\chi^3}} R_{\eta_D}(0)(\not{p} + m_\chi)\gamma_5(\not{p} - m_\chi), \tag{3}$$

to select the η_D bound state [32]. We find a leading-order differential cross section

$$\frac{d\sigma_{e^+e^-\to\eta_D V}}{d\cos\theta} = \frac{4\pi\alpha\alpha_D^2\kappa^2[R_{\eta_D}(0)]^2(1+\cos^2\theta)}{m_\chi s^{3/2}(s - 4m_\chi^2 + m_V^2)^2}|\mathbf{p}|^3, \tag{4}$$

where θ is the angle between η_D and the initial e^- in the center-of-mass (c.m.) frame, and $|\mathbf{p}|$ is the spatial momentum

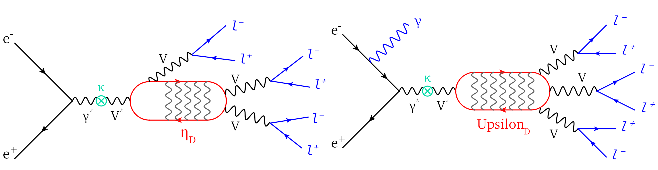

FIG. 1. Diagram for η_D and Υ_D production and decay at B factories.

of η_D, $|\mathbf{p}| = \sqrt{[s - (2m_\chi + m_V)^2][s - (2m_\chi - m_V)^2]}/(2\sqrt{s})$. We neglect the binding energy for η_D, and set $m_{\eta_D} \simeq 2m_\chi$.

The value of $R_{\eta_D}(0)$ can be analytically estimated using the Hulthén potential $V(r) = -\alpha_D \delta e^{-\delta r}/(1 - e^{-\delta r})$ with $\delta = (\pi^2/6)m_V$, known as a good approximation of the Yukawa potential $V(r) = -\alpha_D e^{-m_V r}/r$ [33]. In that case, $R_{\eta_D}(0) = (4 - \delta^2 a_0^2)^{1/2} a_0^{-3/2}$, where $a_0 = 2/(\alpha_D m_\chi)$.

The scalar bound state η_D dominantly decays into two dark photons, each subsequently decaying into a pair of SM particles via kinetic mixing. These decays are all prompt for the relevant region of parameter space. The above decay chain eventually results in the final states containing six charged tracks, which can be electrons, muons, or pions, depending on the dark photon mass.

We turn to the calculation of Υ_D production via initial state radiation (Fig. 1). In the Υ_D rest frame, the non-relativistic expansion can be used, taking the dark matter field in the form $\chi = e^{im_\chi t}[\xi, \sigma\cdot\mathbf{p}/(2m_\chi)\xi]^T + e^{-im_\chi t}[\sigma\cdot\mathbf{p}/(2m_\chi)\zeta, \zeta]^T$, where ξ, ζ are the 2-spinor annihilation (creation) operators for particle (antiparticle). We use the relation between matrix element and wave function [34],

$$\langle 0|\zeta^\dagger\sigma^\mu\xi|\Upsilon_D\rangle = \sqrt{\frac{1}{2\pi}} R_{\Upsilon_D}(0)\varepsilon^\mu_{\Upsilon_D}, \tag{5}$$

where $\varepsilon^\mu_{\Upsilon_D}$ is the polarization vector of Υ_D and $R_{\Upsilon_D}(0) \simeq R_{\eta_D}(0)$ is the radial wave function at origin. Taking into account the kinetic mixing between dark photon and the photon, we derive the effective kinetic mixing term between Υ_D and the photon,

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\kappa\kappa_D F_{\mu\nu}\Upsilon_D^{\mu\nu}, \qquad \kappa_D = \sqrt{\frac{\alpha_D}{2m_\chi^3}} R_{\Upsilon_D}(0). \tag{6}$$

In the limit $m_V \ll \alpha_D m_\chi$, the term κ_D reduces to $\kappa_D = \alpha_D^2/2$. We obtain a differential cross section:

$$\begin{aligned} \frac{d\sigma_{e^+e^-\to\gamma\Upsilon_D}}{d\cos\theta} &\simeq \frac{2\pi\alpha^2\kappa^2\kappa_D^2}{s}\left(1 - \frac{4m_\chi^2}{s}\right) \\ &\times \left[\frac{8s^2(s^2 + 16m_\chi^4)\sin^2\theta}{(s - 4m_\chi)^2(s + 4m_e^2 - (s - 4m_e^2)\cos 2\theta)^2} - 1\right], \end{aligned} \tag{7}$$

where θ is the angle between γ and the initial e^- in the c.m. frame. In the denominator, the electron mass must be retained in order to regularize the θ integral, as for $m_e = 0$ the cross section is divergent in the forward direction [35].

Compared to the $e^+e^- \to \eta_D V$ process, the $e^+e^- \to \gamma\Upsilon_D$ cross section is suppressed by a factor α/α_D, although the latter contains a logarithmic enhancement from the angular integral. Moreover, the cross section $e^+e^- \to \eta_D V$ contains an additional m_χ^2/s factor, which brings additional suppression of lighter dark matter. For $\alpha_D \gtrsim 0.1$ and

$m_\chi \sim \sqrt{s}$, the two processes have similar cross sections, and we will combine them to set the limit on this model.

The Υ_D particle will subsequently decay into three dark photons. Similarly to derivation of Eq. (4), we calculate the differential decay rate of Υ_D into three massive dark photons,

$$\frac{d\Gamma(\Upsilon_D \to 3V)}{dx_1 dx_2} = \frac{2\alpha_D^3[R_{\Upsilon_D}(0)]^2}{3\pi m_\chi^2} \times \frac{39x^8 + 4x^6F_6 - 16x^4F_4 + 32x^2F_2 + 256F_0}{(x^2-2x_1)^2(x^2-2x_2)^2(x^2+2(x_1+x_2-2))^2}, \quad (8)$$

where $x_{1,2} = E_{1,2}/m_\chi$, $x = m_V/m_\chi$, and

$$\begin{aligned} F_6 &= x_1^2 + (x_1+x_2)(x_2-2) - 30, \\ F_4 &= (x_1^2 + x_1x_2 - 2x_1)(3x_2-10) - 10x_2(x_2-2) - 21, \\ F_2 &= x_1^4 + 2x_1^3(x_2-2) + x_1^2(x_2(3x_2-22)+28) \\ &\quad + 2x_1(x_2-2)(x_2(x_2-9)+12) \\ &\quad + x_2(x_2-2)(x_2(x_2-2)+24) + 24, \\ F_0 &= x_1^4 + 2x_1^3(x_2-2) + x_1^2(3x_2(x_2-3)+7) \\ &\quad + x_1(x_2-1)(x_2-2)(2x_2-3) \\ &\quad + (x_2-1)^2(x_2(x_2-2)+2). \end{aligned} \quad (9)$$

When x_1, x_2 are fixed, the relative angles between the dark photons are also fixed in the rest frame of Υ_D. Their ranges are $x \le x_1 \le 1-(3/4)x^2$, and $(x_2)_{\rm min}^{\rm max} = \pm\sqrt{(4-3x^2-4x_1)(x_1^2-x^2)/(4+x^2-4x_1)}/2 + (2-x_1)/2$. This channel eventually results in the final states containing 3 pairs of electrons, muons, or pions, and one photon.

To estimate the limit from searches at B factories, we simulate events according to Eqs. (4) and (7), and apply the kinematic constraints used for dark Higgsstrahlung searches [36–38]. We select events containing six charged tracks (made of e, μ, or π), excluding the 6π final states due to the presence of larger SM backgrounds. For each track, we require $p_T > 150$ MeV, and $-0.95 < \cos\theta_{\rm cm} < 0.85$ in the c.m. frame. We include a 95% efficiency for reconstructing each of the charged tracks. For the η_D channel, we require the invariant mass of the six charged tracks to be close to the c.m. energy. For the Υ_D channel, we do not require to find the photon, but impose the condition that the missing mass recoiling against the six tracks is around zero. We assume negligible SM background, similar to the dark Higgsstrahlung searches [36–38]. In the left plot of Fig. 2, we present the 90% C.L. exclusion in the dark photon parameter space based the existing *BABAR* luminosity of 516 fb^{-1} [37]. We expect the current Belle data to give a similar limit. If dark bound states exist, the limit on the kinetic mixing is more than 1 order of magnitude stronger than the direct dark photon search via dilepton resonance at *BABAR* [39] and the pion decay search at NA48/2 [40]. We also show the expected sensitivity of future B factories (Belle-II) assuming 100 times more luminosity and a similar search strategy.

These results provide useful constraints on self-interacting dark matter (SIDM) scenarios. In the right plot of Fig. 2, the green region is favored for SIDM models solving the small-scale structure problem, which satisfies the condition $0.1\ {\rm cm}^2/{\rm g} \le \langle\sigma_T\rangle/m_\chi \le 10\ {\rm cm}^2/{\rm g}$ [3]. For this parameter space, the s-partial wave gives the dominant contribution to

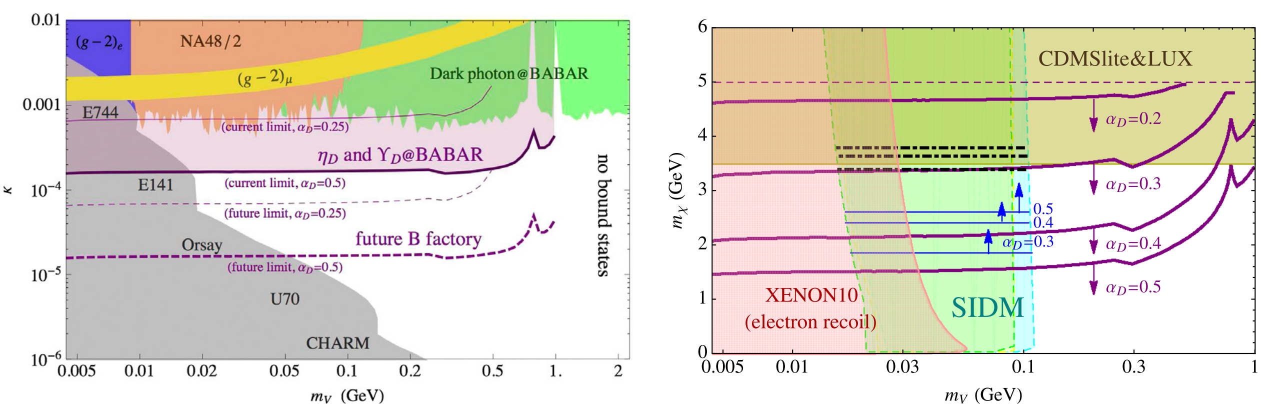

FIG. 2. Left: Constraint on the dark photon parameter space from the *BABAR* dark Higgsstrahlung searches, adapted to the production and decay of dark bound states η_D and Υ_D. The solid purple curve corresponds to the current *BABAR* limit for the parameters $\alpha_D = 0.5$, $m_\chi = 3.5$ GeV. The dashed purple curve shows the future reach of B factories. Right: Current constraints on the $m_\chi - m_V$ plane for the SIDM scenario are shown with $\kappa^2 = 10^{-7}$ and different values of α_D. The green (blue) region is favored for SIDM solving the galactic small-scale structure problems [3] for $\alpha_D = 0.3(0.5)$. The combined constraints via the $e^+e^- \to (\eta_D V, \Upsilon_D) \to 3V$ channels are shown as thick purple curves, and the constraints via the $e^+e^- \to \chi\bar{\chi} + 3V$ channel are shown in thin blue curves. Allowed regions are in the arrow direction. Assuming no SM background, the constraints via the $e^+e^- \to \chi\bar{\chi} + 2V$ channel are shown in dot-dashed black curves for $\alpha_D = 0.3$, 0.4, 0.5 (bottom up). The brown region is excluded by CDMSlite [41] and LUX [42]. The region $m_V \lesssim 30$ MeV is ruled out by the XENON10 electron recoil analysis [12] for $\alpha_D = 0.3$.

the dark matter elastic scattering cross section. The purple curves show the current *BABAR* 90% C.L. exclusion contours for the SIDM model. Compared to direct detection experiments, dark bound states at *BABAR* further constrain the allowed dark matter mass down to sub-GeV, if α_D is sufficiently large.

In the case of scalar dark matter charged under $U(1)_D$, the ground state χ_D formed by a pair $\chi\chi^*$ has quantum numbers ${}^1S_0$ (0^{++}) [43], and will be produced in a similar process as η_D in Eq. (2). On the other hand, the counterpart of Υ_D is a p-wave state, and its production rate is further suppressed by the derivative of its wave function at the origin. Therefore, we expect slightly weaker bounds on scalar dark matter compared to the fermion case.

Multimediator final state radiation.—Smaller values of α_D or larger m_V/m_χ ratios may prevent the existence of $\chi\bar{\chi}$ bound states. In that case, mediator states can still be produced through the FSR process $e^+e^- \to \chi\bar{\chi} + nV$. (One could also study this process in high-energy proton collisions [10], should a new efficient channel for $\chi\bar{\chi}$ production exist.) The FSR dark photons further decay into pairs of charged SM particles. Therefore, the typical signal consists of multiple charged tracks plus missing energy, taken away by the $\chi\bar{\chi}$ pair. The *BABAR* experiment did not trigger on two charged leptons due to overwhelming QED backgrounds. The channel with four charged leptons plus missing energy is, however, quite promising, and we suggest performing a corresponding search at both *BABAR* and Belle. The dominant SM backgrounds for the $4l+$ missing energy signature may come from the $\tau^+\tau^-l^+l^-$ final states, and one would expect over 10^4 such events at *BABAR*. If, however, the two invariant mass $m_{l^+l^-} = m_V$ conditions are imposed, this background can be considerably reduced. With the assumption of negligible background, the whole low mass dark matter window for the SIDM can be potentially ruled out for $\alpha_D \gtrsim 0.3$, as shown in black dot-dashed curves of Fig. 2. Six charged lepton final states, similar to the case of the bound state study, have been searched for, and we generate $e^+e^- \to \chi\bar{\chi} + 3V$ events using MadGraph5 [44]. With the same kinematic requirements described in the previous section, the lower bounds on m_χ in the region favored by the SIDM model are shown by the thin blue curves in Fig. 2 for several choices of α_D. For this channel, we only show the constraint in the region of interests to the SIDM scenario. For smaller m_V, the leading order simulation becomes less accurate due to the large logarithms from the soft dark photons. Moreover, this region has already been excluded by the direct detection experiment with electron recoils [12]. For $m_V >$ few100 MeV the sensitivity is expected to worsen due to a shrinking phases space.

The search for FSR production of dark photons by dark matter pair-production has additional kinematic limitations. The phase space for producing energetic charged leptons becomes smaller for larger m_χ, resulting in softer final state leptons. This feature can be read from Fig. 2, as for $m_\chi \gtrsim 2.5$ GeV, producing charged leptons energetic enough to pass the cuts becomes difficult. As a result, the potential lower bound on m_χ does not change very much with the increase of α_D. On the other hand, the production and decay of dark bound states Υ_D and η_D create more energetic leptons for larger m_χ. Therefore, the two search strategies are complementary to each other.

Hadronic probes of dark sector.—Fixed target experiments with proton beams can also be used to probe a dark sector. For realistic energies of available proton beams, the most important production channel is from the quark–antiquark fusion, $q\bar{q} \to \Upsilon_D$. Generalizing calculations of Ref. [45], the production cross section is given by

$$\sigma_{pp(n)\to\Upsilon_D} = \frac{4\pi^2\alpha\kappa^2\kappa_D^2}{s}\sum_q Q_q^2 \int_\tau^1 \frac{dx}{x} \times \left[f_{q/p}(x) f_{\bar{q}/p(n)}\left(\frac{\tau}{x}\right) + f_{\bar{q}/p}(x) f_{q/p(n)}\left(\frac{\tau}{x}\right)\right], \quad (10)$$

where $\tau = m_V^2/s$, $f_{q/p(n)}$ and $f_{\bar{q}/p(n)}$ are the relevant structure functions for this process, and Q_q is the quark charge in units of e. Unlike B factories, only muonic decays of dark bound states, such as $\Upsilon_D \to 3V \to 3(\mu^+\mu^-)$, constitute a useful signature, as backgrounds in other channels are likely to be too large. The multidark photon FSR channels can also be relevant for the proton beam experiments.

Among the possible candidates of proton-on-target experiments, we focus our discussion on SeaQuest [46] and the planned SHiP [47] facilities. Note that only a fixed target mode of operation, rather than a beam dump mode that would try to remove prompt muons, is suitable for the search of Υ_D. Taking a point in the parameter space, $m_\chi = 2$ GeV, $\kappa^2 = 10^{-7}$, $m_V = 300$ MeV, $\alpha_D = 0.5$ and the energy of incoming proton beam of 400 GeV, we estimate a probability of producing a Υ_D decaying to $3(\mu^+\mu^-)$ for a 1 mm tungsten target, $P = n\sigma\ell \sim 2 \times 10^{-17}$. With $O(10^{20})$ particles on target, one could potentially expect up to 2×10^3 six muon events. The large multiplicity of signal events gives some hope that this signal could be extracted from a large number of muons produced per each proton spill. Given the current uncertainties in estimating the background, we refrain from showing the potential reach of proton experiments in Fig. 2, noting that in any case, it would not cover the most interesting region for SIDM, namely, $m_V < 200$ MeV.

Outlook.—Among the various probes of dark sectors suggested and conducted in recent years, only a few are sensitive to both the dark force and dark matter at the same time. We have pointed out that in the case of relatively strong self-interaction, the presence of dark force greatly facilitates the discovery of the entire sector, as it leads to the formation of dark bound states, and causes dark FSR radiation that decays into multiple charged particles of the SM. The existing searches at *BABAR* and Belle already limit this possibility, and further advance in sensitivity can be made by searching for the missing energy plus pairs of charged particles.

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[1] M. Pospelov, A. Ritz, and M. B. Voloshin, Phys. Lett. B **662**, 53 (2008).
[2] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D **79**, 015014 (2009).
[3] S. Tulin, H. B. Yu, and K. M. Zurek, Phys. Rev. D **87**, 115007 (2013).
[4] M. Kaplinghat, S. Tulin, and H. B. Yu, Phys. Rev. Lett. **116**, 041302 (2016).
[5] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[6] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, Mon. Not. R. Astron. Soc. **415**, L40 (2011).
[7] W. Shepherd, T. M. P. Tait, and G. Zaharijas, Phys. Rev. D **79**, 055022 (2009).
[8] M. Autran, K. Bauer, T. Lin, and D. Whiteson, Phys. Rev. D **92**, 035007 (2015).
[9] Y. Bai, J. Bourbeau, and T. Lin, J. High Energy Phys. 06 (2015) 205.
[10] M. Buschmann, J. Kopp, J. Liu, and P. A. N. Machado, J. High Energy Phys. 07 (2015) 045.
[11] Y. Tsai, L. T. Wang, and Y. Zhao, Phys. Rev. D **93**, 035024 (2016).
[12] R. Essig, A. Manalaysay, J. Mardon, P. Sorensen, and T. Volansky, Phys. Rev. Lett. **109**, 021301 (2012).
[13] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1502.01589.
[14] T. R. Slatyer, Phys. Rev. D **93**, 023527 (2016).
[15] P. Fayet, Phys. Rev. D **74**, 054034 (2006).
[16] E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Phys. Rev. D **91**, 094026 (2015).
[17] R. Essig, J. Mardon, M. Papucci, T. Volansky, and Y. M. Zhong, J. High Energy Phys. 11 (2013) 167.
[18] B. Batell, M. Pospelov, and A. Ritz, Phys. Rev. D **80**, 095024 (2009).
[19] E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Phys. Rev. D **88**, 114015 (2013).
[20] Y. Zhang, J. Cosmol. Astropart. Phys. 05 (2015) 008.
[21] R. Essig *et al.*, arXiv:1311.0029.
[22] M. Pospelov and A. Ritz, Phys. Lett. B **671**, 391 (2009).
[23] J. D. March-Russell and S. M. West, Phys. Lett. B **676**, 133 (2009).
[24] D. E. Kaplan, G. Z. Krnjaic, K. R. Rehermann, and C. M. Wells, J. Cosmol. Astropart. Phys. 05 (2010) 021.
[25] E. Braaten and H.-W. Hammer, Phys. Rev. D **88**, 063511 (2013).
[26] L. Pearce and A. Kusenko, Phys. Rev. D **87**, 123531 (2013).
[27] M. B. Wise and Y. Zhang, Phys. Rev. D **90**, 055030 (2014); Phys. Rev. D **91**, 039907 (2015).
[28] M. B. Wise and Y. Zhang, J. High Energy Phys. 02 (2015) 023.
[29] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Phys. Rev. Lett. **113**, 171301 (2014).
[30] F. J. Rogers, H. C. Graboske, Jr., and D. J. Harwood, Phys. Rev. A **1**, 1577 (1970).
[31] H. Davoudiasl and W. J. Marciano, Phys. Rev. D **92**, 035008 (2015).
[32] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, and M. L. Mangano, Nucl. Phys. **B514**, 245 (1998).
[33] C. S. Lam and Y. P. Varshni, Phys. Rev. A **4**, 1875 (1971).
[34] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); Phys. Rev. D **55**, 5853 (1997).
[35] P. Fayet, Phys. Rev. D **75**, 115017 (2007).
[36] B. Batell, M. Pospelov, and A. Ritz, Phys. Rev. D **79**, 115008 (2009).
[37] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **108**, 211801 (2012).
[38] I. Jaegle (Belle Collaboration), Phys. Rev. Lett. **114**, 211801 (2015).
[39] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **113**, 201801 (2014).
[40] J. R. Batley *et al.* (NA48/2 Collaboration), Phys. Lett. B **746**, 178 (2015).
[41] R. Agnese *et al.* (SuperCDMS Collaboration), Phys. Rev. Lett. **112**, 041302 (2014).
[42] D. S. Akerib *et al.* (LUX Collaboration), Phys. Rev. Lett. **112**, 091303 (2014).
[43] Chong-shou Gao and Jin-yan Zeng, *Lectures in Particle and Nuclear Physics* (High Education Press, Beijing, 1990).
[44] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, and T. Stelzer, J. High Energy Phys. 06 (2011) 128.
[45] P. deNiverville, D. McKeen, and A. Ritz, Phys. Rev. D **86**, 035022 (2012).
[46] M. Liu, Proceedings of CIPANP 2015, http://indico.wlab.yale.edu/indico/event/2/session/18/contribution/284/material/slides/0.pdf.
[47] S. Alekhin *et al.*, arXiv:1504.04855.