The author is glad to acknowledge his indebtedness to Professor G. N. Lewis for his interest and helpful suggestions; to Dr. Simon Freed for his kind advice throughout the work in a field with which the author was not familiar; and to Beatrice Wulf with whom much of the experimental work was done.

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[1] Becquerel, *Compt. rend.*, **92,** 348 (1881).

[2] Schumeister, *Sitz. Akad. Wiss. Wien,* II, **83,** 45 (1881).

[3] Lewis, *Valence and the Structure of Atoms and Molecules*, Chemical Catalog Co., **1923,** pages 130, 147 et seq; *Chemical Reviews,* **1,** 234 (1924).

[4] See, for example, Stoner, *Magnetism and Atomic Structure,* E. P. Dutton and Co., **1926,** page 40.

[5] The expression for oxygen was calculated from an average value of the volume susceptibility using the three recent values listed by Stoner in Table IX of his book, see ref. 4. The expression for air was obtained from this and from the composition of air, taking account of the oxygen only.

GENERAL ARITHMETIC

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Communicated October 15, 1927

The abstract theory of a mathematical system consisting of a set of elements and two operations "multiplication," and, later, "addition" is developed by postulational methods with examples.

The more important results are the following. An "arithmetic" may be roughly described as a system in which

1. Every element is completely specified by a finite number of cardinal numbers.
2. "Division" is not always possible, and we can find when one element divides another element in a finite number of steps.
3. Unique resolution into "prime factors" is always possible.

For the case when multiplication is commutative we replace these vague requirements by an exact set of postulates, the necessary and sufficient conditions for a system to form an arithmetic. We give a general theory of recurring sequences, exhibiting the connections with the Dedekind field theory which we develop following Kronecker, as an arithmetic of forms without assuming the so-called "fundamental theorem of algebra." We next show that all systems of ideals and ideal numbers are abstractly equivalent and may be replaced by the system of rational integers, making ideals and ideal numbers unnecessary in algebraic arithmetic.

An arithmetic may be defined over any arbitrary class of distinct denumerable elements, and conversely any arithmetic determines such a class, giving connections with the algebra of logic. These connections are destroyed if we assume "multiplication" is not commutative.

For the case when multiplication is not a commutative, we replace "division" by "left division" and "right division," with analogous changes for other relations such as "equivalence." We then develop the theory of the greatest common divisor, least common multiple, equivalence with respect to unit factors and so on. It is shown that for an arithmetic, we must assume the units of the system are commutative with all the other elements of the system. Any "arithmetic" may be converted into a group by adjunction, and the theory of congruence and the fundamentals of Kronecker's theory of forms carry over almost unchanged. The complete development will be published shortly in a mathematical journal.

ON THE STRUCTURE OF A PLANE CONTINUOUS CURVE[1]

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Communicated October 11, 1927

If x and y are distinct points of a continuous curve M, the set of all points $[z]$, such that z lies on some arc of M with end-points x and y, is called the *arc-curve* xy and is denoted by $M(x + y)$. This was defined in a previous paper, "Concerning the Arc-Curves and Basic Sets of a Continuous Curve."[2] Among other results this paper contained the following theorems for the case where M lies in a plane:

A.—The arc-curve xy is itself a continuous curve.

B.—If K is a maximal connected subset of $M - M(x + y)$, then K has only one limit point in $M(x + y)$.

C.—If P is a point of a set K such that $K - P$ is the sum of two non-vacuous mutually separated sets K_1 and K_2, and A and B are distinct points of $K_1 + P$, then no point of K_2 is a point of any arc whose end-points are A and B and which lies wholly in K.

It is the purpose of this paper to characterize types of points of a continuous curve and types of continuous curves by a set which is the limit of the arc-curve xy as y approaches x. Throughout the paper the letter M is used to denote a plane continuous curve and all point sets mentioned are considered as subsets of M. The theorems listed above will be referred to as "Theorem A," etc.

THEOREM 1.—*If the point x lies on no simple closed curve of M and α is any arc of M whose end-points are x and any other point z of M, then x is a limit point of the points of M which lie on α and separate x and z in M.*[3]