THE SECOND POSTULATE OF RELATIVITY.

BY RICHARD C. TOLMAN.

IN a recent article by Lewis and Tolman[1] a non-analytical method was developed for obtaining the more important conclusions which can be drawn from the principle of relativity. Our reasoning was based *only* upon the first and second postulates of relativity, and those fundamental conservation laws of mass, energy and momentum which science has never in a single instance been forced to abandon. Since the method of attack avoided any use of involved mathematical analysis, restricting itself to the simplest processes of logical reasoning, and, further, made no use of the assumptions of electromagnetic theory, it may be concluded that the unexpected nature of the results of the theory of relativity is due to something unusual in the two postulates of relativity themselves.

No objections have ever been made to the first postulate of relativity, as stated in its original form by Newton, that it is impossible to measure or detect absolute translatory motion through space. In the development of the theory of relativity, this postulate has been modified to include the impossibility of detecting translatory motion through any ether or medium which might be assumed to pervade space. In support of this principle is the general fact that no "ether drift" has ever been detected, but,

[1] Lewis and Tolman, Proc. Amer. Acad., 44, 711–724, 1909; Phil. Mag., 18, 510–23, 1909. A novel method of proof was adopted in this article which consisted in the consideration of certain experiments which might be performed by two observers situated on similar systems which are in relative motion. The reasoning was based on the supposition that the results obtained in such experiments would not contradict either of the two postulates of relativity nor the conservation laws of mass, energy and momentum. These supposed experiments are analogous to the cyclical processes used in thermodynamic proofs, and bear the same relation to the analytical method used by Einstein in his treatment of the theory of relativity, as the "cyclical process" method bears to the more elegant considerations of the analytically inclined thermodynamist.

especially, the conclusive experiments of Michelson and Morley, and Trouton and Noble, in which, a motion through the ether, of the earth in its path around the sun would certainly have been detected. For the purposes of this article we shall consider that the first postulate of relativity needs no further proof.

It is Einstein (to whom, indeed we owe the development of relativity along its present broad lines) who first stated the second postulate of relativity in a general form, namely, that the velocity of light in free space appears the same to all observers, regardless of the relative motion of the source of light and the observer. This is the assumption which has forced the theory of relativity to its

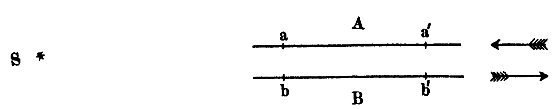

Fig. 1.

strange conclusions, and it is for its further consideration that this paper is designed.

A simple example will make the extraordinary nature of the second postulate evident.

S is a source of light and A and B two moving systems. A is moving towards the source S, and B away from it. Observers on the systems mark off equal distances aa' and bb' along the path of the light and determine the time taken for light to pass from a to a' and b to b' respectively. Contrary to what seem the simple conclusions of common sense, the second postulate requires that the time taken for the light to pass from a to a' shall measure the same as the time for the light to go from b to b'. Such a consideration makes the path obvious by which the theory of relativity has been led to strange conclusions as to the units of length and time in a moving system.

The second postulate of relativity is obtained by a combination of the first postulate with a principle which has long been familiar in the theory of light. This principle states that the velocity of light is unaffected by a motion of the emitting source, in other words, that the velocity with which light travels past any observer is not increased by a motion of the source of light towards the observer. The first postulate of relativity adds the idea that a motion of the source of light towards the observer is identical with a motion of the observer towards the source. The second postulate of relativity is seen to be merely the combination of these two principles, since it states that the velocity of light in free space appears the same to all observers regardless *both* of the motion of the source of light and of the observer.[1] Since the first postulate of relativity has already been considered as sufficiently proved, we shall proceed at once to present certain evidence in favor of the assumption that the velocity of light is independent of the motion of the source. In the latter part of the paper, we shall also consider an entirely independent proof of the second postulate based on the Kaufmann-Bucherer experiment.

The principle that the velocity of light is independent of the velocity of its source has hitherto lacked experimental justification. It was obtained, however, as a direct consequence of the ether theory of light, which makes the velocity depend only upon the properties (*i. e.*, elasticity or electrical nature) of a stationary transmitting medium, and therefore, as with sound or other wave motions, independent of the velocity of the source. Until within a few years the ether theory had been so extraordinarily successful in explaining even the most complicated phenomena of optics, that we should have accepted any of its experimentally unproved conclusions without hesitation. At the present time, however, since the experiments of Michelson and Morley and of Trouton and Noble stand in such direct contradiction to the predictions of the ether theory, we have no hesitation in considering any other assumption as to the velocity of light, which, although not in accord with the ether theory, would free us from the complications introduced by the theory of relativity.

Such an alternative assumption, as to the velocity of light, which

[1] The first postulate of relativity practically denies the existence of any stationary ether through which the earth for instance might be moving. On the other hand, the principle that the velocity of light is unaffected by a motion of the source is closely bound up with the idea that light is transmitted by a stationary ether which does not partake in the motion of the source. It is not surprising that the combination of two principles based on seemingly contradictory ideas should give to the second postulate its extraordinary content.

would cause none of the complications introduced by the second postulate, *is* possible. The velocity of light and other electromagnetic propagations might not be independent of the motion of the source, but their velocity and that of the source might be additive. This assumption would be very simple, would be no contradiction to the first postulate of relativity, and would directly explain all our failures to detect an ether drift. It is not difficult, for example, to see that this assumption does directly explain the Michelson-Morley experiment. If O is a source of light and A and B are mirrors placed a meter away from O, the Michelson-Morley experiment shows that the time taken for light to travel to A and back is the same as for the light to travel to B and back, in spite of the fact that the whole apparatus is moving through space in the direction of $O-B$, due to the earth's motion around the sun.

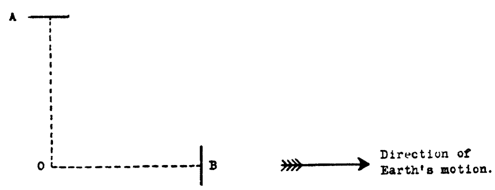

Fig. 2.

The above assumption, however, would require exactly this result, since it says that light travels out from O with a constant velocity in all directions with respect to O, and not with respect to some ether through which O is supposed to be moving. It is in fact obvious, in general, that this principal if true would lead to the simplest kind of relativity. For, if light or any electromagnetic disturbance which is being emitted from a source, partakes in the motion of that source in such a way that the velocity of the source is added to the velocity of emission, then a system consisting of the source and its surrounding disturbances acts as a whole and suffers no change in configuration when the velocity of the source is changed.[1] The possibility of such an assumption has already been pointed out in various places.[2] Nevertheless, in spite of the ap-

[1] There would, of course, be a temporary change in configuration during acceleration.

[2] Lewis and Tolman, loc. cit. Comstock, Phys. Rev., 30, 267, 1910. Tolman, Phys. Rev., 30, 291, 1910. The assumption has also been adopted in a modified form

parent relief which would follow the adoption of this idea, the evidence which we are about to present is all in favor of the older idea of the velocity of light which has led to the second postulate and the complicated theory of relativity.

The Doppler Effect.

The Doppler effect, that is the influence which a motion of the source of light has upon the *frequency* of the emitted ray, has been the subject of much refined experimental work. It is natural to consider what relation this effect bears to our own problem, concerning the influence which a motion of the source of light has upon the *velocity* of the emitted ray.

If we have a body which is emitting the periodic disturbance which we call light, and then start the body in motion towards an observer, it is evident that the frequency with which the disturbances will reach the observer will be increased, since, in order to get to him, each successive disturbance has to travel a less distance than the one preceding it. As we shall see, this change, in the frequency of the light, will be produced in nearly the same amount, whether or not the velocity with which the light travels towards the observer is affected by the motion of the source. We shall find, however, that, with regard to the actual distance in space between successive disturbances (*i. e.*, wave-length) the two hypotheses as to the velocity of light lead to quite different conclusions. This difference presents a method of deciding between the two rival hypotheses.

We shall proceed to an actual derivation of the effect which a motion of the emitting source has upon the wave-length of light, first, assuming that the velocity of the light is independent of the motion of the source, and then that the velocities of the light and source are additive.

It is evident that we must make no use of arguments based on preconceived notions as to the nature of light, but must restrict ourselves to purely kinematic considerations which would be equally

by Campbell who considers that light is a transverse vibration in the Faraday tubes attached to a vibrating electron, and since the tubes partake in the motion of the electron, the velocity of light is evidently dependent on the velocity of the source. (See Modern Electrical Theory, University Press, Cambridge, 1907.)

true of any periodic disturbance. Let c be the velocity with which the disturbance travels, in this case that of light, λ_0 and n_0 the observed wave-length and frequency (of some particular line in the spectrum), when the source is at rest with respect to the observer, and λ and n the same quantities after the source has been set in motion towards the observer with the velocity v. Let us first consider—

Case I.—The velocity of light is independent of the velocity of the source. Then,

$$\lambda = \lambda_0 - \frac{v}{c}\lambda_0 = \lambda_0\left(1 - \frac{v}{c}\right).$$

This is the ordinary formula for the Doppler effect and the derivation is simple, since it is evident that while an emitted wave-front is moving forward the distance λ_0, the source itself has moved forward the distance $\lambda_0 \cdot v/c$, and the next wave-front to leave the source will have gained this distance over the earlier one. The frequency of the light will be equal to the velocity divided by the wave-length.

$$n = \frac{c}{\lambda} = \frac{c}{\lambda_0}\left(\frac{c}{c - v}\right) = n_0 \frac{c}{c - v}$$

$$= n_0\left(1 + \frac{v}{c} + \frac{v^2}{c^2} + \cdots\right).$$

Case II.—The velocity of light and that of the source are additive, the velocity with which light passes the observer is $v + c$.

$$\lambda = \lambda_0.$$

This result is evident on inspection, since, under the conditions assumed, the velocity of light relative to the source is always the same, and the source and its surrounding disturbances move together as a whole, suffering no permanent change in configuration when the velocity of the source is changed. In detail, however, we see that, with respect to the observer, an emitted wave front moves forward the distance $\lambda_0 \cdot (c + v)/c$ during the interval of time which elapses before the next wave front leaves the source, and during that time the source has moved forward the distance

$$\frac{v}{c + v}\lambda_0 \frac{c + v}{c} = \lambda_0 \frac{v}{c},$$

making

$$\lambda = \lambda_0 \frac{c + v}{c} - \lambda_0 \frac{v}{c} = \lambda_0 .$$

The frequency of the light is evidently

$$\frac{c + v}{\lambda} = \frac{c + v}{\lambda_0} = \frac{c}{\lambda_0}\left(\frac{c + v}{c}\right) = n_0 \left(1 + \frac{v}{c}\right),$$

a result which except for second order terms is identical with that obtained for Case I.

To sum the matter up, when the source is set in motion, whichever of the two hypotheses as to the velocity of light is true, the frequency of the light will be changed by practically the same amount, but the wave-length will be changed if one of the hypotheses is true and entirely unaffected if the other is true. Light emitted from moving sources, whether they are astronomical bodies or the moving mirrors arranged by Belopolsky, unquestionably does show the Doppler effect. We must investigate whether both the frequency and the wave-length are changed by the motion of the source.

The determination of the Doppler effect is made by a measurement of the displacement of some particular line from its normal position in the spectrum. When the spectrum is produced by a prism it is difficult to say whether the position of a given line would depend upon its frequency or its wave-length. Spectroscopic measurements made with the help of a grating are, however, actual determinations of wave-length. Since such measurements do show a change in wave-length of the light from many of the stars, and especially in light coming from the approaching and receding limbs of the sun, where the velocity of rotation is known from observations on the sun spots, we have, at first sight, strong evidence in favor of our first hypothesis that the velocity of light is independent of the source.

We must notice in these experiments, however, that the measurements of wave-length are made only after the light has been reflected from the surface of the grating and consider the possibility that this reflecting surface would act as a new source, giving to the reflected beam the ordinary velocity of light as from any stationary source. If the surface of the grating should act in this way, then the reflected light would have the same velocity whichever of our hypotheses were true. Moreover, except for second-order terms, both hypotheses have led to the same conclusions as to the frequency of the light,[1] and since the wave-length of light is completely determined by its frequency and velocity, such an action of a reflecting surface would prevent our distinguishing between the two hypotheses. We can, however, draw one useful conclusion from our consideration, namely, that if the velocity of light does depend on the velocity of the source, then a reflecting mirror acts as a new source, and the velocity of the reflected beam depends only on the motion of the mirror.

Before proceeding to the consideration of an experiment in which we shall make use of the principle just derived, let us consider whether a transmission grating would also act as a new source of light and destroy any original difference between the velocities of light, for example, from the two limbs of the sun. In the next section we shall consider more in detail certain experiments of Fizeau and of Michelson concerning the velocity of light in media which are moving with respect to the source of light. We are, however, led by them to the conclusion that an original difference in the velocity of light from the two limbs of the sun would be only about one half obliterated by passage through a plate of glass, and hence the use of transmission gratings for deciding between the two postulates as to the velocity of light would be possible. At the present time very excellent transmission gratings are obtainable, being replicas of original Rowland gratings. It is desirable that observations be made with apparatus in which the light suffers no reflection in order to definitely settle the matter.

Experiments on the Velocity of Light from the Two Limbs of the Sun.

The fact that a mirror acts as a new source of light led me to the construction of a very simple apparatus for comparing the velocity of light from the approaching and receding limbs of the

[1] The frequency, would, of course, not be changed by reflection.

sun. A tube mounted on a small telescope stand was provided with a slit, an observing eye piece, and a mirror (plate glass) as shown in the diagram. The apparatus was adjusted to produce interference fringes between the light from the slit and from its mirror image, a method employed by Dr. Lloyd. The mirror was about 25 cm. in length and the distance from the end of the mirror to the eye piece was about 10 cm.

Let us suppose an interference fringe at the point *f* produced by the combination of a ray of light *sf*, coming direct from the slit and another *smf* which has suffered reflection from the mirror. If now

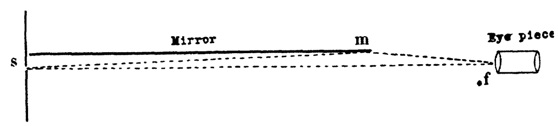

Fig. 3.

the velocity of light *were* dependent on the velocity of its source, and we set the source in motion towards the slit, the time taken for a given wave-front to travel from the slit to the eye piece along the path *sf* would be increased by a greater amount than along the path *smf*, since for the distance *mf*, which in the apparatus was at least 10 cm., the ray is traveling with the normal velocity of light as from a stationary source, while for the whole path *sf* the light is traveling with an increased velocity. We should thus expect a shift in the fringes to accompany a change in the velocity of the source.

A lens of about 9 cm. aperture and 80 cm. focus was used to throw an image of the sun (about 8 mm. in diameter) on the apparatus, and light first from one and then from the other limb of the sun allowed to enter the slit. No shift in the fringes was observed. We may easily calculate the magnitude of the expected effect. Suppose we are receiving light from the approaching limb of the sun, the velocity of the ray *sf* would be greater than that of the reflected ray by about 1.5 km. per second,[1] so that a given wave-front in traveling the ten centimeters from *m* to *f* would fall behind the corresponding one which travels along *sf* by

[1] The peripheral velocity of the sun due to its rotation is a little under 2 km. per second.

$$\frac{1.5}{300,000} \times 10 = .5 \times 10^{-4}\,\text{cm}.$$

We obtain double the effect, or 10^{-4} cm., in changing from the approaching to the receding limb of the sun, and considering 5×10^{-5} as an average wave-length, we should expect a shift of about two fringes. Since no shift was observed, we have strong evidence that the velocity of light is independent of the velocity of its source.

To complete the discussion, it is necessary to consider the effect of the lens (and the earth's atmosphere) through which the light has to pass before reaching the slit. At first sight, it might seem as if an original difference between the velocities of light from the two sources would be obliterated by passage through a stationary medium. The experiments of Fizeau and of Michelson, however, give us data for calculating the effect upon the velocity of light of relative motion between the source of light and a transmitting medium.

If c is the velocity of light in vacuo, μ the index of refraction of the medium and v the velocity with which the medium is moving towards the source, we have the velocity of light in the medium equal to $c/\mu \mp v\theta$, where θ is some fraction which must be determined experimentally. The considerations of Fresnel and others have led to the expression for any medium $\theta = (u^2 - 1)/\mu^2$, an equation which was very closely verified for water in the experiments referred to. The velocity of light in the lens may now be calculated.

$c/\mu \mp v\theta$ is the velocity of light with respect to the source; with respect to the medium, it will be $c/\mu \pm v(1 - \theta)$. For glass, putting $\mu = 1.5$, we have $\theta = (\mu^2 - 1)/\mu^2 = 0.51$. The velocity of light from the approaching limb will be $c/\mu + 0.49v$ and from the receding limb $c/\mu - 0.49v$, where v may be taken as 1.5 km. per second. We now see that, even if the light after leaving the lens did not regain its original velocity, the difference in the velocities of the light from the two limbs of the sun would still be about 1.5 km. per second, which would give a shift of one fringe in the experiment performed.

As a result of the experiment, we conclude that the velocity of light from the two limbs of the sun is the same. The possibility that an original difference in velocity would be destroyed when the light reached the neighborhood of the earth is not entirely excluded. Nevertheless, the experiments of Fizeau and Michelson which we have just discussed seem to show that the presence of air or other transmitting medium *would not* completely destroy such a difference. Furthermore, the experiment of Sir Oliver Lodge has led us to expect no change in the velocity of light produced by the neighborhood of large masses such as the earth. It may also be pointed out in this connection, that, up to the present time, no astronomical data of any kind have been found which are in disagreement with the principle that the velocity of light is independent of that of the source. For example, it has been shown by Comstock[1] that a difference in the velocity of light from approaching and receding stars would lead us to expect irregularities in the observed orbital motion of double stars which have never been detected. Certainly, until further evidence is presented, we may best accept that principle regarding the velocity of light which has led to the second postulate of relativity.

We shall now consider an entirely different method of proving the second postulate of relativity.

The Kaufmann-Bucherer Experiment.

Certain conclusions of the theory of relativity have been quantitatively verified by the experiments of Kaufmann and of Bucherer on the mass of the β particle. It has already been stated in the article of Lewis and Tolman referred to above that this experimental fact may itself be used for the reverse process of deducing the second postulate, and a method of proof was worked out at the time that paper was published. It is very desirable to consider this proof since it includes *a deduction, without the help of the second postulate, of all the changes in the units of length and time*, to which the theory of relativity has led, and finally gives a proof of the second postulate itself.

Let us suppose an electron ε at rest and an electron ε' moving past it with the velocity v.

[1] Comstock, loc. cit.

Let ε' be originally so far away that the electrons do not appreciably repel each other, and let the experiment continue until they are again far apart. Owing to their mutual repulsion, while they are within each other's sphere of influence, electron ε receives a certain component velocity μ in a direction perpendicular to the general line of motion of the two systems, and ε' also receives a transverse velocity μ'. These transverse velocities are made small compared with the relative velocity of the two systems. If the mass of an electron at rest is m, then by the Bucherer experiment, the electron ε', which we consider in motion, will have the mass $m/\sqrt{1-\beta^2}$, where β is v/c, so that the electrons have respectively received the transverse momenta mu and $mu' \cdot /\sqrt{1-\beta^2}$.

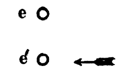

Fig. 4.

By the principle of the conservation of momenta

$$mu = m\frac{1}{\sqrt{1-\beta^2}}u'$$

and

$$u' = \sqrt{1-\beta^2}u.$$

In other words, the electron ε', since it has a larger mass than electron ε does not receive so great a transverse velocity. If, however, an observer had been traveling along with ε', he would have been entirely unable to detect this fact that his electron had received a smaller velocity than the other one, since the first postulate of relativity states that no measurements are possible by which an observer may detect that he is in absolute motion. Since, now, to this moving observer, the velocity seems larger than it does to an observer at rest in the ratio $1 : \sqrt{1-\beta^2}$, the second which the moving observer uses must be longer than a "stationary" second in the same ratio $1 : \sqrt{1-\beta^2}$.[1]

Having obtained the ratio between the units of time in a moving and stationary system, let us deduce the Lorentz shortening. Suppose two systems a and b moving past each other with the velocity v and two observers A and B on the systems. A makes two marks

[1] It is evident that the difference of opinion of the two observers as to the transverse velocity of the electron could not be reconciled by assuming a difference in the transverse units of length, since it is perfectly possible for the observers to make a direct comparison of meter sticks held perpendicular to the line of motion of the systems. (See Lewis and Tolman, loc. cit.)

on his system a centimeter apart, in the line of motion of the system, and requests B to determine the time it takes for a point on b to pass from one mark to the other. B also makes two marks on his system a centimeter apart, and A finds that it takes the same number of seconds to pass from one mark to the other as B found in his own similar experiment. Any other outcome of the trial would be contradictory to the first postulate of relativity. If, however, we again arbitrarily consider A to be at rest, A's seconds are shorter than B's in the ratio $\sqrt{1-\beta^2}:1$, and hence the two points on B's system must have been nearer together than those on A's in this same ratio $\sqrt{1-\beta^2}:1$. In other words, the "moving" centimeter in the longitudinal direction is shorter than a "stationary" one in the ratio $\sqrt{1-\beta^2}:1$.

We have now derived the change in the units of length and time in a moving system, with the help of the Bucherer experiment. Before we can derive the desired principle regarding the velocity of light, we must go one step further and find out how the clocks are set in a moving system.

The observer B who is in motion with the velocity v past a system a which we consider at rest, lays off a length of one centimeter on his system in a longitudinal direction, and with the help of two clocks, one at each end of the centimeter, notes the time taken for a point on a to pass from one end of this centimeter to the other. He obtains, of course, the time $1/v$. Since, however, his seconds are longer and his centimeters shorter than stationary ones in the ratio $1:\sqrt{1-\beta^2}$ we would have expected him to obtain the time $(1-\beta^2)\cdot 1/v$, and we can account for his obtaining the longer time $1/v$, only by the assumption, that, in a moving system, a clock 1 cm. to the rear of another is set ahead by the amount

$$\frac{1}{v}-\frac{1}{v}(1-\beta^2)=\frac{v}{c^2}\text{ seconds.}$$

We are now ready to deduce our principle as to the velocity of light. Consider a source of light and an observer B who measures the velocity of the light coming from this source. If the observer B is at rest and marks off a length of one centimeter in the path of light, he finds, of course, that the light takes the time $1/c$ to pass from one mark to the other. We wish to prove, however, that the relation which we have just derived, requires that the observer will also obtain the time $1/c$ even if he is in motion towards or away from the source.

Let us suppose that the observer B is moving towards the light with the velocity v, the velocity with which light is passing his system is $v + c$, the time taken for it to travel over a centimeter length is $1/(v + c)$. Since, however, the centimeters which B marks off are shorter than "stationary" ones in the ratio $\sqrt{1 - \beta^2} : 1$, and the seconds which he uses are longer in the inverse ratio, the time required in his units would be $(1 - \beta^2) \cdot 1/(v + c)$, and further since the rearmost clock is set ahead by the amount v/c^2, we finally conclude that B will obtain the time $(1 - \beta^2) \cdot 1/(v + c) + v/c^2$ which reduces to $1/c$. We thus conclude that the velocity of light appears the same whatever the motion of the observer, or by the first postulate of relativity whatever the relative motion of the source of light and the observer, and have obtained a proof of the second postulate of relativity with the help of the Bucherer experiment.

In this connection, it must be again pointed out that the Kaufmann-Bucherer experiment may not really indicate an increase in the mass of an electron in motion. It is, at first sight, equally possible that the forces acting on an electron in rapid motion through electrostatic or magnetic fields are not as large as those calculated on the basis of Maxwell's fifth equation, since its application to high velocities certainly lacks experimental justification. The balance of all the evidence which has been presented, however, is in favor of the second postulate of relativity.

Summary.

In this paper it is shown that the extraordinary conclusions of the theory of relativity are forced on it by the second postulate of relativity. This postulate is obtained by combining the first postulate of relativity with the principle that the velocity of light is independent of the velocity of the source. The alternative hypothesis that the velocity of light and the velocity of its source are additive would lead to none of the complications of the theory of relativity. Two methods are presented for deciding between the two hypotheses.

The first method would be to measure the wave-length of light from some moving source, since it is shown that the first hypothesis would lead us to expect a change in the wave-length of light from moving sources and the alternative hypothesis would not. The existing measurements of the wave-length of light from moving astronomical sources, made with reflection gratings, are not of such a nature that we can definitely decide that the velocity of light is independent of the motion of the source. We can, however, state that, if the alternative hypothesis is true, then a reflecting surface acts as a new source of light and light coming from such a surface has the normal velocity as from any other stationary source. It is further shown that measurements with a transmission grating would more nearly allow a definite decision of the question.

The principle that the velocity of light from a stationary mirror is the same as for light from any stationary source leads to a second method of deciding between the two hypotheses as to the velocity of light. An apparatus is described in which interference fringes were produced between light which had come direct from the sun and light which had first suffered reflection. Since the velocity of the reflected ray would in any case be unaffected by the motion of the original source, we should expect a shift in the position of the fringes to accompany a change in the velocity of the direct ray. No shift in the fringes was observed in examining light first from the approaching and then from the receding limb of the sun. We conclude that the velocity of light from the two limbs of the sun is the same, which confirms the principle that has led to the second postulate of relativity.

An entirely different method of proving the second postulate of relativity is afforded by the results of the Kaufmann-Bucherer experiment. In fact, by a combination of the first postulate of relativity with the principle that the mass of a moving electron is greater than that of a stationary one in the ratio $1 : \sqrt{1 - \beta^2}$, it was found possible to deduce *all the conclusions of the theory of relativity as to the units of length and time in a moving system*, and finally to deduce the second postulate of relativity itself.

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