# Phase-Shift Analysis of T(n,n)T†

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A phase-shift analysis of the scattering and polarization data for the elastic scattering of neutrons from tritium has been made for neutron energies between 1.0 and 6.0 MeV. Although there are insufficient experimental data to allow a unique determination of the phase shifts, a satisfactory solution has been obtained by using as a starting point the phase shifts previously obtained for $He^3(p,p)He^3$. The results are similar for these two reactions and a single-level analysis of the phase shifts allows the level parameters of states in $H^4$ and $Li^4$ to be compared.

## I. INTRODUCTION

SUFFICIENT evidence exists that $H^4$ possesses neither particle-stable nor narrow low-lying resonances with isotopic spin $T=1$.[1] (The existence of $T=2$ states remains undecided, though no compelling proof of their presence has appeared.[2]) This lack of sharp level structure is confirmed by the corresponding absence of bound states or narrow resonances in $Li^4$, which has been studied by the reactions $He^3(d,n)Li^4$ (Ref. 3), $He^3(He^3,d)Li^4$ (Ref. 4), and $Li^6(p,t)Li^4$ (Ref. 5). However, that $Li^4$ has a spectrum of broad, unresolved $P$-wave resonances has been indicated by a phase-shift analysis of the available scattering and polarization data for $He^3(p,p)He^3$ (Ref. 6). In this case, exploitation of the behavior of the phase shifts at low energy together with the excellent polarization data available at higher energies has allowed a unique solution of the phase shifts to be found. In this report, a similar analysis will be described for the reaction T(n,n)T, for which there are at present insufficient data to allow such a unique determination of the phase shifts. However, since $H^4$ and $Li^4$ are mirror nuclei, their level spectra should be sufficiently similar that the $He^3(p,p)He^3$ scattering parameters could be used as starting values in the present analysis.

The experimental data for T(n,n)T consist of angular distributions (11 angles each) at neutron energies of 1.0, 2.0, 3.5, and 6.0 MeV.[7] These measurements are consistent with the total cross sections that had previously been obtained.[8] In addition, the polarization of the scattered neutrons was measured for three angles at the 1.0 MeV point.[7] All data are in excellent agreement with a resonating-group calculation by Bransden, Robertson, and Swan,[9] in which no splittlng of the $P$-wave phase shifts due to spin-orbit effects was considered.

## II. PHASE SHIFTS AND SINGLE-LEVEL ANALYSES

Phase shifts were obtained using a gradient-search technique that has been described in an earlier paper.[6] The errors assigned to these phase shifts are necessarily quite large, since the angular distributions are not particularly sensitive to the differences between the various $P$-wave phase shifts. The resulting values are given in Table I and are shown in Figs. 1 and 2. The comparison of the calculated and experimental angular distributions is indicated in Fig. 3.

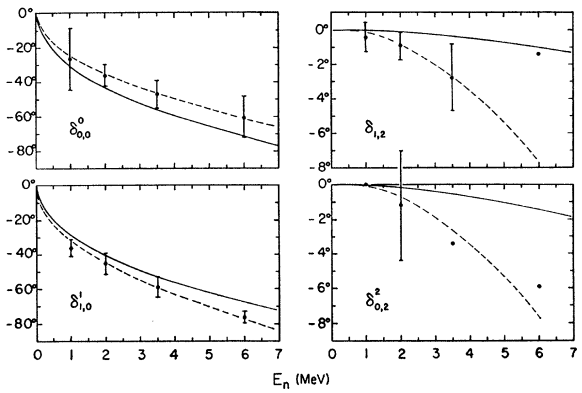


FIG. 1. The $S$- and $D$-wave phase shifts for T(n,n)T. The solid lines in all cases refer to the resonating-group prediction. The dashed lines are for hard-sphere scattering: for the $S$-wave singlet phase shift ($\delta_{0,0^0}$) corresponding to a radius of 2.63 F; for the $S$-wave triplet phase shift ($\delta_{1,0^1}$) corresponding to a radius of 3.33 F; and for both $D$-wave phase shifts corresponding to a radius of 4.0 F. Where error bars are not indicated, one may assume that the value of the parameter is quite uncertain.

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TABLE I. The phase shifts for T($n$,$n$)T.[a]

| $E_n$ (MeV) | $\delta_{0,0}{}^0$ (deg) | $\delta_{1,0}{}^1$ (deg) | $\delta_{0,1}{}^1$ (deg) | $\delta_{1,1}{}^0$ (deg) | $\delta_{1,1}{}^1$ (deg) | $\delta_{1,1}{}^2$ (deg) | $\delta_{0,2}{}^2$ (deg) | $\delta_{1,2}$ (deg) | $\epsilon$ (deg) | $\mathfrak{M}_\sigma$ | $\mathfrak{M}_P$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | −26.6 | −36.2 | 5.7 | 3.0 | 11.1 | 8.1 | 0.1 | −0.4 | 0.5 | 0.70 | 1.01 |
| 2 | −36.1 | −45.5 | 17.6 | 4.8 | 16.2 | 30.6 | −1.2 | −0.9 | 0.0 | 0.70 | ··· |
| 3.5 | −47.0 | −59.2 | 25.0 | 7.0 | 29.0 | 53.2 | −3.4 | −2.8 | 0.0 | 0.44 | ··· |
| 6 | −60.7 | −76.2 | 24.9 | 14.3 | 48.9 | 65.1 | −5.9 | −1.4 | 0.0 | 0.20 | ··· |

[a] The phase shifts $\delta_{S,L}{}^J$ are labeled by the channel spin $S$, the orbital angular momentum $L$, and the total angular momentum $J$. The quantity $\mathfrak{M}_\sigma$ is equal to

$$\frac{1}{N_\sigma}\sum_{i=1}^{N_\sigma}\left\{\frac{\sigma_{\text{expt}}(i)-\sigma_{\text{calc}}(i)}{\Delta\sigma_{\text{expt}}(i)}\right\}^2,$$

where $N_\sigma$ refers to the number of angles at which the cross section $\sigma_{\text{expt}}$ has been measured. $\sigma_{\text{calc}}$ is the cross section calculated from the phase shifts listed, and $\Delta\sigma_{\text{expt}}$ is the rms uncertainty of the experimental measurement. The quantity $\mathfrak{M}_P$ is defined in an analogous way for the neutron polarization data.

These phase shifts vary with the neutron energy in a way similar to that obtained for the phase shifts from $He^3(p,p)He^3$—as one would expect. The $S$-wave phase shifts are in fair agreement with the resonating-group calculations (solid lines). If these $S$-wave phase shifts were described in terms of hard-sphere scattering, then the singlet phase shift $\delta_{0,0}{}^0$ would correspond to a radius of 2.63 F while the triplet phase shift $\delta_{1,0}{}^1$ would correspond to a radius of 3.33 F. Considering the magnitude of the errors, these are in fair agreement with the values of 3.05 and 3.15 F obtained for $He^3(p,p)He^3$.

The $D$-wave phase shifts behave much like the corresponding phase shifts in the mirror reaction, but since the errors are so large no detailed comparison seems justified. As in the previous analysis the $D$-wave triplet phase shifts were assumed to be unsplit.

The channel-spin-mixing parameter $\epsilon$, which connects $J^\pi=1^-$ states for the singlet and triplet channel spins, cannot be easily determined in the absence of extensive polarization data, and its value is subject to considerable uncertainty.

The $P$-wave phase shifts are all positive and show evidence for the presence of broad resonances. Those phase shifts for which the errors were small enough were parameterized in the same way as in the $He^3(p,p)He^3$ analysis using a Breit-Wigner single-level formalism with an interaction radius of 4.0 F:

$$\delta_{S,L}{}^J=-\varphi_L+\tan^{-1}\left\{\frac{(ka/A_L{}^2)\gamma_{\lambda,L}{}^2}{E_\lambda-\gamma_{\lambda,L}{}^2(g_L+L)-E}\right\},$$

where $\gamma_{\lambda,L}{}^2$ is the reduced width of the level, $a$ is the interaction radius, $k$ is the wave number, $E_\lambda$ is the characteristic energy of the level, and $E$ is the center-of-mass energy. The other functions appearing can be defined in terms of the regular and irregular solutions, $F_L$ and $G_L$, of the one-dimensional Schrödinger equation:

$$\varphi_L=\tan^{-1}(F_L/G_L)_{r=a},$$
$$A_L=[(F_L{}^2+G_L{}^2)^{1/2}]_{r=a},$$

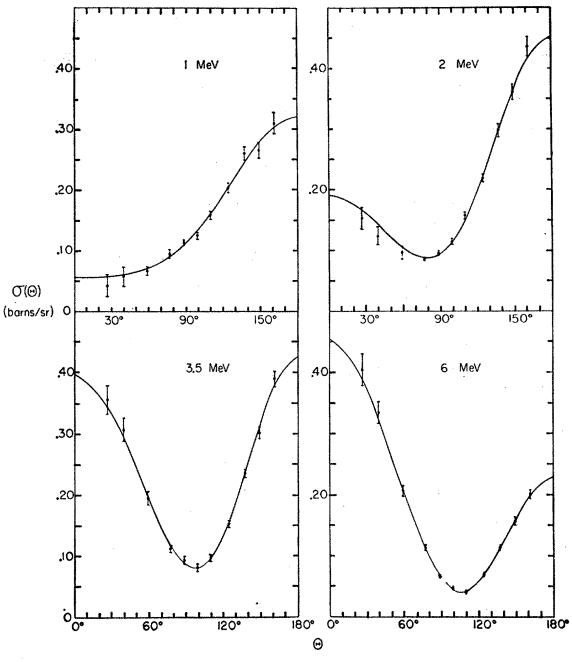


FIG. 3. The angular distributions for T($n$,$n$)T plotted versus the center-of-mass scattering angle. The points and their errors were taken from Ref. 7; the solid lines were calculated using the phase shifts in Table I.

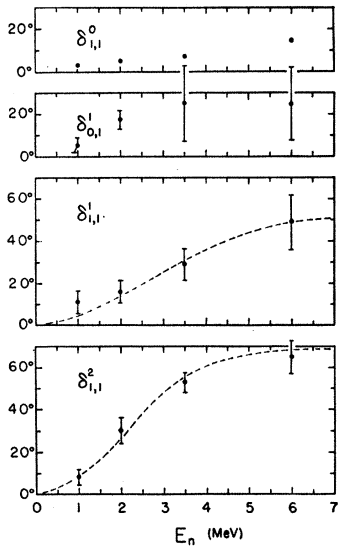


FIG. 2. The $P$-wave phase shifts for T($n$,$n$)T. The dashed lines correspond to the resonance parameters given in Table II. ($H^4$ data.)

TABLE II. Resonance parameters for the lowest states of $H^4$ and $Li^4$ corresponding to an interaction radius of 4.0 F. The quantity $E_x$ refers to the position of a level relative to the ground state of $He^4$ and has been corrected for Coulomb and mass differences.

| | $H^4$ | | | | $Li^4$ | | | |
|---|---|---|---|---|---|---|---|---|
| $J^\pi$ | $E_\lambda$ (MeV) | $\gamma_{\lambda,1}^2$ (MeV) | $E_R$ (MeV) | $E_x$ (MeV) | $E_\lambda$ (MeV) | $\gamma_{\lambda,1}^2$ (MeV) | $E_R$ (MeV) | $E_x$ (MeV) |
| $2^-$ | 7.0 | 5.5 | 3.4 | 23.7 | 8.4 | 5.5 | 4.7 | 24.3 |
| $1^-$ | 9.3 | 5.5 | 5.1 | 25.4 | 10.2 | 5.5 | 6.2 | 25.8 |

and

$$g_L=\left[\frac{d(\ln A_L)}{d(\ln kr)}\right]_{r=a}.$$

The resonance energy $E_R$ is defined as that center-of-mass energy where $E_\lambda-\gamma_{\lambda,L}^2(g_L+L)-E$ is equal to zero. These are exactly the same conventions as those used in the analysis of $He^3(p,p)He^3$.

The dashed lines shown in Fig. 2 correspond to the single-level fits using the parameters given in Table II ($H^4$ data). The corresponding values for the $He^3(p,p)He^3$ resonances are given in Table II ($Li^4$ data). The two resonances that can be compared have the same reduced widths in the two reactions. Taking into account Coulomb corrections and the neutron-proton mass difference the positions of the lowest levels in $H^4$ and $Li^4$ are compared by referring them to the corresponding excitation energies in $He^4$, which are given under $E_x$ in Table II. The magnitude of the Coulomb correction was obtained from the positions of the ground states of $Li^5$ and $He^5$, which correspond to a similar nuclear configuration, i.e., that of a $P$-wave nucleon outside the $1S$ core. The Coulomb correction for $Li^4$ relative to $He^4$ was taken equal to that for $Li^5$ relative to $He^5$, because the change in radius is not appreciable. The correction for $H^4$ relative to $He^4$ was taken to be half that for $He^4$ relative to $Li^4$—a consequence of the $Z(Z-1)$ factor in the Coulomb energy.

## III. CONCLUSIONS

The phase-shift analysis of $T(n,n)T$ is in substantial agreement with that from $He^3(p,p)He^3$ and indicates that the lowest $T=1$ levels of $H^4$ are broad $P$-wave triplet resonances that have assignments of $2^-$ and $1^-$. The lack of polarization data has prevented a more precise examination of this reaction; in particular the large errors on the phase shifts preclude any statement concerning the presence of a $P$-wave triplet $0^-$ state or the $1^-$ $P$-wave singlet state. The neutron polarization has been calculated from the derived phase shifts at each energy (Fig. 4) with the hope that additional measurements will be undertaken in this energy range.

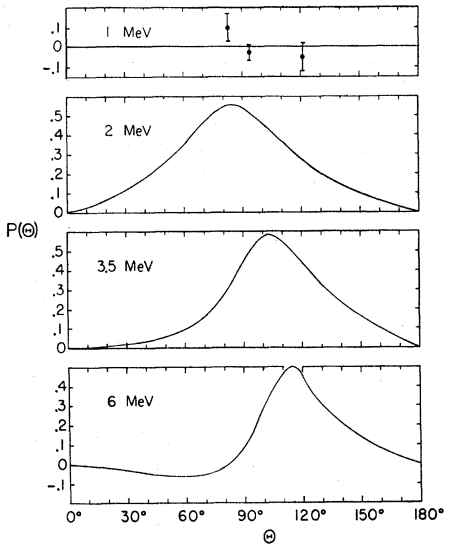


FIG. 4. The polarization of neutrons scattered from tritium plotted versus the center-of-mass scattering angle. The solid curves were calculated from the phase shifts in Table I; the data points are from Ref. 7. (The Basel convention on the sign of the polarization has been used.)

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