RF LOSSES IN HELICAL RESONATORS*

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Introduction

Recently, increased interest has been shown in the use of helically loaded wave guides for the acceleration of heavy ions.[1] These structures seem especially suited to this purpose in that they allow low phase velocities for relatively small transverse dimensions.

The behavior of such wave guides has been discussed in various approximations in the literature.[2] In a simplified sense, the propagation in the wave guide is now coupled to the propagation of a disturbance down the helix. Winding the helix tighter allows lower phase velocities to be obtained. However, in a helix wound with round tubing as the tubing diameter approaches the pitch, the shunt impedance decreases due to increased losses in the structure. These stem mainly from the prevention of radial magnetic flux penetration in the vicinity of the helix, causing an increase in Joule-heating. The shunt impedance may be improved by decreasing the tube diameter, but this consequently decreases the interior area of the helix available for cooling.

Various suggestions have been put forth to decrease the losses without detriment to the cooling capacity. The most profitable method at present seems to be deforming the helix tubing by flattening it in the axial direction, giving it an oblong, or "racetrack" cross section.[3] It is the purpose of this report to investigate the behavior of a $\lambda/2$-wave structure under such deformation, using a modified sheath helix analysis; and to discuss other methods of improving the characteristics of such wave guides.

Calculation of Fields

A method for determining the fields in a $\lambda/2$ helically loaded resonator has been presented in an earlier paper.[4] This method has been modified for the present purpose, in order to obtain a better fit to the magnetic fields.

Cylindrical coordinates are used, with the z-axis along the axis of the wave guide, and the usual definition of r and ϕ. The end plates of the cavity are at 0 and L', with the helix of length $L \le L'$ placed symmetrically inside. The side wall of the structure is at $r = c$: the helix at $r = a < c$ with a pitch $= s$. We assume a solution of the form:

$$E_z = \sum_{n=1}^{\infty} R_n(g_n r) \cos\left(\frac{n\pi}{L}\left(z - \frac{(L' - L)}{2}\right)\right),$$

$$g_n = \sqrt{\frac{n^2\pi^2}{L^2} - k^2} \quad ; \tag{1}$$

$$B_z = \sum_{n=1}^{\infty} S_n(g'_n r) \sin\left(\frac{n\pi}{L'}\right)z \quad ,$$

$$g_n' = \sqrt{\frac{n^2\pi^2}{L'^2} - k^2} \quad . \tag{2}$$

with an implied $e^{-i\omega t}$ time dependence. Where the R_n and S_n are the appropriate linear combinations of the modified Bessel functions in the regions $r < a$ and $r > a$. The other components of E and B may be obtained through Maxwell's equations.

The boundary conditions are:

(1) $r = c$: $E_z = B_r = E_\phi = 0$.
(2) $z = 0,\ L'$: $E_r = E_q = B_z = 0$.
(3) $r = a$: E_z and E_q continuous; $E_z + E_q \cot\theta = 0$; $B_z + B_\phi \cot\theta$ is continuous; where $\cot\theta = \frac{2\pi a}{s}$.

The second boundary condition is automatically satisfied by the solution chosen. The coefficients and the resonant frequency are obtained in the same way as in ref. 4.

Figure 1 shows a comparison for a typical resonator of the theoretical and measured B^2 fields, with measurements obtained from the group at LASL.[3] It is worth noting that the original method set L' = L, and that the modification, while it does improve the magnetic fields, gives a poorer prediction of the electric fields than the original method.

Calculation of Losses

The method used here is a modification of that used by Sierk et al., who considered only tubing with a circular cross section.[5] Due to the frequency of the fields involved (~ 50 MHz), and the size of the tubing (~ .5 cm), a pseudo static situation exists and a conformal map is used to find the fields in the neighborhood of the helix. The transformation,

$$Z = \frac{s}{\pi(1 + \xi)}\left[\sin^{-1}\frac{z'}{a'} + \xi \sin^{-1}\sqrt{\frac{z'^2 - 1}{a'^2 - 1}}\right] \tag{3}$$

transforms the upper half of the z' plane into a region in the upper half of the Z plane extending from - s/2 to s/2, with an approximately elliptical half section removed, centered at the origin (Figure 2). Letting

$$B_- = \tfrac{1}{2}(B_z{}^o - B_z{}^i),\ B_+ = \tfrac{1}{2}(B_z{}^o + B_z{}^i),\ B_r = B_r \tag{4}$$

(the superscript refers to evaluation either just outside, or just inside the helix). The fields associated with these components may be associated with the respective potentials:[5]

$$(1)\ W = \frac{s}{2\pi} B_- \sin^{-1}(1 - 2z'^2) \tag{5}$$

$$(2)\ W = \frac{s}{2\pi} B_+ \sin^{-1}(1 - 2z'^2/a'^2) \tag{6}$$

$$(3)\ W = \frac{s}{2\pi} B_r \sin^{-1}\left(1 + 2\,\frac{1 - z'^2}{a'^2 - 1}\right) \quad . \tag{7}$$

Such potentials give a contribution to $|B|$ that is

$$|B| = \left|\frac{\partial W}{\partial Z}\right| = \left|\frac{\partial W}{\partial z'}\right| / \left|\frac{\partial Z}{\partial z'}\right| \quad . \tag{8}$$

The spatial variations of B_-, B_+, B_r are such that the "cross terms" vanish in forming the integral of the square of the surface current density over the helix. The Joule heating loss may therefore be determined by forming the sums of the integrated contributions over the surface:

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$$S = (c^2 R_s)\sqrt{(2\pi a)^2 + s^2} \times \left[\alpha_- \int_0^L B_-^2\, dz + \alpha_+ \int_0^L B_+^2\, dz + \alpha_r \int_0^L B_r^2\, dz\right] \quad (9)$$

where S is the loss due to the helix, c the speed of light, R_s = RF surface resistance, and

$$\alpha_- = \int_0^1 p(x)\, q(x)\, dx,\ \alpha_+ = \int_0^1 [p(x)/q(x)]dx \text{ and} \quad (10)$$

$$\alpha_r = \int_0^1 [p(x)/q(x)]x^2 dx\ . \text{ Where} \quad (11)$$

$$p(x) = \frac{1+\xi}{4\pi^3}\ \frac{1}{\sqrt{1-(1-\xi^2)x^2}} \text{ and} \quad (12)$$

$$q(x) = \frac{\sqrt{a'^2 - x^2}}{\sqrt{1-x^2}}\ . \quad (13)$$

Results

Calculations have been made for various wave guide configurations, and comparisons made to experimental measurements made by the LASL group.[3] The results may be seen in the table. Group 1 major dimensions were: a = 4.2 cm, c = 12.68 cm, L = 14.6 cm, L' = 17.8 cm, s = .782 cm, f_0 ~ 50 MHz. In group 2, some of the major dimensions were allowed to vary: for A, s = .81 cm; for B and C, L = 13.8 cm, all other parameters remaining the same as in group 1. (In all cases the helical coil was terminated into the side wall of the resonator.)

The table shows the improvement of the new boundary conditions in predicting the relative losses in the helix. However, while good qualitative agreement exists, quantitative prediction cannot be made, especially for large tubing (here, d/s = .406). This is thought to be partially caused by deviations in the shape predicted by the mapping from the true cross section of the tube. A comparison of the two shapes is shown for various tube dimensions in Figure 3. The deviation for the large tube is especially apparent. An improvement in the mapping to better fit the measured cross sections should lead to a better estimate of the losses.

Conclusions

The preceding sections have shown that the technique described therein may be extended from field and frequency calculations to the prediction of the RF losses and shunt impedance of a helically loaded $\lambda/2$ resonator. Although quantitative improvement is still possible, the method clearly points out the relationship between helix configuration and Joule heating. These results should provide aid in the design of short helical resonators for linacs.

It should be pointed out that peak fields in the cavity may be calculated by making use of the conformal map and potentials, by much the same method as was used to calculate the magnetic fields at the helix for the loss estimates.

The results show that the behavior of the helix, as regards heating losses and cooling capacity, may be improved by deforming the helix tubing as suggested. The shape of the B_r^2 field at r = 4.13 cm in Figure 1 hints at another way to improve the characteristics of the resonator. Opening up the pitch at the position along the z-axis where this field is large should reduce further the losses connected with the B_r-induced Joule heating, although at present there are no measurements to substantiate this hypothesis.

References

1. W. Muller and J. Rembser, Nucl. Instr. Meth. 4, 202 (1959).
2. J. R. Pierce, Traveling Wave Tubes, (D. Van Nostrand Co., New York, 1950).
3. B. H. Erkkila, P. J. Bendt and R. H. Stokes, preceeding paper.
4. T. A. Tombrello, K. S. Jancaitis, P. J. Bendt, B. H. Erkkila and R. H. Stokes, to be published in Particle Accelerators (1973).
5. A. J. Sierk, C. J. Hamer and T. A. Tombrello, Particle Accelerators 2, 149 (1971).

Table 1

Comparison of theoretical and experimental losses in helix structures. d/s and b/s are measures of the deformed tube; major dimensions of the structures are as given in the text. $(Z_{REF}/Z)_{EXP}$ is a measure of the relative loss determined experimentally, referenced to the A helix in both groups. S/S_{REF} and S'/S'_{REF} are measures of the relative theoretical loss, predicted by the modified and unmodified (ref. 4) calculations for the B-fields, respectively.

	d/s	b/s	$\left(\frac{Z_{REF}}{Z}\right)_{EXP}$	$\frac{S}{S_{REF}}$	$\frac{S'}{S'_{REF}}$
GROUP 1:					
A(REF)	.304	.304	1.00	1.00	1.00
B	.200	.526	.98	.92	.84
C	.250	.507	.99	1.01	.99
D	.300	.475	1.13	1.17	1.25
E	.406	.406	1.82	3.52	3.69
GROUP 2:					
A(REF)	.294	.294	1.00	1.00	1.00
B	.340	.463	1.28	1.40	1.75
C	.375	.442	1.35	1.72	2.48
D	.300	.475	1.14	1.22	1.31
E	.406	.406	1.69	2.96	3.86

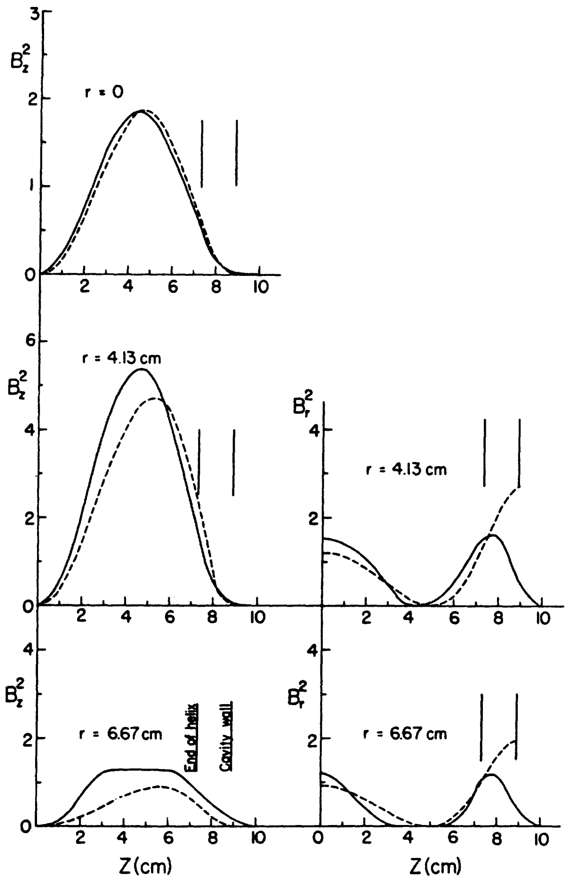

Fig. 1: The squares of the major B-fields are shown as functions of z for various values of r, for a typical resonator (a = 5.40 cm, c = 16.19 cm, L = 14.6 cm, L' = 17.78 cm, s = 1.043 cm). The point z = 0 corresponds to the center of the cavity. The magnitude of the field is expressed in arbitrary units. The solid curve is the experimental result; the dashed curve is the prediction of the modified theory. The curves are normalized such that the theoretical and experimental maxima at r = 0 are equal.

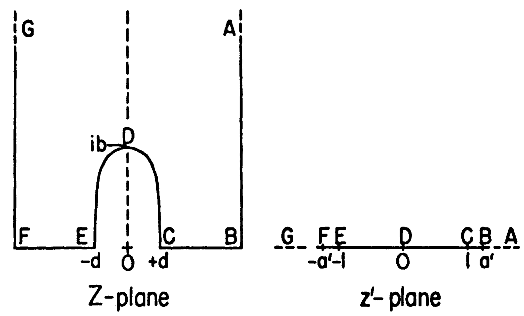

Fig. 2: Diagrammatic representation of the transformation used to describe the conductor contours. The lower case values are as defined in the text; upper case letters relate points between the two planes.

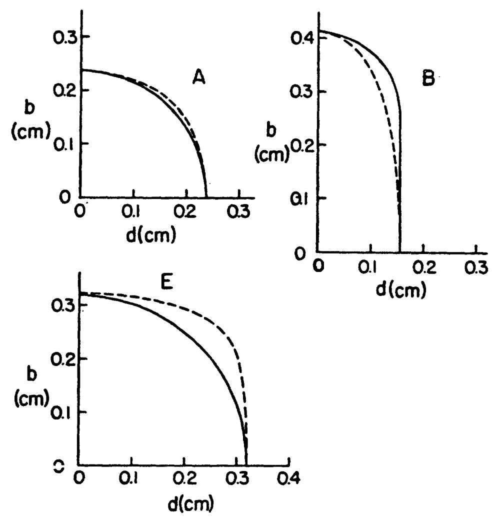

Fig. 3: Contours obtained from conformal mapping shown superimposed in the actual conductor coutour for three resonators in group 1. One quadrant is pictured; the solid curve is the measured shape, and the dashed curve the shape predicted by the transformation.