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Analysis of a Precambrian resonance-stabilized day length

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During the Precambrian era, Earth's decelerating rotation would have passed a 21-hour period; this rotational frequency would have been resonant with the semidiurnal atmospheric tide. Near this point, the atmospheric torque would have been maximized, being comparable in magnitude but opposite in direction to the lunar torque, halting Earth's angular deceleration, as first detailed by Zahnle and Walker [1987]. Computational simulations of this scenario indicate that, depending on the atmospheric Q-factor, a persistent increase in temperature larger than 10K over a period of time less than 10^7 years will break resonance, such as the deglaciation following a possible "snowball Earth" near the end of the Precambrian. The resonance was found to be resilient to comparatively high frequency thermal noise. Our model provides a simulated day length over time that resembles existing paleorotational data, though these are not thought to be reliable; further data is needed to verify this hypothesis.

1. Introduction

At some point during the Precambrian, the Earth would have decelerated such that it had a rotational period of 21 hours, which would have been resonant with the semidiurnal atmospheric tide, with its period of 10.5 hours. At this point, the atmospheric tidal force would have been comparable in magnitude but opposite in sign to the lunar torque, which could create a stabilizing effect on the day length, preserving the 21 hour day length until the resonance was broken, as first discussed in Zahnle and Walker [1987].

The question then arises as to how the Earth broke out of its resonance-stabilized day length of 21hr to progress to its current day length of 24hr. In general, any sufficiently large sudden increase in temperature will shift the resonant period of the atmosphere by thermal expansion (resulting in a change of atmospheric column height) to a lower value and could potentially break resonance. This paper aims to address the specific conditions necessary to preserve or break resonance - namely, how quickly the warmup period must occur for a given temperature change and set of atmospheric properties and how stable the system is to thermal noise.

In our model of atmospheric resonance, there are effectively three options as to the importance and outcome of the constant day length.

First, the Earth could have entered a stable resonant state which lasted for some extended period of time before being interrupted, presumably at approximately 650Ma., by a global temperature increase, such as, for example, the deglaciation period following a possible "snowball Earth" event. Specifically, the Sturtian or Marinoan glaciations make a good candidate for this. [Pierrehumbert, et al. , 2011; Rooney, et al. , 2014]

Second, the resonant stabilization could have never occurred, as the Q-factor of the atmosphere, defined as $2\pi \cdot \frac{\text{total energy}}{\text{energy dissipated per cycle}}$, could have been too low for the magnitude of the atmospheric torque to exceed that of lunar torque, a necessary condition for the formation of a stabilized day length.

Third, the resonance could have been of no interest, as atmospheric and temperature fluctuations could have been too high to allow a stable resonance to form for an extended period of time.

We discuss the plausibility of each of these scenarios in greater detail below and ultimately conclude, based on computational simulations, that the first scenario is the most likely to have occurred.

Existing stromatolite data as compiled in Williams [2000] put the point of breaking resonanace at sometime between 2Ga and 600Ma, likely toward the much more recent end of this range. After 600Ma, stromatolite, coral, and bivalve data indicate that the day length increases to its current 24 hours day length comparatively quickly after a period of relatively constant day length (though paleorotational data is nearly absent during most of this range, only available near the endpoints). However, this data, particularly the stromatolite data[Panella , 1972], should not be taken too seriously. [Zahnle and Walker , 1987] Paleontologists Scrutton [1978], and Hofmann [1973] also found these data to be unreliable and unsuitable for precise quantitative analysis. Regrettably, no significant additional data has emerged in the past several decades. Though we do eventually compare our modeled results to this data as a kind of sanity check, the data is inconsistent and scant; further and more reliable data will be needed to test both Zahnle's and Walker's hypothesis and our developments on mechanisms of breaking resonance.

2. Analysis of atmospheric resonance

The details of the atmospheric tide are quite complex, but the essential features can be appreciated with the following toy model of the torque.

Given a fluid with column density ρ_0 and column height h_0 under gravitational acceleration g, with Lamb waves of amplitude $h \ll h_0$ and wavelength $\lambda \gg h_0$, wave speed of $\sqrt{gh_0}$, Cartesian spatial coordinates of x, a forced heating term h_f, and a damping factor $\Gamma = \frac{1}{t_Q}$ (with t_Q defined as the total energy over power loss of the system, such that $Q = \omega_0 t_Q$), we first start with the forced wave equation without drag (we will add this in later):

$$\frac{\partial^2 h}{\partial t^2} = gh_0 \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h_f}{\partial t^2} \tag{1}$$

We are interested in a heating term of the form $F = F_0 \cos(2\omega t + 2kx)$, with F_0 as the average heating per unit area, ω is the angular frequency, and $k = \frac{2\pi}{2\pi R_\oplus}$ at the equator, with $R_\oplus$ the Earth's equatorial radius. Thus, for C_p as

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the specific heat at constant pressure and T_0 as mean surface temperature, we have $\rho_0 C_p T_0 \frac{dh_f}{dt} = F_0 \cos(2\omega t + 2kx)$, or:

$$h_f = \frac{F_0 \sin(2\omega t + 2kx)}{2\rho_0 C_p T_0 \omega} \tag{2}$$

Expressing $h = A\sin(2\omega t + 2kx)$ and defining $gh_0 \equiv \frac{\omega_0^2}{k^2}$, we obtain via Equation 1 that:

$$A = -\frac{\omega F_0}{2\rho_0 C_p T_0(\omega_0^2 - \omega^2)} \tag{3}$$

At present, $\omega < \omega_0$, making A negative, so the positive peak of $A\sin(2\omega t + 2kx)$ is at $2\omega t + 2kx = -\frac{\pi}{2}$. At noon ($t = 0$), this occurs spatially at $x = -\frac{\pi}{4k} = -\frac{\pi R}{4}$, or $-45°$. This result determines the sign of the torque, as the mass excess closer to the sun exists such that it is being pulled in the prograde rotational direction. Note that for period of time where the length of day is less that the resonant period of 21hr, that is, for $\omega > \omega_0$, the resultant torque of A will exert a decelerating effect on the earth. However, at the point of resonance in question, where the lunar torque is cancelled by the atmospheric torque, $\omega < \omega_0$ by a small factor.

Addressing the drag in our model, if we assume that any excess velocity formed from the tidal acceleration in the atmosphere is quickly dissipated into the Earth through surface interactions with a damping factor Γ, and that this surface motion is relatively quickly dissipated into the rotational motion of the entire Earth, as given by Hide, et al. [1996], writing the dissipative Lamb wave forces, we have:

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial x} - v\Gamma \qquad h_0\frac{\partial v}{\partial x} = -\frac{\partial h}{\partial t} + \frac{\partial h_f}{\partial t} \tag{4}$$

from which we obtain:

$$A = \frac{F_0}{\rho_0 C_p T_0} \cdot \frac{(2\omega - i\Gamma)(4(\omega^2 - \omega_0^2) + 2i\omega\Gamma)}{16(\omega^2 - \omega_0^2)^2 + 4\omega^2\Gamma^2}. \tag{5}$$

In this model, the imaginary component $\Im(A)$ represents amplitude which would create a force with angle of $\frac{\pi}{2}$ with respect to the sun, and thus does not exert any torque on the Earth. We need only concern ourselves with the real part $\Re(A)$, then. Thus, we have:

$$\Re(A) = \frac{F_0}{2\rho_0 C_p T_0} \cdot \frac{4\omega(\omega^2 - \omega_0^2) + \omega\Gamma^2}{4(\omega^2 - \omega_0^2)^2 + \omega^2\Gamma^2}. \tag{6}$$

Since we know the atmospheric displacement A to be directly proportional to the torque exerted by said displacement, we can use the fact that the present day accelerative atmospheric torque, 2.5×10^{19}Nm, is approximately $\frac{1}{16}$ that of the present decelerative lunar torque, 4×10^{20}Nm, as given in Lambeck [1980], to scale the atmospheric torque along the curve following $\Re(A)$, thus solving for the total atmospheric torque $\tau_{atm}(\omega)$ as a function of the Earth's rotational frequency, as detailed in Figure 1.

Given a sufficiently high atmospheric Q with an initial day length of much less than 21 hours, we can see that, near the resonance point in question, as the Earth's rotation slows, increasing the length of day, the atmospheric torque increases until it eventually matches the lunar torque, so the length of day remains constant at this stable equilibrium. While there are two frequencies at which the torques are balanced, only the higher frequency (shorter day length) is linearly stable. That is, infinitesimally perturbing the system in the low frequency direction about the unstable point will cause the system to migrate indefinitely to even lower frequencies (longer day length).

It should be noted that, to the authors' knowledge, there is no consensus on a value of Q for the atmosphere; however, one can reasonably assume it is within the range of $10 - 500$, like many physical systems. We ultimately solve the problems in this paper using all possible values of Q within this range, though for some example calculations involving a specific value of Q, we arbitrarily assume Q is a reasonable value of 100. Ultimately, we establish a critical (relatively low) threshold, dependent on the lunar torque, that Q must exceed for resonance to form and this paper to be relevant - all values of Q sufficiently past this threshold result in similar conclusions.

3. Estimation of resonance-breaking conditions

Before developing a more complete computational model, we first detail a less sophisticated analytical solution to approximate the warming timescale necessary to break resonance. We then verify this with our computational model, noting that the key features are present, albeit at different values.

Given some increase in global temperature ΔT from an initial "average" temperature T_0, we would expect a corresponding increase in atmospheric volume, which, since the atmosphere is horizontally constrained, should result in a nearly linear increase in the column height of the atmosphere. This, in turn, would change the propagation speed of an atmospheric Kelvin wave, given by $v = \sqrt{gh_0}$, and thus the resonance frequency of the atmosphere. A decrease in global temperature serves to increase the resonant frequency (thus decreasing the length of day at which the lunar and atmospheric torques meet, shifting the curves to the left on Figure 1), while an increase in global temperature would decrease this value.

A large, sudden increase in global temperature could shift the atmospheric torque curves given in Figure 1 sufficiently far to the left so as to surpass the unstable equilibrium on the right side of the curve, allowing the Earth to decelerate past the points near resonance. This change would need to be sudden enough that the Earth's rotation could not track this change, and would need to be sustained at a minimum of this temperature for some time following the change, so as to avoid recapture from the atmosphere.

We can see from Figure 1 that very near resonance, the atmospheric torque can be approximated linearly. We know that for a steady-state, comparably large temperature change to preserve resonance throughout a temperature change, excluding edge effects (assuming a resonant curve width of zero, to simplify the calculations), the rotational frequency of the Earth must track the change in resonance frequency of the atmosphere, so $\frac{d\omega}{dt} = \frac{d\omega_0}{dt}$. Since the resonance frequency of the atmosphere $\omega_0 = \frac{\sqrt{gh_0}}{R_\oplus}$, and $h_0 \propto T$ for temperature T, we can express $\omega_0(T)$ as $\omega_0 = \frac{\sqrt{gh_0\frac{T}{T_0}}}{R_\oplus}$, for T_0 the original temperature. Since, for any realistic changes in atmospheric temperature, $T \approx T_0$, and denoting the time over which the temperature changes by an amount ΔT as t_w, we obtain:

$$\frac{d\omega_0}{dt} = \frac{dT(t)}{dt} \cdot \frac{\sqrt{gh\frac{T(t)}{T_0}}}{T(t)R_\oplus} \approx \frac{\Delta T\sqrt{gh_0}}{t_w T_0 R_\oplus}. \tag{7}$$

Following the amplitude-scaling technique mentioned in the previous section, we know the angular acceleration of the Earth to be:

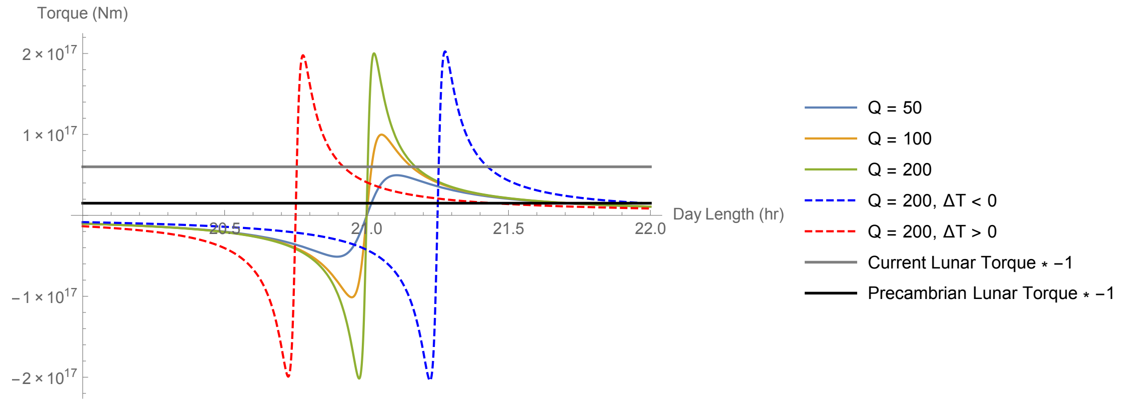

Figure 1. Torque values for atmospheric torques assuming various Q-factors compared to lunar torque. Torques are scaled at the 24hr. endpoint such that they have a value $\frac{1}{16}$ that of the lunar torque, while the contour of the curve is determined by the A term derived in section 2. Note that the minimum value of Q required to form a resonance (the value such that its magnitude exceeds the lunar torque) varies linearly (to a first approximation) with the lunar torque. During the Precambrian, when the lunar torque was thought to be approximately a fourth of its current value [Zahnle and Walker , 1987], very low values of Q could have permitted a resonance.

$$\frac{d\omega}{dt} = \frac{\tau_{atm} - \tau_{moon}}{I_{\oplus}} = \frac{\tau_{moon}(\frac{A(\omega_{max})}{16\cdot A(\frac{2\pi}{24hr})} - 1)}{I_{\oplus}} \quad (8)$$

where ω_{max} is the rotational frequency associated with the global maximum of τ_{atm}, and $\frac{2\pi}{24hr}$ is the current rotational frequency of the earth. Abbreviating $A(\omega_{max})$ as A_{max} and $A(\frac{2\pi}{24hr})$ as A_{24}, we obtain that:

$$\frac{\Delta T\sqrt{gh_0}}{t_w T_0 R_{\oplus}} = \frac{\tau_{moon}(\frac{A_{max}}{16\cdot A_{24}} - 1)}{I_{\oplus}} \quad (9)$$

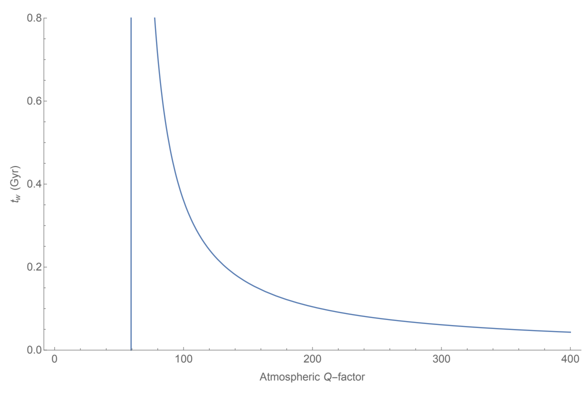

Figure 2. The fastest possible stability-preserving warming time t_w (the stability-instability barrier such that any faster change will break resonance) for a given Q with a fixed $\Delta T = 10K$, as derived in section 3. It should be noted that the asymptote arising at $Q \approx 60$ is a result of no resonance-stabilizing effect occuring, as the maximum value of τ_{atm} does not surpass the lunar torque. As this simple model serves only as an upper bound for the conditions required to break resonance (as it treats the length of day interval in Figure 1 where the solar torque exceeds the lunar torque as having zero width), this value of $Q \approx 60$ will also vary in the computational model, but the asymptote for some low value of Q should still be present.

Since, for reasonable values of Q (say, Q is somewhere between $10-500$), A_{max} will scale linearly with Q, we need only attain one value of A_{max} and scale it accordingly with Q. For example, at $Q = 100$, $A_{max} \approx 27.01 \cdot A_{24}$, and at $Q = 200$, $A_{max} \approx 53.78 \cdot A_{24}$. So our expression becomes:

$$t_w \approx \frac{\Delta T I_{\oplus}\omega_0}{T_0\tau_{moon}(\frac{27}{16}\frac{Q}{100} - 1)}. \quad (10)$$

As shown in Figure 2, this expression indicates asymptotes for stability-preserving (Q, t_w) pairs as $Q \to 60$ and as $t_w \to 0$. For a plausible atmospheric Q-factor of 100, any significant change in temperature (on the order of 10K) faster than on the order of 10^8 years will break resonance if no other opposing temperature changes follow.

Note that this model simply provides a lower bound on how fast the temperature must be changed and an upper bound on the minimum threshold for Q for resonance to form - the model effectively assumes the curves in Figure 1 have a half-maximum width of zero by using a steady-state method, and that resonance will be broken if the rate of change of resonant frequency at all exceeds the angular deceleration of the Earth. In reality, the width of the curves provides a buffer; for example, small changes of resonance frequency about the mean value of ω_0 will not break resonance, as it will not exceed the width of the curve. Thus, temperature changes may need to occur significantly faster than the values shown in Figure 2 to actually break resonance and Q may be allowable at lower values while allowing resonance to form. These problems are more precisely addressed with our computational model outlined in the next section.

4. Computational model

To determine the allowable timescale for an atmospheric temperature change to break resonance, a computational model was implemented in Python to iterate over the exact solutions to the equations developed in section 2 for a given $(\Delta T, Q, t_w)$ tuple. This would generate a stability regime diagram depicting stable and resonance-breaking (unstable) conditions for temperature changes involving each combination of these values.

At the program's core is a simulation function which iterates the Earth's rotational frequency a over a torque-scaled version of equation 6 as global temperature rises from

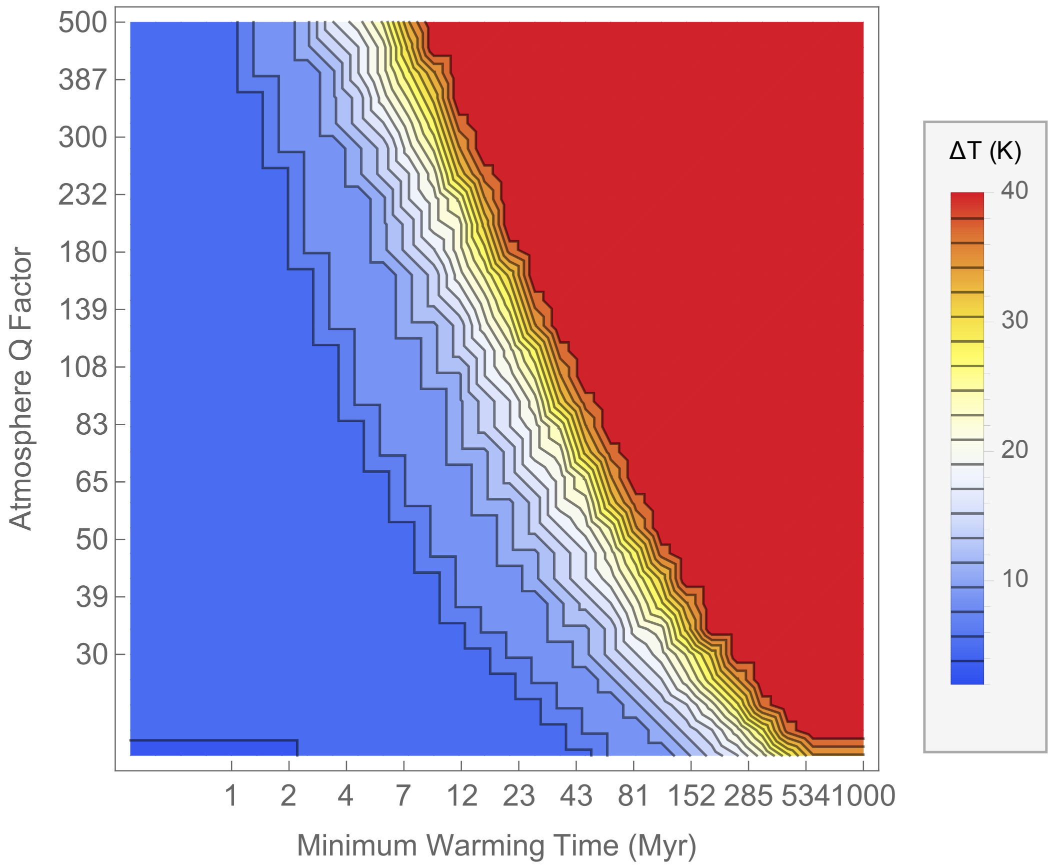

Figure 3. The stability-instability boundary calculated along varying ΔT, Q, and t_w values. At a given Q-factor and warming time (Q_0, t_{w0}), the resulting temperature change ΔT_0 represents the maximum stability-*preserving* temperature change such that any larger change over the same period of time will break resonance. Higher Q-factors permit larger temperature changes per unit time, as the system is more responsive to external torques, and are in a sense "more resilient" to atmospheric changes than scenarios with lower values of Q, so any point with $\Delta T > \Delta T_0$, $t_w < t_{w0}$ or $Q < Q_0$ will break resonance.
It should be noted that, regardless of Q and t_w, there exists a nonzero minimum value of ΔT_0 required to break resonance (5.7 K in the simulation). The existence of an asymptote for very low Q is consistent with Figure 2, as Q approaches a value such that the maximum value of the atmospheric torque can no longer exceed the lunar torque.

$T_0 - \Delta T$ to T_0 (with T_0 being an average global temperature of 287K, though this precise value is unimportant) over a period of t_w years, simulating the warmup following a period of low global temperatures, effectively solving the differential equation for changing resonance frequencies, temperature values, and varying Q-factors.

A very small step size (50yr) was used to ensure accurate results at very high Q values. The simulation function returned whether the result was stable (still trapped in a resonance-stabilizing region) after a warmup period and a subsequent rest period to allow for ω to settle had passed.

To increase computational efficiency, only the stability-instability boundary was solved for using a multiprocessed binary search, such that the entire simulation ran in a more feasible $\mathcal{O}(n^2 \log n)$ time.

5. Results - t_w timescale

A regime analysis was performed using the above computational model to determine which combinations of atmospheric Q, total temperature change ΔT, and warming time t_w resulted in a break of resonance.

As expected, for very fast (small) t_w, temperature changes greater than a critical threshold $\Delta T_0 \approx 5K$ will always break resonance. Similarly, the required t_w to preserve resonance varies inversely with Q: with lower Q, temperature changes must take place over a larger period of time, as the system does not track changes as easily. Additionally, the simulation results suggest an asymptote for t_w slightly below $Q = 30$, with Q-factors below this value prohibiting resonance from forming in the first place (the solar torque fails to exceed the lunar torque in Figure 1). This is in accordance with the results of the approximate analytical method described in section 3, though the exact value for the required Q is different, as expected.

The overall timescale for the required t_w to break resonance was smaller than the rough estimation from section 3: for a ΔT of 10K and a Q of 100, temperature changes occurring on a timescale faster than 10^7 years would be sufficient to break resonance, as shown in Figure 3. Note that the break in resonance is, of course, conditional on the temperature staying near or above this increased temperature long enough for the Earth's rotational velocity to decelerate sufficiently away from the area near resonance - a process which would also take on the order of 10^7 years. This would indicate that, had the rotational velocity and temperature of the Earth previously reached an equilibrium, virtually any deglaciation period following a sufficiently lengthy snowball event would break resonance, as discussed further in the discussion section, though resonant recapture from later temperature decreases is possible, as shown in the final figure in this paper.

6. Results - effects of thermal noise on resonant stability

In addition to a systematic global climate change following a cool-constant-warm pattern, the computational model outlined in section 4 was also further developed to test the resilience of the resonance to atmospheric thermal noise - higher-frequency fluctuations occurring at a variety of amplitudes. The temperature was driven sinusoidally across a very large range of frequencies and amplitudes encompassing all reasonable values for small-scale temperature fluctuations. These results are detailed in Figure 4. It was found that, for a sinusoidally driven mean atmospheric temperature, the optimal fluctuation period to break resonance - that is, the frequency whereby the required amplitude to break resonance is minimized - was on the order of 10000 years. However, the required thermal amplitude was approximately 25K. At this point, these conditions are not so much thermal noise as a large global cyclic temperature change, so the possibility of resonant break due to random thermal fluctuations was discarded. (Further evidence for this decision is also provided by the results from the final figure in this paper.)

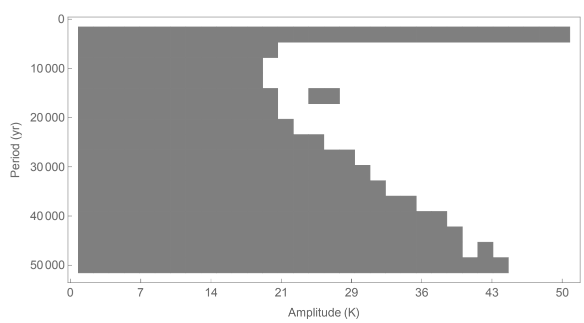

Figure 4. Regime analysis of sinusoidally driven atmospheric temperature fluctuations across (half-wave) amplitude and frequency for an initial phase of zero. Grey regions indicate resonance-preserving scenarios, while white regions break resonance. The "noise" in the diagram, such as the small island of stability in the white region is due to the fact that breakage also depends weakly on initial phase of the sinusoidal driver. However, phase was found not to change the overall shape of the curve, aside from small changes near the edge, so the resilience of the atmospheric resonance to realistic thermal noise is independent of phase.

7. Results - simulated length of day over time

Finally, we used the model from the above two sections to create a simulation of Earth's length of day over time, starting from 4500Ma. to the present time. Given the plausibility of a snowball event breaking resonance, we simulated a sequence of several snowball events, corresponding in time and duration to the three main "snowball Earth" events during the late Precambrian. Throughout this time, random atmospheric noise was also simulated as the sum of several sinusoidal drivers, with a maximum amplitude of approximately 5K.

It should be noted that the Earth is remarkably insensitive to this noise, except at resonance. The first simulated "snowball Earth" failed to break resonance due to a duration of depressed temperature that was too short before warming up again. The second snowball event seemed to break resonance, but was recaptured by the third event, which broke resonance, as shown. The data points at approximately $2Ga.$, while unreliable, could very tentatively be used to establish a lower bound on the formation of this resonance. More reliable data is, of course, needed to confirm this.

8. Discussion

Our model supports the first scenario presented in the introduction - that the Earth entered a resonant state, perhaps at 2Ga. before present, though this is highly uncertain, as it depends on an unknown evolution of lunar tidal Q for that epoch. It then escaped resonance at about 600Ma. (this value also depends on Q), when it was broken by a global temperature change, presumably the deglaciation following a snowball event, though any sharp rise in temperature followed by a period of comparatively constant temperature could have broken resonance.

As shown in the second analytical model presented and as evident in the computational results shown in Figure 3, an asymptote dependent on lunar torque exists such that there is a critical value of Q below which resonance will not form. Near this value, the resonance is quite unstable. Computationally, this asymptotic value was found to be very low: $Q \approx 15$ for the present lunar torque, and even lower for some estimated Precambrian torques [Zahnle and Walker , 1987], making resonance formation likely.

The minimum warming time t_w required to break a resonance state was found to be within values that would be broken by a deglaciation event; assuming $Q \sim 100$, the deglaciation period would need to be at most 10^7 years, easily accepting the t_w estimates presented by Hofmann and Schrag [2002]. Snowball events with depressed, relatively stable temperatures lasting for a period of around 10^7 years (also similar timespans as in Hofmann and Schrag [2002]) were found to provide sufficient time for an equilibrium of ω and ω_0 to be reached such that the following deglaciation breaks resonance, though this value also depends inversely with the lunar torque.

The mid-Precambrian was lacking in global or near-global glaciations, with the exception of the Huronian glaciation ca. 2.4-2.2 Ga., which likely occurred before resonance had formed. The fact that there is little evidence of any glaciation for almost a billion years prior to the Sturtian glaciation [Rooney, et al. , 2014] lends credence to the idea that the deglaciation of a "snowball Earth" was the likely trigger that broke resonance after allowing it to persist for a length of time on the order of a billion years.

It should be noted that while a reasonable choice of atmospheric and lunar variables makes this scenario possible and likely, the paleorotational data available is not sufficient to confirm the hypotheses of resonance formation or breakage. Further data is required; it is our hope that this work will encourage developments in this area.

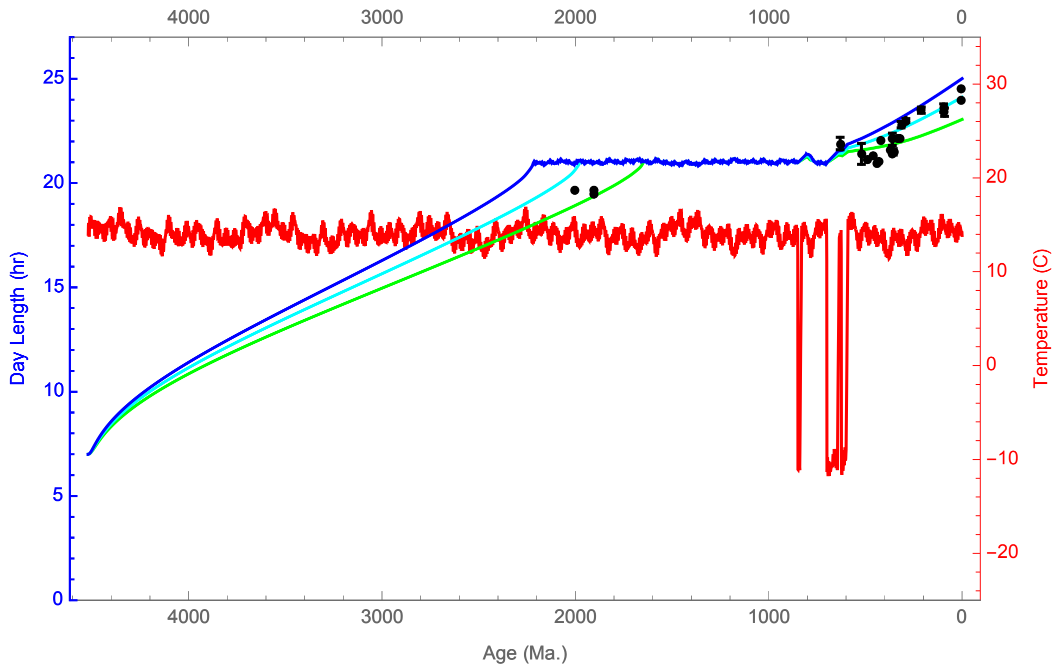

Figure 5. Simulated day length (varying choices of "base", unscaled, lunar torques colored in various blues and greens) and temperature values (red) over the lifetime of the Earth, scaled over time along the -6^{th} power of lunar the orbital radius. Note that atmospheric thermal noise does not influence the day length value except very near resonance, and that the resonance effect remain unbroken until two successive simulated snowball Earths at the end of the Precambrian 720Ma. and 640Ma., with the location and duration of these simulated events picked to coincide with recent estimates of the Sturtian Marinoan glaciations. [Rooney, et al. , 2014] A simulated recapture event can be seen at 870Ma. Approximate empirical day length data from a compilation in Williams [2000] are overlayed in black (error bars included where present), and resemble the modeled data, though the reader should not take these to be too reliable, particularly the points prior to 600Ma.

Appendix A: Source code

All of the code used in this paper is available upon request from the corresponding author.

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