Reaction $Na^{23}(p,\alpha)Ne^{20}$ for Proton Bombarding Energies from 100 to 450 keV*

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The $Na^{23}(p,\alpha)Ne^{20}$ reaction has been studied for proton bombarding energies in the range 100 to 450 keV. Four narrow resonances were observed at proton bombarding energies $E_p=286$, 338, 374, and 445 keV. Excitation functions were taken at each of these resonances, and the alpha yield, width, and resonance energy was determined for each resonance. On the basis of angular distribution measurements, spin and parity assignments have been made for the resonances at $E_p=286$, 338, and 374 keV. Upper limits have been established for the nonresonant cross-section factor S and for the alpha yield from resonances not observed. The $Na^{23}(p,\alpha)Ne^{20}$ reaction rate in stars is computed for temperatures $(5\text{–}10)\times10^8$ °K, the temperature range in which carbon burning takes place

I. INTRODUCTION

THE process of carbon burning in stellar interiors has been analyzed recently by Reeves and Salpeter.[1] In this process the primary reactions $C^{12}(C^{12},p)Na^{23}$ and $C^{12}(C^{12},\alpha)Ne^{20}$ furnish the energy which stabilizes the star against gravitational contraction. As the process progresses, secondary reactions take place involving the protons and alpha particles liberated by the C^{12} collisions. Reeves and Salpeter have set up the differential equations describing this complicated network of interactions and solved for the final relative abundances of elements remaining when the original carbon supply is exhausted. They have pointed out the importance of the reaction $Na^{23}(p,\alpha)Ne^{20}$ in determining the final relative abundances of Na^{23}, Mg^{24}, and Ne^{20}.

Carbon burning is believed to occur in stars in which the temperature has reached $(5\text{–}10)\times10^8$ °K. The Gamow peak in the reaction $Na^{23}(p,\alpha)Ne^{20}$ occurs at a proton bombarding energy $E_p\approx400$ keV for temperatures in this range. The only previous cross-section measurements on the $Na^{23}(p,\alpha)Ne^{20}$ reaction for $E_p<450$ keV are those of Flack *et al.*,[2] who observed resonances at $E_p=287$ and 338 keV and measured the alpha yield from these resonances at 90° in the laboratory. Reeves and Salpeter used Flack's data to calculate the $Na^{23}(p,\alpha)Ne^{20}$ reaction rate, but estimated that their calculated value could be increased by as much as a factor of 20 by the presence of resonances at $E_p<287$ keV. We have, therefore, been led to make a more careful study of the $Na^{23}(p,\alpha)Ne^{20}$ reaction for low-proton bombarding energies.

We have observed four resonances at $E_p=286$, 338, 374, and 445 keV, and measured the yields, widths, and resonance energies. On the basis of angular distribution measurements, we have made spin and parity assignments for the resonances at 286, 338, and 374 keV. We have studied the off-resonance regions to obtain upper limits on the nonresonant cross section and on yields from any resonances which we failed to observe. Using this data, together with the data of other investigators for $E_p>450$ keV, we have calculated values of the $Na^{23}(p,\alpha)Ne^{20}$ reaction rate which are accurate to within 25% over the temperature range $(5\text{–}10)\times10^8$ °K.

II. EXPERIMENTAL PROCEDURE

The proton beam from a 600-keV electrostatic accelerator was deflected magnetically to separate protons from other mass components present, then passed through a 90° electrostatic analyzer with an energy resolution of one part in 1000. The analyzer was calibrated by observing gamma rays from the resonance at $E_p=340.5\pm0.3$ keV in the reaction $F^{19}(p,\alpha\gamma)O^{16}$. An electronic current integrator was used to monitor the beam current.

Sodium chloride targets were prepared by vacuum evaporation of the salt onto thick copper and silver backings. These targets held up better than pure sodium targets under the heavy bombardments required because of the low yield of the reaction. The target was surrounded by a trap at liquid-nitrogen temperature to reduce the deposition of contaminants on the target surface during bombardment.

The Q value of the $Na^{23}(p,\alpha)Ne^{20}$ reaction is 2.38 MeV, and it was necessary to detect alpha particles of approximately this energy against a large background of elastically scattered protons. The alpha detector, a silicon p-n junction counter, was mounted at a distance of $\frac{5}{8}$ in. from the target and could be rotated about a vertical axis through the target. A window immediately in front of the counter defined a solid angle of 0.100 ±0.005 sr; it was covered by a gold foil which stopped elastically scattered protons. These foils were prepared by superimposing several layers of commercial gold leaf, and the foil thickness was chosen to give the best separation between the alpha particles and the elastically scattered protons.

Figure 1 shows the spectrum of pulses from the detector for a proton bombarding energy slightly above the 338-keV resonance in the Na^{23} $(p,\alpha)Ne^{20}$ reaction. The counts below channel 25 are due to counter noise and protons which straggle through the gold foil. The Na^{23} alpha peak occurs at channel 55. The broad peak

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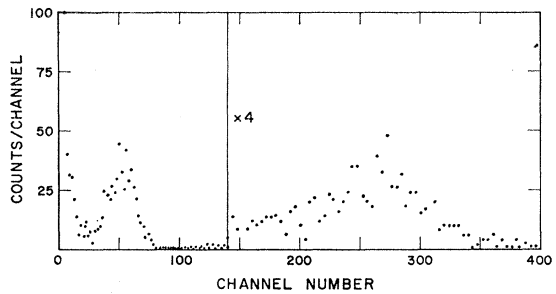

FIG. 1. Pulse-height distribution of alpha particles from a NaCl target for a proton-bombarding energy slightly above the 338-keV resonance in the $Na^{23}(p,\alpha)Ne^{20}$ reaction. Counts below channel 25 are due to counter noise and elastically scattered protons which straggle through the gold foil. The alpha peak from the $Na^{23}(p,\alpha)Ne^{20}$ reaction falls in channel 55 and the broad peak around channel 260 is due to the reaction $B^{11}(p,\alpha)Be^{8*}(2\alpha)$. Channels 140 to 400 are shown on a different vertical scale so that this peak will show up more clearly.

around channel 260 was identified with the reaction $B^{11}(p,\alpha)Be^{8*}(2\alpha)$ by observing a resonance in this group at 163 keV. Data were taken using two integral pulse-height discriminators biased above and below the Na^{23} alpha peak at channels 25 and 100 in Fig. 1.

III. EXCITATION FUNCTIONS

Thick-target excitation functions were taken at each of the four resonances observed. Since the resonances were narrow and well isolated, it was possible to use targets which were many times thicker than the width of the resonance under observation, but which were thin enough to give negligible contributions from lower energy resonances. These excitation functions are shown in Fig. 2.

The background on which the resonances in Fig. 2 are superimposed was subtracted, and the remaining counts were fitted using the Breit-Wigner single-level formula to obtain the yields, widths, and resonance energies. These results are given in Table I. Since the resonances were narrow, the energy dependence of the parameters in the single-level formula was neglected and the level shift was omitted. The nonresonant background in Fig. 2 is due primarily to alpha particles from the reaction $B^{11}(p,\alpha)Be^{8*}(2\alpha)$ and not to a nonresonant $Na^{23}(p,\alpha)Ne^{20}$ cross section.

TABLE I. Resonance parameters for resonances in the $Na^{23}(p,\alpha)Ne^{20}$ reaction.[a]

E_R (keV)	θ	$Y(\theta)$	Γ (keV)
285.9±0.5	90°	$(1.70\pm0.1)\times10^{-12}$	1.75±0.35
338.6±0.6	90°	$(1.77\pm0.1)\times10^{-12}$	0.7 ±0.2
374 ±1	90°	$(1.3\ \pm0.2)\times10^{-13}$	<5
445 ±1	140°	$(2.2\ \pm0.45)\times10^{-13}$	<2
Unobserved resonances			
120–450	90°	$<3\times10^{-14}$	

[a] θ is the laboratory angle at which the excitation function was taken, $Y(\theta)$ is the alpha yield per steradian per proton at this angle, E_R is the resonance energy, and Γ is the resonance width. The quoted uncertainties include uncertainties in solid-angle determination and current integrator calibration in the case of $Y(\theta)$, the uncertainty in the electrostatic analyzer calibration in the case of E_R, and the finite resolution in the case of Γ.

The yield between resonances sets an upper limit on the nonresonant $Na^{23}(p,\alpha)Ne^{20}$ cross section. If this cross section is written in the form $\sigma_{NR}=(S/E)\exp(-2\pi\eta)$, where E is the incident proton energy in the center-of-mass system, and η is the Coulomb factor for the proton, our upper limit for S is 100 MeV-b. The upper limit on the alpha yield from unobserved resonances is given in Table I, and applies in particular to known resonances at 251 and 308 keV in the reaction $Na^{23}(p,\gamma)Mg^{24}$.

IV. ANGULAR DISTRIBUTIONS

Angular distributions were taken at the 286-, 338-, and 374-keV resonances over the angular range from 60° to 150° in the laboratory. These are shown in Fig. 3. The angular distributions were fitted by least squares to the expression,

$$\frac{d\sigma}{d\Omega}(\theta)=A\left[1+\frac{B_2}{B_0}P_2(\cos\theta)+\frac{B_4}{B_0}P_4(\cos\theta)\right], \qquad (1)$$

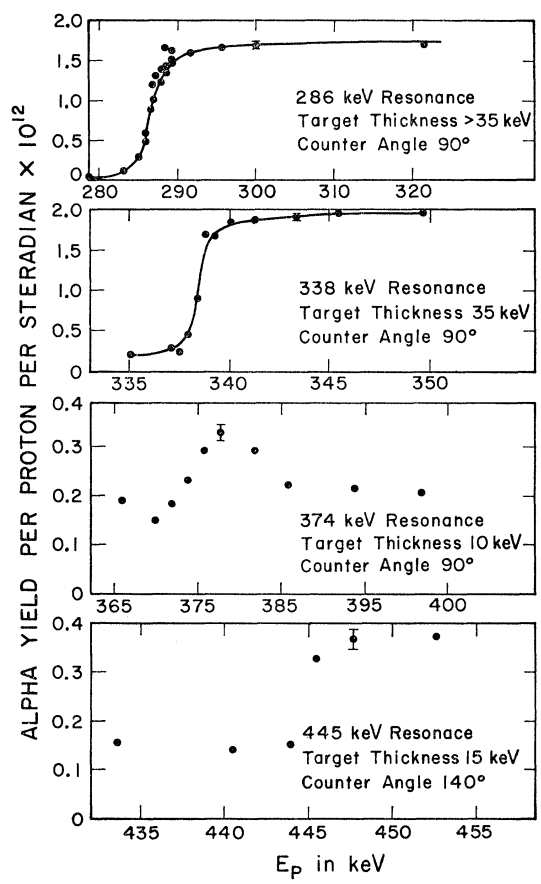

FIG. 2. Excitation functions for resonances at 286, 338, 374, and 445 keV in the reaction $Na^{23}(p,\alpha)Ne^{20}$. The background on which the resonances are superimposed is due mainly to alpha particles from the reaction $B^{11}(p,\alpha)Be^{8*}(2\alpha)$, and is most evident at the 374- and 445-keV resonances where the alpha yield is low. Uncertainties indicated are statistical only. Solid curves are the Breit-Wigner expression with the parameters of Table I.

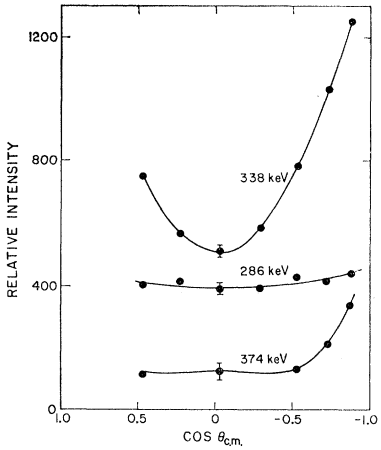

FIG. 3. Angular distribution of alpha particles from the 286-, 338-, and 374-keV resonances in the reaction $Na^{23}(p,\alpha)Ne^{20}$. The ordinate scale is unnormalized, and the relative heights of the three curves have on significance. Center-of-mass corrections to the points have been made. The curves represent least-squares fits to expression (1). Indicated errors are statistical.

where θ is the angle of observation in the center-of-mass system, and A, B_0, B_2, B_4 are constants. The experimental values of the angular distribution coefficients B_i/B_0 have been corrected for finite counter geometry using the procedure described by Rose,[3] and the corrected values are given in Table II.

The theory which relates the numerical values of the coefficients appearing in expression (1) to the spin and parity of the compound nuclear state has been worked out in detail by Blatt and Biedenharn[4] and an application of this theory to the reaction $Na^{23}(p,\alpha)Ne^{20}$ has been made previously by Stelson.[5] We will, therefore, not go into much detail in the discussion which follows. We will use the letters s, l, and J to denote channel spin, the orbital angular momentum of the incident proton, and the spin of the compound nuclear state.

For a particular choice of s, l, and J, the theory predicts unique values for the coefficients appearing in expression (1). Usually, however, a particular value of J can be made by more than one combination of s and l, and in this case the angular-distribution coefficients depend on the partial widths Γ_{sl} which cannot be calculated and must be left in as parameters. The choice of a particular J value does not then lead to a unique prediction for the angular-distribution coefficients, and it may happen that the experimental angular distribution can be fitted by more than one value of J. In this case, the usual procedure is to relate the partial widths Γ_{sl} to the reduced widths θ_{sl}^2 using the formalism of Wigner. Arguments concerning the θ_{sl}^2, such as the well-verified rule $\theta_{sl}^2 \leq 1$, can sometimes be used to reduce the possible choices for J.

Since the Na^{23} ground state has $J^\pi = \frac{3}{2}^+$, the possible values of s which we must consider are 1 and 2. We note also that since both the alpha particle and the residual Ne^{20} nucleus have $J^\pi = 0^+$, the determination of J for a compound Mg^{24} state involved in the $Na^{23}(p,\alpha)Ne^{20}$ reaction also determines the parity which is given by $\pi = (-)^J$.

286-keV Resonance

The experimental angular-distribution coefficients are $B_2/B_0 = 0.09 \pm 0.06$, $B_4/B_0 = 0$; the possible spin and parity assignments are 2^+ and 1^-. If $\theta_{20}^2 = \theta_{22}^2 = \theta_{12}^2$ (where the first subscript denotes s, the second l), a 2^+ assignment gives $B_2/B_0 = 0.13$ and $B_4/B_0 = 0$. To fit the data with a 1^- assignment, we must assume $\theta_{21}^2 = 10\theta_{11}^2$ in which case the 1^- assignment gives $B_2/B_0 = 0.09$, $B_4/B_0 = 0$. Our preference is for the 2^+ assignment since there is reason to expect that the reduced widths for different s and l should be approximately equal, rather than different by a factor of 10.

338-keV Resonance

The experimental angular-distribution coefficients are $B_2/B_0 = 0.84 \pm 0.08$, $B_4/B_0 = 0$; the only possible spin and parity assignment is 3^-. If $\theta_{21}^2 = \theta_{23}^2 = \theta_{13}^2$, this assignment gives $B_2/B_0 = 0.81$, $B_4/B_0 = -0.02$. It is possible to match the experimental values for the angular-distribution coefficients with a 1^- assignment if a large value for the ratio $\theta_{23}^2/\theta_{21}^2$ is assumed, but this possibility can be ruled out since it also requires $\theta_{23}^2 > 1$.

374-keV Resonance

The results of two separate angular distributions gave $B_2/B_0 = 1.09 \pm 0.22$, $B_4/B_0 = 0.70 \pm 0.22$, and $B_2/B_0 = 0.70 \pm 0.18$, $B_4/B_0 = 0.36 \pm 0.17$. Only the first of these is shown in Fig. 3. Although these two angular distributions are not in good agreement, both indicate the presence of a sizeable $P_4(\cos\theta)$ term. Averaging the two, we obtain $B_2/B_0 = 0.90 \pm 0.15$, $B_4/B_0 = 0.59 \pm 0.15$. This result is in reasonable agreement with a 4^+ assignment, which predicts $B_2/B_0 = 1.02$, $B_4/B_0 = 0.55$ if $\theta_{22}^2 = \theta_{24}^2 = \theta_{14}^2$. No assignment having $J < 4$ can explain a $P_4(\cos\theta)$ term of this size in the angular distribution. A 5^- assignment does not fit the data, and any assignment having $J > 5$ would require a reduced proton width greater than the Wigner limit.

V. $Na^{23}(p,\alpha)Ne^{20}$ REACTION RATE

The equations for reaction rates are given by Reeves and Salpeter [Eqs. (11) and (13) of Ref. 1]. For a

TABLE II. Experimental angular-distribution coefficients for resonances in the $Na^{23}(p,\alpha)Ne^{20}$ reaction.

Resonance (keV)	B_2/B_0	B_4/B_0	J^π
286	0.09 ± 0.06	0	$2^+(1^-)$
338	0.84 ± 0.08	0	3^-
374[a]	1.09 ± 0.22	0.70 ± 0.22	4^+
374	0.70 ± 0.18	0.36 ± 0.17	

[a] The table gives two separate sets of experimental angular-distribution coefficients for the 374-keV resonance, where it was difficult to establish the presence of a $P_4(\cos\theta)$ term. The angular distribution from which the first set was obtained is shown in Fig. 3.

[3] M. E. Rose, Phys. Rev. **91**, 610 (1953).
[4] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952).
[5] P. H. Stelson, Phys. Rev. **96**, 1584 (1954).

TABLE III. Total reaction rate[a] P for different temperatures.

	5×10^8 °K	6×10^8 °K	8×10^8 °K	10×10^8 °K
P	$(170_{-24}^{+42})\times10^{-24}$	$(74_{-9}^{+12})\times10^{-23}$	$(70_{-10}^{+11})\times10^{-22}$	$(26_{-3.5}^{+5})\times10^{-21}$
$\log_{10} P$	-21.77	-21.13	-20.16	-19.58
$\log_{10} P$ (Reeves and Salpeter)	-21.9	-21.1	-20.1	-19.6

[a] P is the reaction rate per pair of particles defined by Reeves and Salpeter (see Ref. 1). The values calculated by Reeves and Salpeter are shown for comparison.

narrow resonance which occurs in the region of the Gamow peak, the quantity $\omega(\Gamma_p\Gamma_\alpha/\Gamma)$ determines the contribution of the resonance to the total reaction rate. This quantity can be calculated from a yield measurement at a particular angle and an angular distribution measurement if the stopping cross section of the target is known. We obtained the stopping cross sections of sodium and chlorine by extrapolating between neon, aluminum, and argon.[6] An uncertainty of $\pm10\%$ was assigned to the extrapolated values since extrapolation is of doubtful validity at such low-proton energies. The stopping cross section was checked by measuring the yield from the 286-keV resonance with a pure sodium target, and the ratio of the yields from the pure sodium target and a sodium chloride target agreed with the ratio predicted by the stopping cross sections to within 5%.

For proton energies above 450 keV, we have used the cross section data of Flack *et al.*[2] and Bauman *et al.*[7] Our values for the reaction rate as a function of temperature are tabulated in Table III. The quantity P, called the reaction rate per pair of particles, is the average value $\langle\sigma v\rangle$, averaged over the relative velocity, v. The values calculated previously by Reeves and Salpeter are given for comparison. The upper uncertainty limit in our values for P is greater than the lower uncertainty limit, since we allowed for a possible increase in P due to the presence of a nonresonant cross section and resonances which we failed to observe. The maximum possible contribution P (max) from a hypothetical unobserved resonance is realized if the resonance occurs at 120 keV and has a yield equal to the upper limit quoted in Table I. To arrive at the upper uncertainty limits quoted for our values of P, it was assumed that the contribution of all unobserved resonances was less than P (max) and that the nonresonant cross-section factor S was less than our experimental upper limit of 100 MeV-b.

VI. DISCUSSION

At the 286- and 338-keV resonances, our values for the yield per steradian per incident proton at 90° and the resonance energy are in satisfactory agreement with the results of Flack *et al.*[2] No angular distribution measurements at these two resonances have been published, but angular distributions have been recently measured by Kuperus.[8] He reports $B_2/B_0=0.08\pm0.08$ for the 286-keV resonance and $B_2/B_0=0.97\pm0.08$ for the 338-keV resonance, in reasonable agreement with our values. The preliminary yield measurements which he reports for these two resonances are a factor of 2 larger than ours and those of Flack *et al.*, and the reason for this is not understood at the present time. Our assignment of $J^\pi=2^+$ for the 286-keV resonance is in agreement with the assignment made by Goldberg *et al.*[9] from their data on alpha scattering from Ne^{20}.

The 374-keV resonance has not been observed previously in the reaction $Na^{23}(p,\alpha)Ne^{20}$ due to its low yield. It has been reported in the reaction $Na^{23}(p,\gamma)Mg^{24}$ by Hancock and Verdaguer,[10] Wagner and Heitzman,[11] and others. The small alpha width at this resonance allowing gamma decay to compete favorably with the alpha decay, indicates a high J value for the compound Mg^{24} state and tends to support our assignment $J^\pi=4^+$ for this resonance.

The 445-keV resonance has not been observed previously in the $Na^{23}(p,\alpha)Ne^{20}$ reaction. Hancock and Verdaguer have reported a resonance at this energy in the $Na^{23}(p,\gamma)Mg^{24}$ reaction, but Wagner and Heitzman, in a later study of the reaction, ascribed the appearance of this resonance to chlorine. Our data leave no doubt that a resonance exists at this energy in the reaction $Na^{23}(p,\alpha)Ne^{20}$.

The close agreement between our values for P and those calculated previously by Reeves and Salpeter (see Table II) is due to our failure to discover any new resonances which contribute significantly to P. The contributions of the 374- and 445-keV resonances are negligible due to their low yield. We have, however, removed the large uncertainty which was present in the values of P due to insufficient knowledge about the region of low-proton bombarding energies.

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