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Observation of Torque Exerted by Pure Superflow*†

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Pure superflow is observed to exert a torque in flowing about an object of arbitrary shape. The magnitude of such torque agrees in every respect with expectations based on pure potential flow theory. The present results indicating pure potential flow for superfluid are considered to constitute the complementary experiment to the earlier Craig-Pellam investigation of zero lift for superflow. We consider that these two experiments taken together establish completely the pure potential flow property for superfluid by verifying that (a) zero force, but (b) a prescribed torque are exerted upon an object of arbitrary shape and orientation. As a by-product to the present investigation, an independent evaluation of ρ_s/ρ for liquid helium is obtained in conformity with earlier measurements.

I. INTRODUCTION

(A) Background

ALTHOUGH the hydrodynamic properties expected of perfect fluid flow have long been apparent on a theoretical basis, the possibility of observing such behavior experimentally has become feasible only relatively recently. Appearance of a quantum-mechanical ground state in liquid helium II now makes available a liquid evidently characterized by true perfect fluid nature. The identically zero viscosity $\eta\equiv 0$ of such superfluid component, in contrast to $\eta\rightarrow 0$ for classical fluids of finite viscosity, constitutes the essential distinction for satisfying perfect fluid requirements.

According to classical hydrodynamics, an object of arbitrary shape and orientation exposed to pure potential flow should experience (a) *zero force*, but (b) *prescribed torque*, exerted by the resultant Bernoulli pressure distribution established over its surface. Experimental identification of *superflow* with (a) *zero force*—has already been established[1] by observing "zero lift" characteristics for an airfoil section (within prescribed superfluid velocity limits). Such behavior verifies the irrotational property ($\nabla\times v_s=0$), as required for pure potential flow about an obstacle. The present experiment completes the identification for superfluid by establishing (b) *prescribed torque*—exerted upon an obstacle exposed to pure superflow.

Such capability had been suggested earlier by thermal Rayleigh disk measurements[2] on second sound waves, in which the net torque observed agreed with the predicted effects of counterflowing liquid components comprising such propagation. The ubiquitous normal fluid counterflow basic to temperature waves, however, appears to preclude logical assignment of similar properties uniquely to superfluid (conceivably, the opposing normal flow could "guide" the superfluid into potential flow, or exert other extraneous effect). The question therefore arises[3] whether by investigating the hydrodynamics of pure superflow under the idealized condition of suppressed normal fluid motion such properties could be established unambiguously. The investigation reported here constitutes a direct test of this nature in pure superflow.

(B) Theory

A disk of radius (a) exposed to a perfect fluid of density (ρ) flowing at uniform (undisturbed) velocity

* Preliminary versions of this research have been reported at meetings: T. R. Koehler and J. R. Pellam, Bull. Am. Phys. Soc. 4, 370 (1959); T. R. Koehler and J. R. Pellam, *Proceedings of the Seventh International Conference on Low-Temperature Physics, 1960* (University of Toronto Press, Toronto, 1960), paper 19-2, p. 271.

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[1] P. P. Craig and J. R. Pellam, Phys. Rev. **108**, 1109 (1957).

[2] J. R. Pellam and P. M. Morse, Phys. Rev. **78**, 474 (1950); J. R. Pellam and W. B. Hanson, *ibid.* **85**, 216 (1952).

[3] J. R. Pellam, *Progress in Low-Temperature Physics*, edited by C. J. Gorter, (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, Chap. III, p. 62–3.

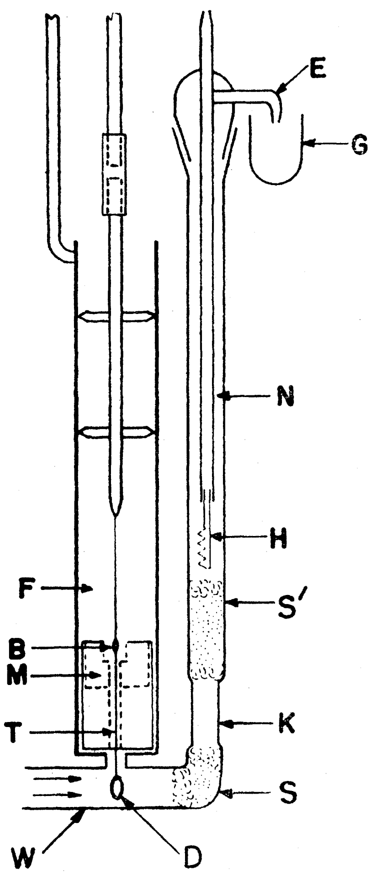

FIG. 1. Equipment. Superfluid (horizontal) wind tunnel: (B) Copper bead, (D) Rayleigh disk, (E) exhaust spout, (F) fiber, (G) measuring flask, (H) heater, (K) Kovar "thermal ground," (M) damping magnet, (N) heater region, (S) and (S') superleaks, (T) glass shaft, and (W) horizontal experimental region.

of magnitude (v) experiences a torque (L) given by the well-known König[4] expression,

$$L=\tfrac{4}{3}a^3\rho v^2 \sin 2\alpha, \qquad (1)$$

where α is the angle between the normal to the disk and velocity vector **v**. The torque operates in a sense tending to align the plane of the disk perpendicular to **v**.

The property of the disk to respond to ordinary fluid flow was first used by Lord Rayleigh[5] for observations on classical sound waves. Ability of the device to detect and measure the intensity of second sound waves was established by Pellam *et al.*[2] some years ago, and the exertion of zero lift [property (a)] by superflow was similarly verified more recently by Craig and Pellam.[1] Finally, the present result completes the identification of superflow with perfect fluid hydrodynamics by verifying the correct torque exerted on a disk [property (b)], according to the König expression modified for superflow,

$$L=\tfrac{4}{3}a^3\rho_s v_s^2 \sin 2\alpha, \qquad (2)$$

where ρ_s and v_s represents the superfluid density and particle migration velocity, respectively.

[4] W. König, Wied. Ann. t. **43**, 51 (1891).
[5] Lord Rayleigh, Phil. Mag. **14**, 186 (1882).

II. EQUIPMENT

(A) Tunnel Operation

The principle and application of the superfluid wind tunnel have been described previously.[1] The present version reflects some essential differences from such prototype and, therefore, warrants some independent description. As evident in the diagram of Fig. 1, the major characteristic is a horizontally oriented "tunnel" section, arranged to expose the vertically suspended disk device (D) to pure horizontal superflow within the experimental region (W). Hydrodynamic interaction with such disk produces a torque (L) [Eq. (2)] about the vertical and resultant observable angular deflection.

Here superleaks (S) and (S') jointly comprise the semipermeable barrier and, operating in conjunction with heater (H), permit a mass flow of liquid helium II through the experimental region (to the right, as indicated) in accordance with the fountain effect phenomenon. Mass flow rate of liquid helium traversing (W) was measured directly in terms of efflux rate from the exhaust spout (E) under equilibrium conditions.

Note also in contrast that the experimental region (W) is located ahead (i.e., "upstream") of the two superleaks in the present version, rather than positioned as before between them. By thus eliminating the superleak preceding (W) as an outer boundary, accessibility to the experimental region—both physical and optical—is improved without compromising any basic operation.[6] Moreover, by employing the "sandwich" configuration of double superleak (S,S') and interconnecting Kovar metal section (K) the inherent leakage defect of simple semipermeable barriers is cured completely. Attributable to natural limitations of physical barriers (either as normal fluid slippage or heat conduction imperfections), such leakage produces normal fluid backflow in simple barrier structures of the type which plagued[7] the zero-lift experiments [reference 1]. By contrast, the equivalent unavoidable normal fluid backflow through S in the present arrangement is "thermally grounded" through K, and thus diverted[8] from disturbing region W.

(B) Disk Operation

The (3-mm diam) disk (D) is supported within W by means of a quartz fiber (torsion constant $k=10^{-5}$ dyne-cm/deg) and adjusted to equilibrium orientation of 45° relative to the direction of superflow [so that $\sin^2\alpha=1$, in Eq. (2)]. This aluminized disk serves the dual function of both test obstacle and deflection sensing

[6] There exist no forces within W tending to displace normal fluid to the left and any motion to the right is prevented by the superleak. As before, accordingly, the entire mass current traversing the system may, within this region, be considered supported by the superfluid component alone.
[7] Application of sandwich structure complex barrier techniques to the zero-lift problem would evidently now permit at least order-of-magnitude better measurements in the pure potential flow zone.
[8] In complete analogy to guard-ring procedure in electropotential applications.

device. A beam of light collimated (from the left) along the axis of (W) and reflected by D permits both alignment for equilibrium orientation and observation of deflections attributable to hydrodynamics.

Despite improvements in tunnel design, the flow instability previously observed[1]—and apparently inherent to such systems—reappeared here as erratic disk behavior. Evidently the flow velocity would suddenly change by as much as 10%, and as often as once a second; besides being kept thus oscillating by periodic impluses, the disk also oscillated about different central positions between impulses. These oscillations were partially controlled by eddy current damping introduced by copper bead (B) and permanent magnet (M). Such damping, of course, could not eliminate the fundamental problem of nonconstant flow rate and the accuracy of all deflection measurements was limited accordingly.

(C) Flow Measurements

Superfluid velocity (v_s) was determined by measuring the time (t) required by the exhausting fluid to fill a small flask (G) of volume $V(=2\text{ cc})$, placed for this duration under E. Within the experimental region, of cross section (A)—where superfluid alone supports the entire mass flow—the (undisturbed) superflow velocity thus measured becomes

$$v_s=(\rho/\rho_s)(V/At), \tag{3}$$

in terms of density (ρ) of liquid helium II.

We emphasize that values computed from Eq. (3) do not represent the full superflow velocity effective within (W). A sizable fraction of the total liquid mass flowing through the system actually evaporates from N, and thus escapes collection in flash (G). Such losses may be accounted for by an additional term introduced to Eq. (3), as follows:

$$v_s=\left(\frac{\rho}{\rho_s}\right)\left(\frac{V}{A}\right)\left(\frac{1}{t}+\frac{1}{t_0}\right). \tag{4}$$

The new quantity (t_0) appearing in this expression represents physically the time required for one "flask-full" of liquid to traverse the experimental region on the basis of evaporative losses alone. As shown under results, analysis of data on the basis of Eq. (4) provides also a numerical evaluation of (t_0).

III. EXPERIMENTAL RESULTS

Measurements in this experiment consisted of observing the angular deflection (θ) produced on the disk by a flow rate filling the flask in time (t). Equation (2) may therefore preferably be re-expressed in a form relating these two quantities directly. Using Eq. (4) for fluid velocity and substituting $L=k\theta$ in Eq. (2), there results

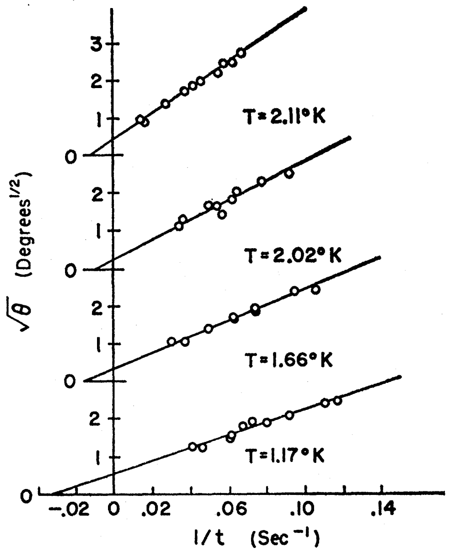

FIG. 2. Dependence of disk deflection (θ) on filling time (t). The quantity $\sqrt{\theta}$ ($\deg^{\frac{1}{2}}$) is plotted as a function of $1/t$ ($\sec^{-1}$) for various temperatures (1.17°K, 1.66°K, 2.02°K, and 2.11°K). Linearity indicated by solid "best fit" straight lines.

$$\sqrt{\theta}=C\left(\frac{1}{t}+\frac{1}{t_0}\right). \tag{5}$$

Here the fixed system parameters (dimensions, etc.) have been absorbed into the "constant" (C) defined as

$$C=\frac{2a^{\frac{3}{2}}}{(3k)^{\frac{1}{2}}}\left(\frac{V}{A}\right)\left(\frac{\rho}{\rho_s}\right). \tag{6}$$

Note that C remains sensitive to the properties of liquid helium II through the factor (ρ_s/ρ). Remember also that $\sin^2\alpha\simeq 1$ for the conditions of the experiment.

Experimental response curves of the disk to superflow are shown in Fig. 2, where measured values of $\sqrt{\theta}$ are plotted versus $1/t$ as observed for four characteristic temperatures.[9] The range of values of $1/t$ for the data on the graph correspond to superfluid velocities of magnitude between 0.01 and 0.1 cm/sec. Linearity of the data plotted on this basis verifies the quadratic dependence of the torque on superfluid velocity. The slope and intercept of the straight line curves fitting these points are determined, for any one temperature, by a statistical analysis. The necessity of adding the constant $(1/t_0)$ is indicated directly by the negative intercepts, and the observed systematic differences in (reciprocal) slope relate to variations in (ρ_s/ρ).

In Fig. 3, reciprocal slope $(1/C)$ is plotted versus temperature and normalized to unity at the low-temperature extreme. With such adjustment in units,[10]

[9] Out of a total of thirteen.

[10] Equivalent to defining $[(3k)^{\frac{1}{2}}/2a^{\frac{3}{2}}](A/V)=1$, so that Eq. (6) becomes $1/C=\rho_s/\rho$.

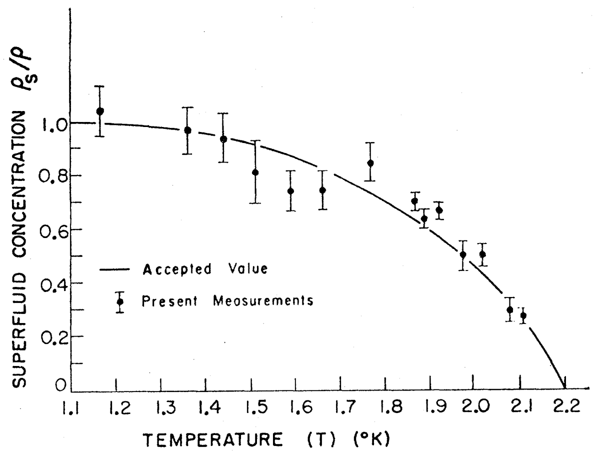

FIG. 3. Experimental values of superfluid concentration (ρ_s/ρ). Values of superfluid concentration (ρ_s/ρ) as determined from disk deflection measurements are plotted as a function of temperature (T). The experimental points (●) for the present experiment are normalized to $\rho_s/\rho=1$ as $T\rightarrow 0°K$. Solid line indicates accepted value.[11]

Fig. 3 becomes basically a plot of the superfluid concentration (ρ_s/ρ) as a function of temperature (T) for liquid helium II. For comparison with measurements by well-known methods, the solid line curve represents the accepted value[11] of ρ_s/ρ. Confidence limits assigned to the present determinations, as indicated, reflect the inherent instabilities of the method. In this connection, we emphasize that the above measurements are not intended as improvements in the determination of ρ_s/ρ. Rather they constitute an independent evaluation adding to the self-consistency of the two-fluid concept, and as a by-product of the present investigation.

IV. DISCUSSION

(A) Wall Effects

In the above treatment no account has been taken of possible hydrodynamic interactions between the disk and the adjacent wall forming (W). Although in the procedure of normalizing the slopes of Fig. 2 such corrections were automatically lumped into the final result, the wall effects are significant to the actual magnitudes observed for torque. In this connection, the values of $\sqrt{\theta}$ obtained during the measurements were found consistently about five percent higher than the uncorrected value predicted by Eq. (6). We considered the exact calculation of wall effect corrections unwarranted[12] for the present experiment, but crude "outside limit" estimates indicate that the observed excess torques are thereby accountable.

(B) Evidence of ρ_s/ρ

An interesting aspect of the method concerns the manner in which selective flow within (W) affects the magnitude of torque. Support of mass flow by superfluid alone always produces greater torque on the disk than for equivalent liquid mass flow as a whole [see Eqs. (5–6)]. Otherwise stated, a spectator witnessing the experiments with no preknowledge of liquid helium would be forced to presume a two-fluid situation. In no other logical way could one rationalize the excessive (and variable) torque observed for known quantities of liquid observed flowing past the disk.

Pursuing similar logic to the low-temperature extreme, where the liquid becomes essentially all superfluid, further justification for the slope-normalizing procedure applied to Eq. (6) appears. The low-temperature limit of superfluid alone provides a natural calibration environment for the over-all disk-tunnel system.

CONCLUSIONS

Pure superflow is observed to exert a torque in flowing about an object of arbitrary shape. The magnitude of such torque agrees in every respect with expectations based on pure potential flow theory. The present results indicating pure potential flow for superfluid are considered to constitute the complementary experiment to the earlier Craig-Pellam investigation of zero lift for superflow. We consider that these two experiments taken together establish completely the pure potential flow property for superfluid by verifying that (a) zero force, but (b) a prescribed torque are exerted upon an object of arbitrary shape and orientation. As a by-product to the present investigation, an independent evaluation of ρ_s/ρ for liquid helium is obtained in conformity with earlier measurements.

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[11] E. Andronikashvili, J. Exptl. Theoret. Phys. USSR **18**, 424 (1948).

[12] Calculations of flow past the 3-mm diam disk (D) within the 3.6-mm diameter cylinder (W) become complicated by effects of an $\simeq$3-mm top opening above the disk.