PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Electroproduction scaling in an asymptotically free theory of strong interactions *

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(Received 30 July 1973)

We analyze electroproduction in a non-Abelian gauge model of the strong interactions using the techniques of Christ, Hasslacher, and Mueller. The theory is asymptotically free and consistent with scaling up to logarithms. All logarithmic factors appear as inverse powers of $\ln(-q^2)$ and hence vanish as $-q^2\to\infty$. When $-q^2$ gets very large, the structure functions become strongly peaked near $x=0$, and as $-q^2\to\infty$ approach the singular scaling functions $2xF_1(x)=F_2(x)=a\delta(x)$, where the constant a is determined. In a strong-interaction model based on the gauge group SU(3) with three triplets of fractionally charged quarks, $a=0.16$.

The recent discovery that non-Abelian Yang-Mills field theories are asymptotically free has opened up an exciting new field in theoretical physics.[1] If the strong interactions are described by a field theory of this kind, it may be possible to reliably calculate some quantities which depend on the details of strong-interaction dynamics using perturbation theory, suitably "improved" with the renormalization group (or Callan-Symanzik equations). These ideas have already been applied to e^+e^- annihilation into hadrons.[2] In this paper we take the next logical step and consider electroproduction.

The strong-interaction model we have in mind is a non-Abelian Yang-Mills theory with fermions, where the gauge group of the strong interactions commutes with the electromagnetic (and weak) charge. Thus the vector mesons associated with the strong gauge group are neutral and the fermions in each gauge multiplet have the same charge. For definiteness, we can imagine that the gauge group is SU(3) and that there are three triplets of fermions (quarks) with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$. The fermions may have a bare mass consistent with the gauge symmetry, but the vector mesons are massless in the tree approximation. At this time, it is not known whether it is possible to introduce scalar mesons into the Lagrangian which give mass to all the vector mesons by the Higgs mechanism without destroying the ultraviolet stability of the theory; so we will assume that the vector mesons develop a mass due to some nonperturbative mechanism. In fact, for simplicity in the analysis which follows, we assume that the fermion mass terms are also absent, and that the only dimensional parameter in the theory is an arbitrary renormalization mass M.[3,4]

The calculational techniques which we use to analyze electroproduction have been worked out by Christ, Hasslacher, and Mueller.[5] They derive Callan-Symanzik equations for the c-number coefficients in an operator-product expansion of the product of two electromagnetic currents, and solve them to exhibit the asymptotic behavior of the coefficients. The relevant coefficients are then related to certain moments of the structure functions.[6] In this paper we give a simple review of these techniques, as applicable to the theory outlined in the preceding paragraph.

The first thing to notice is that since the electromagnetic current is invariant under the strong gauge group, only gauge-invariant operators appear in the operator-product expansion. So we are interested in Callan-Symanzik equations for matrix elements involving gauge-invariant operators. Suppose we were calculating in a general gauge. Then the Callan-Symanzik equation for a matrix element Γ_α would look like

$$\left(M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}+\lambda_i\frac{\partial}{\partial \xi_i}\right)\Gamma_\alpha+\sum_\beta \gamma_{\alpha\beta}\Gamma_\beta=0\,,$$

where g is the gauge coupling constant and ξ_i are additional dimensionless parameters specifying the gauge. In general, the matrix γ will depend on ξ; but if Γ_α involves only gauge-invariant operators, Γ will be ξ-independent.[7] This simple fact has two important corollaries: The matrix $\gamma_{\alpha\beta}$ (and also β) is gauge-independent and depends only on g; and all of the Γ_β appearing in the equation involve only gauge-invariant operators. In practice, it is convenient to calculate in Landau gauge, where no additional gauge parameters are necessary because of stability under mass and coupling renormalization, and to consider matrix elements which are not gauge-invariant. But it is still true that the γ matrix associated with a gauge-invariant operator acts only on the space of gauge-invariant operators.

The model of strong interactions we have discussed above is described by a non-Abelian gauge Lagrangian with m fermion multiplets:

$$\mathcal{L} = -\tfrac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + i\sum_{j=1}^{m}\bar{\psi}_j \not{D}\psi_j\,,$$

where

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc}A_b^\mu A_c^\nu\,,$$

$$D^\mu\psi_j = (\partial^\mu - igT_a A_a^\mu)\psi_j\,.$$

The f_{abc} are the structure constants of the gauge group and the T_a are representation matrices satisfying $[T_a, T_b] = if_{abc}T_c$. For simplicity (and also because it seems reasonable) we assume that each of the fermion multiplets transforms under the same representation of the gauge group.

The electromagnetic current is

$$J^\mu = \sum_{j=1}^{m} Q_j\bar{\psi}_j\gamma^\mu\psi_j\,.$$

The operators appearing in the operator-product expansion of two electromagnetic currents which are relevant to electroproduction in the Bjorken limit are the gauge-invariant symmetric tensor operators of minimum twist[8]:

$$O_0^{\mu_1\cdots\mu_n} = \frac{i^{n-2}}{2n!}\,(F^{\alpha\mu_1}D^{\mu_2}\cdots D^{\mu_{n-1}}F_\alpha^{\mu_n} + \text{permutations of vector indices})\,,$$

and

$$O_j^{\mu_1\cdots\mu_n} = \frac{i^{n-1}}{n!}\,(\bar{\psi}_j\gamma^{\mu_1}D^{\mu_2}\cdots D^{\mu_n}\psi_j + \text{permutations})\,.$$

In O_0,

$$D^\mu F_a^{\nu\lambda} = (\partial^\mu\delta_{ac} + gf_{abc}A_b^\mu)F_c^{\nu\lambda}\,.$$

We will compute the following matrix elements of these operators to the second order in g for even $n \geqslant 2$:

$$\langle 0|T(A_a^\mu(-p)O_k^{\mu_1\cdots\mu_n}(0)A_b^\nu(p))|0\rangle_{\text{amputated}} = \delta_{ab}g^{\mu\nu}p^{\mu_1}\cdots p^{\mu_n}\left[\delta_{0k} + b_{0k}^n\ln\left(\frac{-p^2}{M^2}\right)\right] + \cdots\,,$$

where we have subtracted at $-p^2 = M^2$ and $\cdots$ are terms with different tensor structure, and

$$\langle 0|T(\psi_j(-p)O_k^{\mu_1\cdots\mu_n}(0)\bar{\psi}_j(p))|0\rangle_{\text{amputated}} = \sum_l \frac{1}{n}\gamma^{\mu_l}p^{\mu_1}\cdots p^{\mu_{l-1}}p^{\mu_{l+1}}\cdots p^{\mu_n} \times\left[\delta_{jk} + b_{jk}^n\ln\left(\frac{-p^2}{M^2}\right)\right] + \cdots\,.$$

For convenience, we use the notation $\phi_0 = A$ and $\phi_j = \psi_j$ or $\bar{\psi}_j$ and abbreviate the above as follows:

$$\langle\phi_j(-p)O_k^n(0)\phi_j(p)\rangle = \delta_{jk} + b_{jk}^n\ln\left(\frac{-p^2}{M^2}\right). \tag{1}$$

Now we give a schematic review of the techniques of Christ, Hasslacher, and Mueller as applied to this theory. The expansion of the product of two electromagnetic currents has the form (suppressing tensor indices)

$$J(x)J(0) = \sum_{n,k} F_k^n(x)O_k^n(0) + \cdots\,.$$

Let

$$D = M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial g}\,.$$

Then

$$(D + 2\gamma_j)\langle\phi_j(-p)J(x)J(0)\phi_j(p)\rangle = 0 = (D + 2\gamma_j)\sum_{n,k}F_k^n(x)\langle\phi_j(-p)O_k^n(0)\phi_j(p)\rangle\,.$$

The F's for different n have different tensor structure, so

$$(D + 2\gamma_j)\sum_k F_k^n(x)\langle\phi_j(-p)O_k^n(0)\phi_j(p)\rangle = 0$$

for each n. But

$$(D + 2\gamma_j)\langle\phi_j(-p)O_k^n(0)\phi_j(p)\rangle + \sum_{k'}\gamma_{kk'}^n\langle\phi_j(-p)O_{k'}^n(0)\phi_j(p)\rangle = 0\,. \tag{2}$$

So

$$\sum_k\left[\langle\phi_j(-p)O_k^n(0)\phi_j(p)\rangle DF_k^n(x) - \sum_{k'}\langle\phi_j(-p)O_{k'}^n(0)\phi_j(p)\rangle\gamma_{kk'}^n F_k^n(x)\right] = 0\,.$$

Evaluating this expression at $-p^2 = M^2$, we find

$$DF_j^n(x) - \sum_k\gamma_{kj}^n F_k^n(x) = 0\,.$$

Now from (1) and (2) we find

$$\gamma_{kj}^n = 2b_{jk}^n - 2\gamma_j\delta_{jk}\,. \tag{3}$$

Thus the Callan-Symanzik equations for the coefficients $F_j^n(x)$ involve only the gauge-invariant matrix γ_{kj}^n associated with the operators O_k^n. This matrix can be calculated from the quantities b_{jk}^n, which we compute.

It is useful to rewrite the operator-product expansion in terms of eigenstates of the matrices γ^n. Let u_{nj}^α and v_{nj}^α be left and right eigenvectors, respectively, satisfying

$$\sum_j u_{nj}^\alpha \gamma_{jk}^n = \gamma_n^\alpha u_{nk}^\alpha ,$$

$$\sum_k \gamma_{jk}^n v_{nj}^\alpha = \gamma_n^\alpha v_{nj}^\alpha ,$$

$$\sum_j u_{nj}^\alpha v_{nj}^\beta = \delta^{\alpha\beta} ,$$

where the γ_n^α are the eigenvalues (not necessarily all distinct) of γ^n. Then we can write

$$J(x)J(0) = \sum_{n,\alpha} \mathfrak{F}_\alpha^n(x)\mathcal{O}_\alpha^n(0) ,$$

where

$$\mathfrak{F}_\alpha^n(x) = \sum_j F_j^n(x) v_{nj}^\alpha$$

and

$$\mathcal{O}_\alpha^n(0) = \sum_j u_{nj}^\alpha O_j^n(0) .$$

The $\mathfrak{F}$'s satisfy

$$(D - \gamma_n^\alpha)\mathfrak{F}_\alpha^n(x) = 0 .$$

To apply this to electroproduction, we must write down the operator-product expansion in more detail:

$$iT(J^\mu(x)J^\nu(0)) = \sum_{n,\alpha} [-g^{\mu\nu} i^n \partial_{\mu_1} \cdots \partial_{\mu_n} \mathfrak{F}_{1,\alpha}^n(x^2, M, g) + g^\mu{}_{\mu_1} g^\nu{}_{\mu_2} i^{n-2} \partial_{\mu_3} \cdots \partial_{\mu_n} \mathfrak{F}_{2,\alpha}^n(x^2, M, g)]\mathcal{O}_\alpha^{\mu_1 \cdots \mu_n}(0) + \cdots .$$

Dimensional analysis gives, for the Fourier transforms,

$$\mathfrak{F}_{1,\alpha}^n(q^2, M, g) = \frac{1}{(-q^2)^n} f_{1,\alpha}^n\left(\frac{-q^2}{M^2}, g\right),$$

$$\mathfrak{F}_{2,\alpha}^n(q^2, M, g) = \frac{1}{(-q^2)^{n-1}} f_{2,\alpha}^n\left(\frac{-q^2}{M^2}, g\right).$$

The f's satisfy[9]

$$(D - \gamma_n^\alpha) f_{1\,\mathrm{or}\,2,\alpha}^n\left(-\frac{q^2}{M^2}, g\right) = 0 .$$

These have the solution

$$f_{1\,\mathrm{or}\,2,\alpha}^n\left(\frac{-q^2}{M^2}, g\right) = f_{1\,\mathrm{or}\,2,\alpha}^n(1, \bar{g}(g,t)) \times \exp\left[-\int_0^t dt' \gamma_n^\alpha(\bar{g}(g,t'))\right],$$

where $t = \frac{1}{2}\ln(-q^2/M^2)$ and $\bar{g}$ satisfies

$$\frac{\partial}{\partial t}\bar{g}(g,t) = \beta(\bar{g}(g,t)) ,$$

$$\bar{g}(g,0) = g .$$

To lowest nontrivial order in g, γ_n^α is of the form

$$\gamma_n^\alpha(g) = \frac{g^2}{16\pi^2} d_n^\alpha$$

and

$$\beta(g) = -\frac{g^3 b}{16\pi^2} ,$$

with $b > 0$ for an ultraviolet-stable theory. Then

$$\bar{g}(g,t) = \frac{g}{[1 + (g^2/8\pi^2)bt]^{1/2}}$$

and[10]

$$f_{1\,\mathrm{or}\,2,\alpha}^n\left(\frac{-q^2}{M^2}, g\right) = f_{1\,\mathrm{or}\,2,\alpha}^n(1, \bar{g}(g,t)) \times \left(1 + \frac{g^2}{8\pi^2} bt\right)^{-d_n^\alpha/2b} .$$

So the f's have a calculable logarithmic dependence on q^2.

We can now put the pieces back together to exhibit the forward Compton amplitude, averaged over proton spin. Define the invariant functions T_1 and T_2 by

$$i\int \langle p| T(J^\mu(x)J^\nu(0))|p\rangle e^{iqx} d^4x = -g^{\mu\nu}T_1(q^2,x) + \frac{p^\mu p^\nu}{m_p^2} T_2(q^2,x) + \cdots ,$$

where $x = -q^2/2p\cdot q$. Then

$$T_1(q^2,x) = \sum_{n,\alpha}\left(\frac{1}{2x}\right)^n f_{1,\alpha}^n\left(\frac{-q^2}{M^2}, g\right) A_\alpha^n + O\left(\frac{m_p^2}{q^2}\right),$$

$$T_2(q^2,x) = \frac{m_p^2}{-q^2}\sum_{n,\alpha}\left(\frac{1}{2x}\right)^{n-2} f_{2,\alpha}^n\left(\frac{-q^2}{M^2}, g\right) A_\alpha^n + O\left(\frac{m_p^2}{q^2}\right), \tag{4}$$

where A_α^n is related to the matrix element of $\mathcal{O}_\alpha^n$ between proton states,

$$\langle p|\mathcal{O}_\alpha^{\mu_1\cdots\mu_n}(0)|p\rangle = A_\alpha^n p^{\mu_1}\cdots p^{\mu_n} + \cdots ,$$

and the terms of order m_p^2/q^2, where m_p is the proton mass, come from operators of higher twist.

The structure functions are related to the absorptive parts:

$$W_i(q^2, x) = \frac{1}{\pi}\mathrm{Im}T_i(q^2, x) \tag{5}$$

for $i = 1$ or 2. We can use (4) and (5) and the analytic properties of the T's to derive expressions for certain moments of the structure functions. Ignoring terms of order m_p^2/q^2,

$$\begin{aligned}\int_0^1 W_1(q^2, x)x^{n-1}dx &= \frac{1}{2^{n+1}}\sum_\alpha f^n_{1,\alpha}\left(\frac{-q^2}{M^2}, g\right)A^n_\alpha ,\\ \int_0^1 \nu W_2(q^2, x)x^{n-2}dx &= \frac{m_p}{2^n}\sum_\alpha f^n_{2,\alpha}\left(\frac{-q^2}{M^2}, g\right)A^n_\alpha ,\end{aligned} \tag{6}$$

for even $n \geq 2$. Before calculating the γ^n's, we see that in the region where terms of order m_p^2/q^2 can be neglected, the moments in Eq. (6) have only logarithmic dependence on q^2.

The calculation of b^n_{jk} is straightforward but rather tedious. It involves the graphs illustrated in Fig. 1(a)–1(d). The results are

$$b^n_{00} = \frac{g^2}{16\pi^2}c_1\left[\frac{5}{2} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{l=2}^{n}\frac{1}{l}\right],$$

$$b^n_{0j} = \frac{g^2}{16\pi^2}c_2\left[\frac{8}{n+2} + \frac{16}{n(n+1)(n+2)}\right],$$

$$b^n_{j0} = \frac{g^2}{16\pi^2}c_3\left[\frac{1}{n+1} + \frac{2}{n(n-1)}\right],$$

$$b^n_{jk} = \frac{g^2}{16\pi^2}c_3\left[1 - \frac{2}{n(n+1)} + 4\sum_{l=2}^{n}\frac{1}{l}\right]\delta_{jk},$$

where $j, k = 1$ to m, and the c's are positive constants depending on the group and the fermion representation, defined by

$$c_1\delta_{ab} = f_{acd}f_{bcd},$$

$$c_2\delta_{ab} = \mathrm{tr}(T^aT^b),$$

$$c_3 I = T^aT^a .$$

From Ref. 1, we know $\gamma_j = 0$ for $j \neq 0$ and[11]

$$\gamma_0 = \frac{g^2}{16\pi^2}\left(\tfrac{13}{6}c_1 - \tfrac{4}{3}mc_2\right).$$

Then from (3), we have[12]

$$\gamma^n_{00} = \frac{g^2}{16\pi^2}\left\{2c_1\left[\tfrac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{l=2}^{n}\frac{1}{l}\right] + \tfrac{8}{3}mc_2\right\},$$

$$\gamma^n_{j0} = 2b^n_{0j}, \quad \gamma^n_{0j} = 2b^n_{j0}, \quad \gamma^n_{kj} = 2b^n_{jk}.$$

By inspection, one finds that the d^α_n's for even $n \geq 2$ are increasing with n and are positive, with one exception: γ^2 has one zero eigenvalue corresponding to the energy-momentum tensor. So in the limit $-q^2 \to \infty$, the right-hand sides of Eq. (6) vanish for $n \neq 2$ and only one term survives for $n = 2$. Then for even $n \geq 2$

$$\begin{aligned}\lim_{-q^2\to\infty}\int_0^1 W_1(q^2, x)x^{n-1}dx &= \delta_{n2}\tfrac{1}{8}f_1^2(1,0)A^2 ,\\ \lim_{-q^2\to\infty}\int_0^1 \nu W_2(q^2, x)x^{n-2}dx &= \delta_{n2}\tfrac{1}{4}m_p f_2^2(1,0)A^2 .\end{aligned} \tag{7}$$

Here $f^2_{1\,\mathrm{or}\,2}$ are known from the free-field operator-product expansion *and* A^2 is also known because it is related to a matrix element of the energy-momentum tensor between proton states. The $-q^2 \to \infty$ limit is completely determined.

To calculate these numbers, let us examine γ^2 in more detail. The matrix γ^2 is

$$\frac{g^2}{16\pi^2}\frac{8}{3}\begin{bmatrix} mc_2 & c_3 & c_3 & \cdots & c_3 & c_3 \\ 2c_2 & 2c_3 & 0 & \cdots & 0 & 0 \\ 2c_2 & 0 & 2c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2c_2 & 0 & 0 & \cdots & 2c_3 & 0 \\ 2c_2 & 0 & 0 & \cdots & 0 & 2c_3 \end{bmatrix},$$

where the first row and column refer to the zeroth

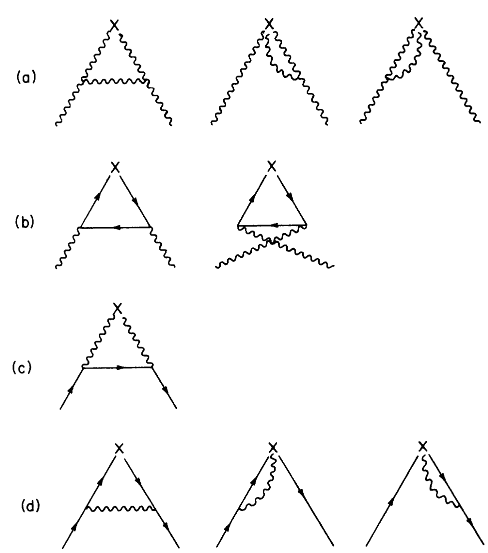

FIG. 1. Graphs contributing to (a) b^n_{00}, (b) b^n_{0j}, (c) b^n_{j0}, and (d) b^n_{jk}.

components. The left-hand and right-hand eigenvectors with zero eigenvalue are

$$u=(-2,1,\ldots,1),$$

and

$$v=\frac{1}{2c_3+mc_2}\begin{bmatrix}-c_3\\ c_2\\ \cdot\\ \cdot\\ \cdot\\ c_2\end{bmatrix}.$$

These are normalized so that

$$\begin{aligned}\Theta^{\mu_1\mu_2}&=\sum_j u_j O_j^{\mu_1\mu_2}\\ &=T^{\mu_1\mu_2}+g^{\mu_1\mu_2}\cdots,\end{aligned}$$

where $T^{\mu\nu}$ is the energy-momentum tensor. With this normalization, $A^2=2$. Now

$$f^2_{1\text{ or }2}(1,0)=\sum_j f^2_{1\text{ or }2,j}(1,0)v_j\,,$$

where the $f^2_{1\text{ or }2,j}$ are related to the coefficients of O_j^2 in the free-field operator-product expansion. Explicitly $f^2_{1,0}=f^2_{2,0}=0$ and $f^2_{1,j}=f^2_{2,j}=4Q_j{}^2$.

So

$$\lim_{-q^2\to\infty}\int_0^1 W_1(q^2,x)x\,dx=\tfrac{1}{2}a\,,$$

$$\lim_{-q^2\to\infty}\int_0^1 \nu W_2(q^2,x)dx=m_p a\,,$$

where the constant a is

$$a=\frac{2mc_2\langle Q_j{}^2\rangle}{2c_3+mc_2}$$

with $\langle Q_j{}^2\rangle$ the average square quark charge.

Equation (7) describes a rather singular situation in the $-q^2\to\infty$ limit. Evidently, as $-q^2$ increases the structure functions become sharply peaked at $x=0$. The limiting form can be described by the singular "scaling functions" $2xF_1(x)=F_2(x)=a\delta(x)$.[12] For the specific model we discussed earlier, with an SU(3) gauge group and three triplets of quarks with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, $a=\frac{4}{25}$.

ACKNOWLEDGMENTS

T. Appelquist has helped us with advice and encouragement from the beginning of this work. We are also grateful for helpful conversations with A. De Rújula, H. Schnitzer, and S. Weinberg. We thank A. Zee and T. Appelquist for pointing out an error in an earlier version of this work.

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*Work supported in part by the Air Force Office of Scientific Research under contract F44620-70-C-0030 and the National Science Foundation under grant GP30819X.

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[2]T. Appelquist and H. Georgi, Phys. Rev. D 8, 4000 (1973); A. Zee, *ibid.* 8, 4038 (1973).

[3]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973), discuss how a massless renormalized field theory might indeed describe massive particles and how, in the process, a dimensionless coupling constant is necessarily fixed.

[4]Using techniques just developed by S. Weinberg [Phys. Rev. D 8, 3497 (1973)], it is possible to include fermion mass terms without complicating the formalism. The results for the leading terms are the same as in the zero-mass theory.

[5]N. Christ, B. Hasslacher, and A. H. Mueller, Phys. Rev. D 6, 3543 (1972).

[6]After this paper was submitted for publication we received preprints by D. Gross and F. Wilczek [Phys. Rev. D 8, 3633 (1973); *ibid.* (to be published] in which these coefficients are calculated, in agreement with our results.

[7]With the conventional choice of the renormalized charge g, this is only true to lowest order in g. Since we only work to lowest order in g here, all our manipulations are justified. Furthermore, it may be possible, with an unconventional choice for g, to make this statement true to all orders.

[8]D. Gross and S. B. Treiman, Phys. Rev. D 4, 1059 (1971).

[9]We use the fact that u_n^α and v_n^α are independent of g, which is true when we only calculate γ^n to second order in g.

[10]Actually this is not quite right. We have used perturbation theory to compute

$$\int_0^t dt'\gamma_n^\alpha(\bar{g}(g,t'))\,,$$

but if γ is not identically zero, this involves contributions from the small-t' region where perturbation theory is not relevant. The net effect is to introduce an unknown multiplicative constant in the asymptotic expression for f (see Ref. 6). Since all terms with nonzero γ appear in the forward Compton amplitude multiplied by unknown matrix elements [see Eq. (4)], this effect can be ignored.

[11]Also $b=\frac{11}{3}c_1-\frac{4}{3}mc_2$.

[12]Here

$$F_1(x)=\lim_{-q^2\to\infty}W_1(q^2,x)$$

and

$$F_2(x)=\frac{1}{m_p}\lim_{-q^2\to\infty}\nu W_2(q^2,x)$$

in the sense of distribution theory. Since these limits do not exist as functions, true scaling is never realized. Note that these F's differ by a factor of two from those of Ref. 6.