

**Hidden  $SU(N)$  glueball dark matter**Amarjit Soni<sup>1</sup> and Yue Zhang<sup>2</sup><sup>1</sup>*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*<sup>2</sup>*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*

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We investigate the possibility that the dark matter candidate is from a pure non-Abelian gauge theory of the hidden sector, motivated in large part by its elegance and simplicity. The dark matter is the lightest bound state made of the confined gauge fields, the hidden glueball. We point out that this simple setup is capable of providing rich and novel phenomena in the dark sector, especially in the parameter space of large  $N$ . They include self-interacting and warm dark matter scenarios, Bose-Einstein condensation leading to massive dark stars possibly millions of times heavier than our sun giving rise to gravitational lensing effects, and indirect detections through higher dimensional operators as well as interesting collider signatures.

DOI: [10.1103/PhysRevD.93.115025](https://doi.org/10.1103/PhysRevD.93.115025)**I. INTRODUCTION**

An outstanding issue of fundamental importance in particle physics is the nature of the dark matter (DM). This question is particularly intriguing and perplexing, given the preponderance of DM over visible matter and its profound gravitational effects throughout the evolution of the universe.

In this work, we investigate the viability of the dark matter candidate from the hidden sector with a non-Abelian gauge symmetry, a minimal theory with nontrivial mass scale. The gauge group is chosen to be  $SU(N)$ , and, for simplicity, neither fermions nor any other particle is introduced in that sector. The dark matter is the lightest hidden glueball state, which is likely a scalar field, and a non-perturbative bound state made of a pair of confined gauge fields. This is a very simple setup with only a handful of parameters, which are the intrinsic scale  $\Lambda$ , the number of colors  $N$ , and  $\theta$ —for the T and P-odd  $\theta$ -term in the hidden sector. They control the mass and all the couplings of the hidden glueball dark matter (GDM), named  $\phi$  hereafter.

In spite of the simplicity of this setup, we will show that the hidden glueball indeed satisfies all the conditions for a dark matter candidate. Moreover, such dark matter could be both self-interacting and warm, thus safely evading all the potential problems of the usual collisionless cold dark matter. The scalar GDM could have the novel feature of Bose-Einstein condensation into compact objects thus plausibly leading to interesting gravitational effects such as microlensing. It could also be tested in particle physics experiments if there exist interactions of it with standard model particles via higher dimensional operators. We will elaborate on these points in order in the following sections.

In passing, we want to briefly say that we are aware of several other works which include a non-Abelian dark

sector in their overall setup. The hidden glueball as dark matter was first mentioned in [1], but at that time the cosmological observation data were very preliminary. There are more recent works which involve, in addition, other elaborate features with significantly different phenomenology from this study; see, e.g. [2–6]. We emphasize that to the best of our knowledge, no existing work in the literature is devoted to the possibility of DM simply being in a pure  $SU(N)$  gauge theory, which is what we are studying here. In our following discussions, we point out the impact of the number-of-color parameter  $N$ , and use the recent results on bullet cluster and Lyman- $\alpha$  forest observations to set important constraints on the GDM parameter space.

**II. HIDDEN GLUEBALL OF DARK MATTER**

In this work, we consider a dark matter candidate from a very simple setup, a hidden sector non-Abelian gauge symmetry with only gauge fields and without fermions. The Lagrangian of the model is

$$\mathcal{L} = -\frac{1}{4} H_{\mu\nu}^a H^{a\mu\nu}, \quad (1)$$

where  $H_{\mu\nu}^a$  is the gauge field strength of the group  $SU(N)$ , with an unspecified value of  $N$  to be determined later. As is well known the gauge coupling  $g_h$  becomes large at low energy scale and dimensional transmutation generates a scale  $\Lambda$  for the theory, similar to the emergence of the QCD scale. Around the scale  $\Lambda$ , the physical degrees of freedom turn into a tower of hidden glueballs. From the knowledge based on existing calculations the lowest lying glueball states when  $\theta = 0$  carry quantum numbers  $J^{PC} = 0^{++}$  or  $0^{-+}$  [7,8]. Their masses depend on the two parameters of the theory,  $\Lambda$  and  $N$ . Also from lattice calculations [9,10], the lightest glueball masses approach a constant at large  $N$ ,

and can be parametrized as  $m = (\alpha + \beta/N^2)\Lambda$  where  $\alpha, \beta$  are order one parameters. In general, we could also introduce the  $\theta$ -term in the above Lagrangian, which is  $C$  even and  $P$  odd. It can mix the  $0^{++}$  and  $0^{-+}$  states and the lightest glueball state is then not an eigenstate under  $P$ .

We argue that within this simple setup the lightest hidden glueball state  $\phi$  could be a candidate for dark matter.<sup>1</sup> It could be cosmologically long lived. As the lightest state, there is nothing in the hidden sector that  $\phi$  could decay into. It is possible for  $\phi$  to decay into two gravitons, and this decay rate can be estimated as  $\Gamma_\phi \sim m^5/M_{pl}^4 \sim \tau_U^{-1}(m/10^7 \text{ GeV})^5$ , where  $\tau_U = 10^{17}$  sec is the age of our Universe. The lifetime of  $\phi$  against gravitational decay can be long enough if its mass is less than  $10^7$  GeV. Moreover, the hidden glueball  $\phi$  particles could have the correct relic density and be (non)relativistic enough as will be elaborated in the next section. So far, we have not written down any interactions between the hidden sector and the visible sector, which by gauge invariance is only possible in the form of higher dimensional operators. We will explore the resulting experimental bounds in an example where the hidden GDM  $\phi$  decays into photons.

### III. SELF-INTERACTING DARK MATTER

The effective potential of a real scalar  $\phi$  takes the form

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{3!}\lambda_3\phi^3 + \frac{1}{4!}\lambda_4\phi^4 + \frac{1}{5!}\lambda_5\phi^5 + \dots, \quad (2)$$

where the  $\dots$  represent higher power terms. It is useful to consider the large  $N$  behavior of these couplings,

$$\lambda_3 = \frac{\kappa_3 m}{N}, \quad \lambda_4 = \frac{\kappa_4}{N^2}, \quad \lambda_5 = \frac{\kappa_5}{mN^3}, \quad (3)$$

where  $\kappa_{3,4,5}$  are order one parameters to be determined from non perturbative calculations. From these interactions, we could obtain the  $2 \rightarrow 2$  elastic scattering cross section of  $\phi$  as a function of the two model parameters,  $m(\Lambda)$  and  $N$ ,

$$\sigma_{2 \rightarrow 2} \sim 1/(m^2 N^4). \quad (4)$$

The self-interacting dark matter scenario has been proposed [11] to reconcile the core/cusp problem in dwarf galaxy observations and simulations. For this scenario to work, the elastic scattering cross section of dark matter must lie in the range  $0.1 \text{ cm}^2/\text{gram} < \sigma_{2 \rightarrow 2}/m < 10 \text{ cm}^2/\text{gram}$ . This requirement puts a correlated constraint on  $m$  and  $N$ ,

$$m \sim 0.1 \text{ GeV} \cdot N^{-4/3}. \quad (5)$$

<sup>1</sup>If after mixing of the  $0^{++}$  and  $0^{-+}$  glueball states, the heavier mass eigenstate is kinematically forbidden to decay into two  $\phi$ 's, it can also be stable and be the dark matter. In this case, we could have two components of dark matter existing in nature.

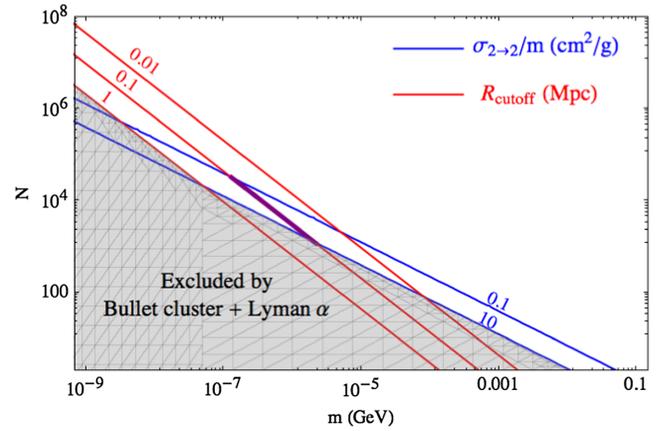


FIG. 1. The parameter space of  $m$  versus  $N$  where the lightest hidden glueball could be a self-interacting and/or warm dark matter candidate. The two blue curves correspond to constant values a DM self-interaction cross section,  $\sigma_{2 \rightarrow 2}/m = 0.1, 10 \text{ cm}^2/\text{gram}$ , respectively. Self-interacting DM lives between the blue curves. The red curves correspond to constant values of damping scale in the power spectrum,  $R_{\text{cutoff}} = 0.01, 0.1, 1 \text{ Mpc}$ , respectively. Warm DM lives along the middle red curve. The glueball dark matter can be both self-interacting and warm at the intersection of the two regions (thick purple curve). In the gray shaded region, the dark matter either has too strong self interaction and is excluded by the bullet cluster observation, or is too warm and excluded by the observation of the Lyman- $\alpha$  forest.

This region is shown between the blue curves in Fig. 1. Below the blue curves in the gray shaded region, the dark matter has too strong of a self-interaction and is excluded by the bullet cluster observation [12].

### IV. SELF-HEATING AND WARM DARK MATTER

In addition to elastic scattering, the effective interactions in (2) also allow  $\phi$  to have the inelastic  $3 \leftrightarrow 2$  annihilation, which changes the  $\phi$  particle number. The analog of the cross section could be estimated as

$$\sigma_{3 \rightarrow 2} \sim 1/(m^5 N^6). \quad (6)$$

The  $3 \rightarrow 2$  reaction rate is given by  $\Gamma_{3 \rightarrow 2} = n_\phi^2 \sigma_{3 \rightarrow 2}$ , where  $n_\phi$  is the  $\phi$  number density in the universe. This interaction could play an important role in the velocity dispersion of dark matter in the early universe, because after each  $3 \rightarrow 2$  reaction, the two outgoing  $\phi$  particles are relativistic. If this process has a larger reaction rate than the Hubble expansion, the annihilation will keep heating up the  $\phi$  particles until it reaches the balance with the inverse process where two energetic  $\phi$ 's annihilate into three.

Gauge invariance dictates the interactions between the SM and hidden sector to take the form  $H_{\mu\nu} H^{\mu\nu} \mathcal{O}_{\text{SM}}$ . They will cause the dark matter  $\phi$  to decay and thus are highly constrained as we show below. This makes the early

universe history of dark matter in our model very different from the one considered in [13]. Next, we assume that there are no interactions for  $\phi$  and SM particles to exchange heat in equilibrium; therefore the entropy of the  $\phi$  particles is conserved,  $\frac{d}{da}[(\rho_\phi + p_\phi)a^3/T] = 0$ . For nonrelativistic  $\phi$ 's, i.e.,  $T_\phi \ll m$ , one could derive

$$T_\phi(a) \simeq T_\phi(a_0) \left(1 + \frac{3T_\phi(a_0)}{m} \ln \frac{a}{a_0}\right)^{-1}, \quad (7)$$

where  $a$  is the Hubble radius at a given time in the early universe ( $a = 1$  today), and  $a_0 < a$  corresponds to an earlier time. This means the  $\phi$  particles thermalize to a temperature which drops more slowly than  $1/(\ln a)$  with the expansion of the universe, as first noted in [1]. In contrast, the temperature of the photons falls as  $T_\gamma \sim 1/a$ , thus leading to the interesting possibility that the hidden and SM sectors have their own temperatures and evolve separately.

It is useful to expand the energy density and pressure of  $\phi$  to next order in  $T_\phi/m$ ,  $\rho_\phi = mn_\phi(1 + 3T_\phi/(2m))$ ,  $p_\phi = n_\phi T_\phi$ . With this one can obtain the evolution equation of  $n_\phi$  as a function of  $a$ ,

$$\frac{d(n_\phi a^3)}{da} \simeq -\frac{(n_\phi a^3) 3T_\phi}{a m}. \quad (8)$$

The message here is that the number density of  $\phi$  dilutes faster than  $a^{-3}$ ; thus the total number of  $\phi$  is still decreasing while the  $3 \rightarrow 2$  annihilation is in equilibrium. The consumption of  $\phi$ 's is used to maintain the temperature of the remaining  $\phi$  particles. The final DM relic density is given by  $n_\phi$  at the decoupling of  $3 \rightarrow 2$  annihilation. In Fig. 2, we

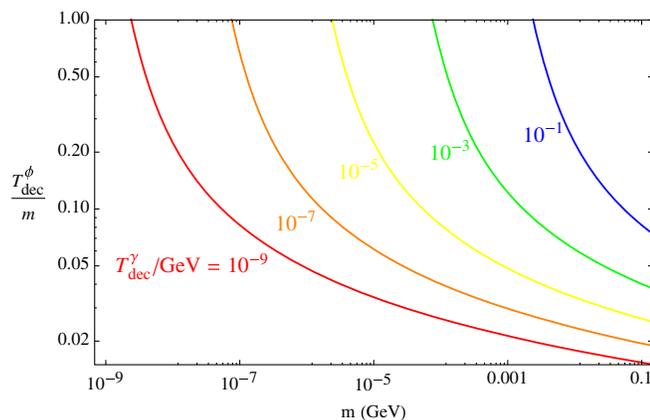


FIG. 2. Ratio of temperature  $T_\phi$  to the mass  $m$  of  $\phi$  particles at the decoupling of  $3 \rightarrow 2$  annihilation that could give the correct dark matter relic density. The curves correspond to different photon temperatures ( $T_{\text{dec}}^\gamma$ ) at this epoch. Roughly,  $T_\phi$  is only 1 order of magnitude below the mass, and the  $\phi$  particles remain heated before the decoupling.

show the ratio of the decoupling temperature  $T_{\text{dec}}^\phi$  to the mass of  $\phi$  that is needed to give the correct dark matter relic density, for different values of the photon temperature at this epoch. The initial conditions that give the desired values of  $T_{\text{dec}}^\phi$  and  $T_{\text{dec}}^\gamma$  might be set by reheating the SM and dark sectors to different temperatures after the inflation. See e.g. [14], or through the freeze-in mechanism.

Before the  $3 \rightarrow 2$  decoupling, the temperature  $T_\phi$  stays roughly 1 order of magnitude below the mass  $m$ . The strongly coupled  $\phi$  particles form a fluid with a large speed of sound  $c_s = \sqrt{2T_\phi/(3m)} \sim 0.3c$ . It allows the perturbations to the density of  $\phi$  within one Hubble patch to be smoothed out efficiently via collisional damping, thus offering the opportunity for  $\phi$  to be a warm dark matter candidate.

To find when the  $3 \rightarrow 2$  process decouples, or the corresponding temperature of photon  $T_{\text{dec}}^\gamma$ , we first express the  $3 \rightarrow 2$  rate in terms of the photon temperature,

$$\Gamma_{3 \rightarrow 2} = n_\phi^2 \sigma_{3 \rightarrow 2} \simeq 10^{-17} \text{ GeV}^2 T_\gamma^6 / (m^7 N^6). \quad (9)$$

When it is equal to the Hubble rate, we get the photon temperature at the decoupling of the  $3 \rightarrow 2$  reaction

$$T_{\text{dec}}^\gamma \simeq 1 \text{ keV} [m/(1 \text{ keV})]^{7/4} [N/(10^4)]^{3/2}. \quad (10)$$

The collisional damping length scale (measured today) is determined by the Hubble radius at the  $3 \rightarrow 2$  decoupling

$$R_{\text{cd}} = \frac{1}{H(T_{\text{dec}}^\gamma)} \frac{T_{\text{dec}}^\gamma}{2.7 \text{ K}} \simeq 0.1 \text{ Mpc} \left( \frac{1 \text{ keV}}{T_{\text{dec}}^\gamma} \right). \quad (11)$$

After the  $3 \rightarrow 2$  decoupling, the temperature of  $\phi$  will drop as  $1/a^2$  such that the velocity redshifts as  $1/a$ . We calculate the free streaming length of  $\phi$  particles from this time,  $t_{\text{dec}}^{3 \rightarrow 2}$ , to the time of matter-radiation equality,  $t_{\text{eq}}$ . This corresponds to the collisionless damping scale,

$$R_{\text{fs}} = \int_{t_{\text{dec}}}^{t_{\text{eq}}} \frac{v(t)}{a(t)} dt = \frac{2v_{\text{eq}} t_{\text{eq}}}{a_{\text{eq}}} \ln \left[ \frac{a_{\text{eq}}}{a_{\text{dec}}} \frac{1 + \sqrt{1 + v_{\text{eq}}^2}}{1 + \sqrt{1 + v_{\text{dec}}^2}} \right].$$

At matter-radiation equality  $t_{\text{eq}} = 2 \times 10^{12}$  sec,  $a_{\text{eq}} = 1/(1 + z_{\text{eq}})$ ,  $z_{\text{eq}} \simeq 3360$ , and  $v_{\text{eq}} = v_{\text{dec}} a_{\text{dec}}/a_{\text{eq}}$ . In principle, the distance  $\phi$  travels would be even shorter than  $R_{\text{fs}}$  because of the  $2 \rightarrow 2$  scatterings which if frequent would make the  $\phi$  particles diffuse rather than free stream. In practice, we find that for most of the parameter space of interest to this study,  $R_{\text{fs}} \lesssim R_{\text{cd}}$ . Therefore, it is  $R_{\text{cd}}$  in (11) that determines the actual damping scale  $R_{\text{cutoff}}$  in the dark matter power spectrum.

For  $\phi$  to be the warm dark matter which solves the missing satellite problem, it is required that  $R_{\text{cutoff}} = R_{\text{cd}} \sim 0.1 \text{ Mpc}$  [15]. The contours of fixed  $R_{\text{cutoff}}$  are shown by

the red curves in Fig. 1. We further find that for  $m \in (0.1, 10)$  keV and  $N \in (10^5, 10^3)$  (along the thick purple curve), the hidden glueball  $\phi$  dark matter qualifies to be both self-interacting and warm, thus plausibly solving all the small scale structure problems. Below the red curves in the shaded region, the damping scale  $R_{\text{cutoff}}$  becomes too large and is in contradiction with the Lyman- $\alpha$  forest observation [16].

Moreover, if the dark matter still has non-negligible velocity and fast  $2 \rightarrow 2$  self-interactions during the formation of the cosmic microwave background (CMB), it might leave an imprint in the CMB spectrum. We leave this interesting possibility for a future detailed study.

## V. COMPACT BOSON STARS

So far, we have not considered any interactions between the hidden  $SU(N)$  sector and SM particles. In the absence of such interactions, we would look for the dark matter only through gravitational effects. It has been shown that the dark scalar field could have Bose-Einstein condensation and form massive compact objects such as boson stars [17,18]. This may result in very dramatic gravitational effects in our universe today such as microlensing [19,20].

The mass range of the boson star depends on whether the self-interaction of  $\phi$  is repulsive or attractive. The size of the boson star is typically much larger than the inverse of the glueball mass. In the hidden glueball model Eq. (2), at low momentum transfer the effective coupling of the  $\phi^4$  self-interaction is

$$\lambda_{\text{eff}} = \lambda_3^2/(2m^2) + \lambda_4 = (\kappa_3^2/2 + \kappa_4)/N^2. \quad (12)$$

Non perturbative calculations are needed to reliably determine the size and signs of  $\kappa_3$ ,  $\kappa_4$ , and in turn the fate of the condensate.

The opportunity to observe the microlensing effect arises if there are repulsive self-interactions for the  $\phi$  field, with  $\lambda_{\text{eff}} > 0$ . In this case, it has been calculated [18] that the boson star mass from condensation lies in the range  $1-10^8 M_\odot$ , for the glueball dark matter with mass from the GeV to 0.1 keV scale. In particular, in the interesting window of Fig. 1 where the dark matter is both self-interacting and warm, the corresponding boson star mass is in the range  $10^6-10^8 M_\odot$ .

The sign of the  $\lambda_4$  term is closely related to the scattering length in glueball-glueball scattering, which could be determined using nonperturbative methods [21]. In Ref. [22], an effective picture is discussed where the intrinsic scale of  $SU(N)$  theory is connected to the vacuum expectation of the scalar glueball. In this case, one finds  $\lambda_4 > 0$ , and this suggests that  $\lambda_{\text{eff}}$  is positive. On the other hand, if  $\lambda_{\text{eff}} < 0$ , the boson star mass would be too small to have an observable effect.

## VI. INTERACTIONS WITH THE SM THROUGH HIGHER DIMENSIONAL OPERATORS

In general, there may exist interactions between the hidden sector and the SM sector. This may allow the glueball dark matter to be discovered through means other than gravitational effects. However, we do not want to introduce other particles just to facilitate these interactions, since as explained before, we want to explore how far our setup with just a simple pure  $SU(N)$  gauge theory can go in addressing the DM issue. So, without introducing additional particles, gauge invariance dictates that these interactions may arise via higher dimensional operators,

$$\mathcal{L}_{\text{int}} = (1/M^n) H_{\mu\nu} H^{\mu\nu} \mathcal{O}_{\text{SM}}, \quad (13)$$

where  $M$  is the cutoff scale. There are many choices for the  $\mathcal{O}_{\text{SM}}$  part. Here we discuss one representative which couples the hidden sector directly to photons,

$$\mathcal{L}_{\text{int}} = \frac{1}{M^4} H_{\mu\nu} H^{\mu\nu} (F_{\alpha\beta} F^{\alpha\beta}) \rightarrow \frac{Nm^3}{M^4} \phi F_{\alpha\beta} F^{\alpha\beta}, \quad (14)$$

where  $F$  is the photon field strength. In the second step, we go to the low scale where  $\phi$  is the lightest glueball field. In the following, we choose the value of  $N$  making  $\phi$  a self-interacting dark matter,  $N \simeq \text{Max}[(m/0.1 \text{ GeV})^{-3/4}, 2]$ . It is also worth noting that the effective interaction of  $\phi$  is proportional to powers of its mass  $m^3$ .

From Eq. (14), the decay rate of  $\phi$  into two photons (see the left diagram in Fig. 3) is

$$\Gamma_{\phi \rightarrow \gamma\gamma} = \frac{N^2 m^9}{4\pi M^8}. \quad (15)$$

There are experimental searches for monochromatic photons from decaying dark matter, from cosmic gamma rays to x rays and even extragalactic background lights [23–27]. They give the strongest constraints on the scale  $M$  for the dark matter  $\phi$  mass above  $\sim 100$  keV. We show these constraints in Fig. 4.

For lower  $\phi$  masses, we find the energy loss constraints of stars place a stronger lower limit. The relevant reaction is the analog of the Primakoff-type process  $e + \gamma \rightarrow e + \phi$ , as shown by the right diagram of Fig. 3. The cross section was calculated in [28],

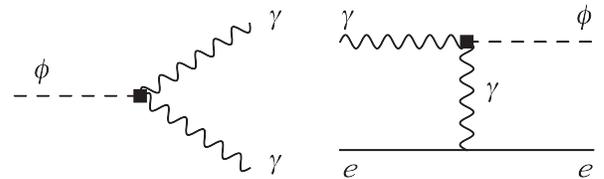


FIG. 3. Feynman diagrams for  $\phi$  decay and production in stars from Eq. (14). The relation between the decay rate and cross section is dictated by Eq. (16).

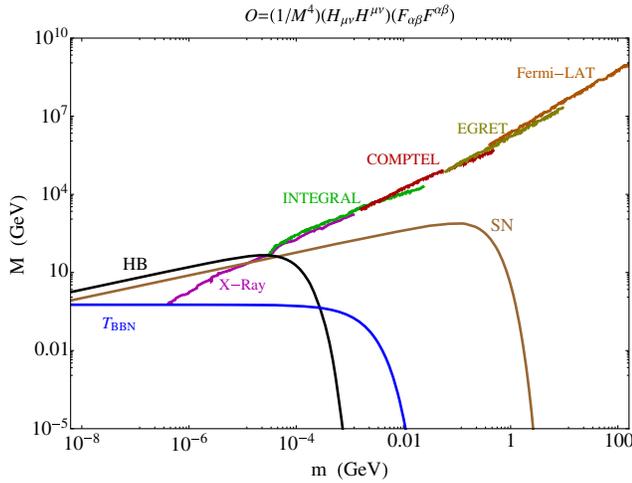


FIG. 4. Lower bounds on the cutoff scale  $M$ . Cosmic ray photon observations constrain glueball dark matter decay into photons, and from right to left, the curves correspond to constraints from Fermi-LAT, EGRET, COMPTEL, INTEGRAL, x ray. The black (brown) solid curve is the lower bound on  $M$  from the energy loss argument of HB (SN). The blue curve represents the requirement that the hidden sector is not thermalized with the SM sector below the BBN temperature.

$$\sigma v = 64\pi\alpha \frac{\omega\Gamma_{\phi\rightarrow\gamma\gamma}}{m^2} \frac{(\omega^2 - m^2)^{1/2}(\omega - m)}{(m^2 - 2\omega m)^2}, \quad (16)$$

where  $\omega$  is the energy of the incoming photon and  $m$  is the mass of glueball dark matter. To calculate the rate of energy loss from the star via  $\phi$  emission, we first average the  $\sigma v \cdot \omega$  over the thermal photon energy distribution, and then the energy loss rate per unit volume is given by  $\Phi = n_e n_\gamma \langle \sigma v \cdot \omega \rangle$ . We consider the energy loss argument [29] of horizontal branch stars (HB) and the cooling of type-II supernova (SN). For HB, the core temperature is 10 keV, the mass density is  $10^4$  gram/cm<sup>3</sup>, and the energy loss rate per unit volume is required to be  $\Phi < 10^{-42}$  MeV<sup>5</sup>. For SN, the core temperature is 30 MeV, both photons and electrons are thermalized, and the energy loss rate is required to be  $\Phi < 10^{-14}$  MeV<sup>5</sup>. Their constraints on  $M$  (lower bound) are shown in Fig. 4. Not-too-much energy loss of HB sets the strongest lower bound on  $M$  for  $\phi$  mass below  $\sim 100$  keV. For the model to be realistic in cosmology, the hidden sector must not thermalize with the SM sector, at least not since the onset of BBN. We find this to be a subdominant constraint (shown by the blue curve in Fig. 4).

The operators in Eq. (14) not only lead the glueball dark matter particle to decay, but also allow it to scatter with SM particles by virtue of the expansion  $H_{\mu\nu}H^{\mu\nu} \sim Nm^3\phi + m^2\phi^2 + \dots$ . Given the above lower bounds on the cutoff scale  $M$ , we find the direct detection cross section for the glueball dark matter is more than 10 orders of magnitude below the current LUX bound [30]. This is consistent with

the null results so far in the direct detections. It also implies that if the future direct detection experiments discover the dark matter, it cannot originate from our dark matter candidate.<sup>2</sup>

From Fig. 4, we find that for the dark matter mass  $m$  in the range keV to MeV, the cutoff  $M$  is allowed to be as low as the weak (or TeV) scale. The effective operator in Eq. (14) could be generated by integrating out a heavy particle  $X$  in the ultraviolet theory, which carries both electromagnetic charge and color under the hidden  $SU(N)$  gauge group. If a pair of  $X\bar{X}$  can be produced at colliders, they would eventually form a heavy  $X$ -onium bound state and annihilate away into the hidden glueball dark matter or photons. The final states will exhibit exotic signatures like the quirks [31,32].

Furthermore, if the heavy  $X$  particle is a fermion and also carries color under the  $SU(3)_c$  of QCD, the effective Lagrangian will contain an operator  $(1/M^4)(H\tilde{H})(G\tilde{G})$  [similar to Eq. (12) of Ref. [33]]. In the presence of the  $\theta H\tilde{H}$  term from the hidden  $SU(N)$  theory, it induces an effective  $\theta_{\text{QCD}}G\tilde{G}$  term, with  $\theta_{\text{QCD}} \sim (m/M)^4\theta$ , and makes a contribution to the neutron electric dipole moment (nEDM). The important point we want to make here is that the nEDM bound does not require the  $\theta$  parameter of  $SU(N)$  to be unnaturally small, unlike  $\theta_{\text{QCD}}$ . The current experimental upper bound on nEDM of around  $10^{-26}$  e cm [34] translates, by the arguments of [35], into  $\theta_{\text{QCD}} \lesssim 10^{-13}$ . From the above relation between  $\theta_{\text{QCD}}$  and  $\theta$ , we find that  $\theta$  is allowed to be order one if  $m/M \lesssim 10^{-3}$ , which is always satisfied from Fig. 4.

## VII. GDM DECAY INSIDE A DARK STAR

In the last section, we have discussed the photon line searches and constraints on hidden glueball dark matter decay, which could most frequently happen at the center of the galaxy. The other possibility is that, if the scalar GDM undergoes the Bose-Einstein condensation and forms the dark stars as we also discussed, their decays could contribute to new (pointlike) sources of cosmic ray emissions.

Here we consider the decay of GDM inside a dark star into SM neutrinos, and use the Super-Kamiokande (SK) results to constrain the distance of the dark star from us as a function of GDM mass and lifetime. The effective operator for GDM decay could be of the form  $(H_{\mu\nu}D_\rho H^{\mu\nu})(\bar{L}\gamma^\rho(a + b\gamma_5)L)/M^4$ , where  $L$  is the SM lepton doublet. The SK experiment has taken data for 1679.6 days with an effective area of  $10^3$  m<sup>2</sup>, and set an upper limit on the number of high energy neutrinos above a GeV beyond the atmospheric neutrino background, which

<sup>2</sup>A positive direct detection signal would imply our hidden glueball dark matter decays too fast and cannot comprise all the dark matter relic density. There then needs to be some other components of dark matter too.

is around  $\sim 10$  [36,37]. We will consider the GDM mass above a few GeV scale. From the discussion of [18], the largest allowed dark star mass as a function of the GDM mass is

$$M_{\text{DS}} \approx 0.01 M_{\odot} \left( \frac{10 \text{ GeV}}{m} \right)^{5/4}. \quad (17)$$

If the GDM particle has a lifetime  $\tau$ , and the dark star is of distance  $d$  away from the Earth, then the number of neutrinos from all the GDM decay within a time interval  $\Delta t \approx 10^8$  sec, and reaches the detector with effective area  $A$ , is

$$N_{\text{SK}}^{\nu} \approx 10^{45} \left( \frac{10^{18} \text{ sec}}{\tau} \right) \left( \frac{10 \text{ GeV}}{m} \right)^{9/4} \frac{A}{4\pi d^2}. \quad (18)$$

The weak interaction cross section for high energy neutrinos to interact with nucleons inside the SK detector is  $\sigma \sim 10^{-38} E_{\nu} \text{ cm}^2/\text{GeV}$  [38]; thus the probably for each neutrino to react to produce a signal is  $P \sim 10^{-13} (m/10 \text{ GeV})$ , where we have traded the neutrino energy for the dark matter mass. The SK bound on the number of events then requires  $N_{\text{SK}}^{\nu} P \lesssim 10$ , which translates into

$$d \gtrsim 10^{-3} \text{ kpc} \left( \frac{10^{18} \text{ sec}}{\tau} \right)^{1/2} \left( \frac{10 \text{ GeV}}{m} \right)^{5/8}. \quad (19)$$

Note that the distance of the Galactic Center to the Earth is around 8 kpc, so the dark star is allowed to be much closer and well within our Galaxy.

## VIII. SUMMARY

In this paper, we investigate the physics of  $SU(N)$  glueball dark matter from a pure gauge theory non-Abelian hidden sector. In spite of the simple setup, with few parameters, there are quite a few novel features of this dark matter candidate. We have discussed the conditions for it to be self-interacting and/or warm dark matter. The glueball dark matter could also condense into more compact objects like boson stars and be observed by gravitational lensing effects. Therefore, our model can naturally accommodate the fact that there is only gravitational evidence for dark matter so far [39,40]. It could also interact with the standard model sector via higher dimensional operators and subject to traditional direct searches for light scalar dark particles. The direct detection cross section of the glueball dark matter is constrained to be well below the experimental sensitivity, now as well as for the foreseeable future. We also comment on the possible UV origin of the higher dimensional operators leading to interesting collider signatures.

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