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Erratum: Post-Newtonian, quasicircular binary inspirals in quadratic modified gravity [Phys. Rev. D 85, 064022 (2012)]

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In this erratum, we point out that the correction to the gravitational energy flux from compact binary inspirals in dynamical Chern-Simons gravity enters at third post-Newtonian order instead of second post-Newtonian order. We also correct the scalar energy flux at second post-Newtonian order by considering terms that we missed in the original paper.

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I. GRAVITATIONAL RADIATION

In [1], we claimed that the correction to the gravitational energy flux, i.e. the energy flux from binary inspirals induced by the metric perturbation in dynamical Chern-Simons (dCS) gravity, enters at second post-Newtonian (2PN) order. Here, we point out that there are terms that we overlooked, and if we take these terms into account properly, the 2PN terms cancel and the correction enters at 3PN order.[1]

First, let us remind ourselves what we did in [1]. Our starting point was perturbing the metric around a flat background as

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{GR}} + \mathfrak{h}_{\mu\nu}, \qquad g_{\mu\nu}^{\mathrm{GR}} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $h_{\mu\nu}$ and $\mathfrak{h}_{\mu\nu}$ are the general relativity (GR) and dCS metric perturbations, respectively. We worked in harmonic gauge, $\bar{h}^{\mu\nu}{}_{,\nu} = 0 = \bar{\mathfrak{h}}^{\mu\nu}{}_{,\nu}$, where we defined the metric perturbations via

$$\bar{h}^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{-g_{\mathrm{GR}}}\, g_{\mathrm{GR}}^{\mu\nu}, \tag{2}$$

$$\bar{\mathfrak{h}}^{\mu\nu} \equiv (\eta^{\mu\nu} - \sqrt{-g}\, g^{\mu\nu}) - \bar{h}^{\mu\nu}. \tag{3}$$

We then expanded the modified Einstein equations, focusing on the dCS correction and keeping terms up to linear order in $h_{\mu\nu}$ and $\bar{\mathfrak{h}}_{\mu\nu}$,

$$\kappa\, \square_\eta\, \bar{\mathfrak{h}}_{\mu\nu} = 2\alpha_4 \tilde{\mathcal{K}}_{\mu\nu}^{(1)} - T_{\mu\nu}^{(\vartheta)}, \tag{4}$$

where $\square_\eta$ is the d'Alembertian operator of flat spacetime and

[1] Recall that a term in an expression is said to be of Nth PN order if it is proportional to v^{2N} relative to the leading-order term in that same expression.

$$\tilde{\mathcal{K}}^{(1)}_{\mu\nu} = \vartheta^{,\delta}{}_{,\sigma}\eta_{\nu\alpha}\epsilon^{\alpha\sigma\beta\gamma}(h_{\mu[\gamma,\beta]\delta} + h_{\delta[\beta,\gamma]\mu}) - 2\vartheta^{,\delta}\epsilon_{\delta\sigma\chi\mu}h^{\sigma}{}_{[\alpha}{}^{,\alpha\chi}{}_{\nu]} + \mathcal{O}(h^2) + (\mu\leftrightarrow\nu), \tag{5}$$

$$T^{(\vartheta)}_{\mu\nu} = \beta\left(\vartheta_{,\mu}\vartheta_{,\nu} - \frac{1}{2}\eta_{\mu\nu}\vartheta_{,\delta}\vartheta^{,\delta}\right) + \mathcal{O}(h). \tag{6}$$

We then solved this wave equation in the far zone (FZ), for field points much larger than the gravitational wave (GW) wavelength of the system, using a Green's function. The Green's function solution requires the integration of $\tilde{\mathcal{K}}^{(1)}_{\mu\nu}$ and $T^{(\vartheta)}_{\mu\nu}$, and thus of the fields $h_{\mu\nu}$ and ϑ evaluated in the near zone (NZ), i.e. at field point distances smaller than the GW wavelength of the system, but far from the location of the objects. To leading PN order, the NZ fields are given by

$$h^{\rm NZ}_{00} = 2\frac{m_1}{r_1} + \mathcal{O}\left(\frac{m_1^2}{r_1^2}\right) + (1\leftrightarrow2), \tag{7}$$

$$h^{\rm NZ}_{ij} = 2\frac{m_1}{r_1}\delta_{ij} + \mathcal{O}\left(\frac{m_1^2}{r_1^2}\right) + (1\leftrightarrow2), \tag{8}$$

$$\vartheta^{\rm NZ} = -\mu_1^i\partial_i\frac{1}{r_1} + \mathcal{O}\left(\frac{m_1^3}{r_1^3}\right) + (1\leftrightarrow2) = \frac{\mu_1^i n_1^i}{r_1^2} + \mathcal{O}\left(\frac{m_1^3}{r_1^3}\right) + (1\leftrightarrow2), \tag{9}$$

where μ_A^i is the NZ scalar dipole charge, with

$$\mu_A^i \equiv \frac{5}{2}\frac{\alpha_4}{\beta}\chi_A\hat{S}_A^i \tag{10}$$

for a black hole, where χ_A and $\hat{S}_A^i$ represent, respectively, the dimensionless spin parameter and the unit spin angular momentum vector of the Ath binary constituent. The dominant contribution to the FZ solution comes from the integration of the $T^{(\vartheta)}_{\mu\nu}$ term in Eq. (4), which is roughly of the following PN order:

$$\bar{\mathfrak{h}}^{\rm FZ}_{ij} \sim \delta_{ij}\int \vartheta^{\rm NZ}_{,k}\vartheta^{\rm NZ}_{,k}\frac{d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \propto \frac{1}{r}\frac{1}{b^3}\frac{1}{b^3}b^3 \propto \frac{v^6}{r}, \tag{11}$$

where b is the binary's separation, v is the binary's (relative) velocity and r is the FZ field point distance. Comparing this with the GR, leading PN contribution, $\bar{h}^{\rm FZ}_{ij} \sim v^2/r$, we concluded that the dCS correction first enters at 2PN order, i.e. at $\mathcal{O}(v^4/c^4)$ relative to the leading PN order GR contribution.

Now, let us look at terms that we neglected in Eq. (4). Let us begin by expanding out this equation to find

$$\kappa\,\Box_\eta\,\bar{\mathfrak{h}}_{\mu\nu} = S_{\mu\nu}, \tag{12}$$

$$S_{\mu\nu} \equiv 2\alpha_4\tilde{\mathcal{K}}^{(1)}_{\mu\nu} - T^{(\vartheta)}_{\mu\nu} - \mathfrak{h}T^{\rm mat}_{\mu\nu} - t^{(1)}_{\mu\nu} + \mathcal{O}(\mathfrak{h}^2). \tag{13}$$

One of the terms we neglected in Eq. (4) is the third one in Eq. (13), which comes from expanding the metric determinant in the term $(gT^{\rm mat}_{\mu\nu})$, where $T^{\rm mat}_{\mu\nu}$ is the matter stress-energy tensor (see the relaxed Einstein equations in GR given by Eqs. (2.4) and (2.5) in [2]). Another term we neglected is the fourth term in Eq. (13), which is the linear order in $\bar{\mathfrak{h}}$ piece of $t_{\mu\nu}$, where the latter nonlinear tensor is defined by

$$t_{\mu\nu} \equiv (-g)t^{\rm LL}_{\mu\nu} + \kappa(\tilde{h}_{\mu\alpha}{}^{,\beta}\tilde{h}_{\nu\beta}{}^{,\alpha} - \tilde{h}^{\alpha\beta}\tilde{h}_{\mu\nu,\alpha\beta}), \tag{14}$$

with $t^{\rm LL}_{\mu\nu}$ the Landau-Lifshitz pseudotensor, which depends on $\tilde{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu} + \bar{\mathfrak{h}}_{\mu\nu}$.

Clearly then, in order to solve Eq. (12) in terms of Green's functions, we need to integrate $(\mathfrak{h}T^{\rm mat}_{\mu\nu})$ and $t^{(1)}_{\mu\nu}$ in the FZ using the NZ solution for $\mathfrak{h}$. The latter is given by [3,4]

$$\mathfrak{h}^{\rm NZ} \sim \zeta_4 \frac{m_1^3}{r_1^3}\chi_1^2 + (1\leftrightarrow 2). \tag{15}$$

Denoting the contribution of the third and fourth terms in Eq. (13) to as $\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{ij}^{\rm mat}}$ and $\bar{\mathfrak{h}}_{ij}^{{\rm FZ},t_{ij}}$, respectively, one then finds

$$\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{ij}^{\rm mat}} \sim \int \mathfrak{h}^{\rm NZ} T_{ij}^{\rm mat}\frac{d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \propto \frac{1}{r}\frac{1}{b^3}v^2 \propto \frac{v^8}{r}, \tag{16}$$

$$\bar{\mathfrak{h}}_{ij}^{{\rm FZ},t_{ij}} \sim \int \mathfrak{h}_{kl}^{\rm NZ} h_{ij,kl}^{\rm NZ}\frac{d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \propto \frac{1}{r}\frac{1}{b^3}\frac{1}{b^3}b^3 \propto \frac{v^6}{r}, \tag{17}$$

where we used that $T_{ij}^{\rm mat} \sim \rho v_i v_j$ with ρ representing the energy density of one of the bodies. From the above equations, we see that the $(\mathfrak{h}T_{\mu\nu}^{\rm mat})$ and $t_{\mu\nu}^{(1)}$ terms give 3PN and 2PN corrections to the gravitational radiation, respectively. Notice that the latter is of the same order as the contribution from $T_{\mu\nu}^{(\vartheta)}$ that we calculated in [1].

Given this, what is the correct dCS corrections to the gravitational energy flux? One can answer this question by using a very useful (and standard) trick employed regularly in the PN expansion of GR. Due to the gauge condition $\bar{\mathfrak{h}}_{\mu\nu}{}^{,\nu} = 0$, the tensor $S_{\mu\nu}$ in Eq. (13) is conserved, i.e. $S_{\mu\nu}{}^{,\nu} = 0$. Thanks to this conservation law, one can write (see e.g. [5])

$$\bar{\mathfrak{h}}_{ij}^{\rm FZ} \sim \frac{1}{r}\int S_{ij} d^3x' \sim \frac{1}{r}\frac{\partial^2}{\partial t^2}\int S_{00} x'^i x'^j d^3x', \tag{18}$$

after integrating by parts a few times. Then, denoting the contribution of $T_{00}^{(\vartheta)}$, $T_{00}^{\rm mat}$ and t_{00} to $\bar{\mathfrak{h}}_{ij}^{\rm FZ}$ as $\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{00}^{(\vartheta)}}$, $\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{00}^{\rm mat}}$ and $\bar{\mathfrak{h}}_{ij}^{{\rm FZ},t_{00}}$, respectively, one finds

$$\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{00}^{(\vartheta)}} \sim \frac{1}{r}\delta_{00}\frac{\partial^2}{\partial t^2}\int \vartheta_{,k}^{\rm NZ}\vartheta_{,k}^{\rm NZ} x'^i x'^j d^3x' \propto \frac{1}{r}\omega^2\frac{1}{b^3}\frac{1}{b^3}b^2 b^3 \propto \frac{v^8}{r}, \tag{19}$$

$$\bar{\mathfrak{h}}_{ij}^{{\rm FZ},T_{00}^{\rm mat}} \sim \frac{1}{r}\frac{\partial^2}{\partial t^2}\int \mathfrak{h}^{\rm NZ} T_{00}^{\rm mat} x'^i x'^j d^3x' \propto \frac{1}{r}\omega^2\frac{1}{b^3}b^2 \propto \frac{v^8}{r}, \tag{20}$$

$$\bar{\mathfrak{h}}_{ij}^{{\rm FZ},t_{00}} \sim \frac{1}{r}\frac{\partial^2}{\partial t^2}\int \mathfrak{h}_{kl}^{\rm NZ} h_{00,kl}^{\rm NZ} x'^i x'^j d^3x' \propto \frac{1}{r}\omega^2\frac{1}{b^3}\frac{1}{b^3}b^2 b^3 \propto \frac{v^8}{r}, \tag{21}$$

where we used $T_{00}^{\rm mat} \sim \rho$. Observe that all of these contributions enter at 3PN order relative to the leading GR expression, and not at 2PN order. This implies that certain terms we neglected actually cancel the 2PN correction to the metric perturbation, forcing the correction to enter at 3PN order. A similar result can be obtained in GR when deriving an expression for the FZ metric perturbation in terms of the source multipole moments. Using the conservation law of $S_{\mu\nu}$ in GR, one can show that $t_{\mu\nu}$ in GR sources higher PN contributions compared to the leading quadrupolar term sourced by the matter stress-energy tensor.

A very important consequence of all of this is that the contribution of the dCS correction to the metric perturbation to the energy flux is now smaller (in a PN sense) than that induced by the scalar field. Therefore, one does not need to include the dCS correction to the metric perturbation when computing the energy flux to leading dCS and PN order in compact binary inspirals [6] and in the orbital decay rate of binary pulsars [4].

II. SCALAR RADIATION

We now look at the scalar energy flux. The scalar field evolution equation is

$$\Box\,\vartheta = -\alpha_4 R^* R, \tag{22}$$

where the source is the Pontryagin density (see e.g. Eq. (8) in [1]). When one solves this equation through a Green's function approach, one must employ finite part regularization to avoid divergences introduced by the point-particle approximation. A matching calculation gives rise to an effective source term to the scalar field evolution equation that gives the correct scalar field solution. To leading PN order and in *covariant form*, the evolution equation then becomes

$$\square\,\vartheta = 4\pi \sum_{A=1,2} \int d\tau \mu_A^\alpha \left(\frac{\delta^{(4)}[x^\mu - x_A^\mu(\tau)]}{\sqrt{-g}}\right)_{,\alpha} - \alpha_4 R^* R, \tag{23}$$

where x_A^μ is the trajectory of body A. We will neglect $-\alpha_4 R^* R$ on the right-hand side because it only sources higher PN-order contributions to the FZ scalar field, as shown in [1]. In [1], we did not use the covariant form of the effective source term (compare Eq. (23) to Eq. (76) in [1]); as we will see, there are terms that arise from the covariant form of the source that enter at the same PN order as those computed in [1].

As we are interested in $\vartheta^{\rm FZ}$, μ_A^α in Eq. (23) corresponds to the FZ scalar dipole moment. In principle, this moment must be determined by matching it to the NZ moment in the buffer zone, i.e. where the FZ and the NZ overlap. As in the case of the (source and radiative) multipole moments of metric perturbations in GR, the NZ scalar dipole moment is equal to the FZ scalar dipole moment up to nonlinear terms. These correspond, for example, to the propagation of the scalar field on a nonflat background, leading to scalar field scattering analogous to the "tail" terms of the metric perturbation in GR. As such, these nonlinear terms enter at higher PN order and, thus, they can be neglected to the PN order we consider here. The FZ scalar dipole moment is then equal to the NZ scalar dipole moment, which was related in [1] to the inner zone (IZ) integral of the self interaction contribution of both the Pontryagin density and the $\square\,\vartheta$ terms in the scalar field equation. Therefore, the scalar dipole moment μ_A^α can be obtained by promoting the spin 3-vector $\hat{S}^i$ in Eq. (10) to a spin 4-vector $\hat{S}^\alpha$, where the condition that determines μ_A^0 will be discussed later.

Let us now manipulate Eq. (23) so that one can solve it using the Green's function approach for a flat metric. We decompose the left-hand side of Eq. (23) as $\square = \square_\eta + \square_h$, where $\square_\eta$ is the flat space d'Alembertian operator. Then, Eq. (23) becomes

$$\begin{aligned}\square_\eta\,\vartheta &= 4\pi \sum_{A=1,2} \int d\tau \left\{\mu_A^i \left(\frac{\delta^{(4)}[x^\mu - x_A^\mu(\tau)]}{\sqrt{-g}}\right)_{,i} + \mu_A^0 \left(\frac{\delta^{(4)}[x^\mu - x_A^\mu(\tau)]}{\sqrt{-g}}\right)_{,0}\right\} - \square_h\,\vartheta \\ &\approx 4\pi \sum_{A=1,2} \left\{\mu_A^i \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_A)_{,i} + \int d\tau \mu_A^0 \delta^{(4)}[x^\mu - x_A^\mu(\tau)]_{,0}\right\} - \square_h\,\vartheta,\end{aligned} \tag{24}$$

where we have only kept leading PN-order terms. In [1], we only included the first term on the right-hand side of Eq. (24), which gives a quadrupolar scalar radiation term from a cross interaction contribution:

$$\vartheta_{(1)}^{\rm FZ,cross} = \frac{1}{r}\ddot{\mu}_{ij} n^{ij} = -\frac{1}{r}\omega^2 \mu_{ij} n^{ij} \sim \frac{1}{r} v^6 b \sim \frac{1}{r} v^4, \tag{25}$$

where $\mu_{ij} \equiv x_1^{(i}\mu_1^{j)} + (1\leftrightarrow 2)$. Below, we look at the contribution from the other terms that we did not consider in [1].

We first look at the FZ scalar field $\vartheta_{(2)}^{\rm FZ}$ sourced by the second term in Eq. (24):

$$\begin{aligned} 4\pi \sum_{A=1,2} \int d\tau \mu_A^0 \delta^{(4)}[x^\mu - x_A^\mu(\tau)]_{,0} &= 4\pi \sum_{A=1,2} \int d\tau \mu_A^0 \delta^{(3)}[x^i - x_A^i(\tau)]\delta[t - t_A(\tau)]_{,0} \\ &= -4\pi \sum_{A=1,2} \int d\tau \mu_A^0 \delta^{(3)}[x^i - x_A^i(\tau)]\delta[t - t_A(\tau)]_{,\tau} \left(\frac{dt_A}{d\tau}\right)^{-1} \\ &\approx -4\pi \sum_{A=1,2} \int d\tau \mu_A^0 \delta^{(3)}[x^i - x_A^i(\tau)]\delta[t - \tau]_{,\tau} + \mathcal{O}(v^2) \\ &\approx 4\pi \sum_{A=1,2} \int d\tau \{\mu_A^0 \delta^{(3)}[x^i - x_A^i(\tau)]\}_{,\tau} \delta[t - \tau] \\ &\approx 4\pi \sum_{A=1,2} \{\mu_A^0 \delta^{(3)}[x^i - x_A^i(t)]\}_{,0} \\ &\approx 4\pi \sum_{A=1,2} \left\{\frac{d\mu_A^0}{dt}\delta^{(3)}[x^i - x_A^i(t)] - \mu_A^0 \delta^{(3)}[x^i - x_A^i(t)]_{,j} v_A^j(t)\right\}, \end{aligned} \tag{26}$$

where in the third line, we used $t_A \approx \tau$ to leading PN order. To obtain μ_A^0, we follow [7] and assume that the spin vector is purely spatial in the stellar rest frame. Then, in any frame, one has

$$\mu_A^\mu u_\mu^A = 0, \tag{27}$$

because the spin vector is assumed orthogonal to the four-velocity and such an assumption gives $\mu_a^0 \propto S^0 = 0$ in the rest frame; this leads to

$$\mu_A^0 = \mu_A^i v_i^A [1 + \mathcal{O}(v^2)], \tag{28}$$

where we used $u_0^A = -1$ to leading PN order. Substituting this and Eq. (26) into Eq. (24), the field equation for $\vartheta_{(2)}^{\rm FZ}$ becomes

$$\Box_\eta \, \vartheta_{(2)}^{\rm FZ} = 4\pi \sum_{A=1,2} \left[\frac{d}{dt} (\mu_A^i v_i^A) \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_A) - \mu_A^i v_i^A v_A^j \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_A)_{,j} \right]. \tag{29}$$

When solving this equation for the FZ solution, the first term gives a monopolar contribution while the second term gives a dipolar one, and hence, the former is dominant. One then finds

$$\vartheta_{(2)}^{\rm FZ} = -\frac{1}{r} \ddot{\mu}_{ii}. \tag{30}$$

We next look at the FZ scalar field $\vartheta_{(3)}^{\rm FZ}$ sourced by the third term in Eq. (24). Since

$$\begin{aligned} \Box \, \vartheta &= \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \partial^\mu \vartheta) = \left(1 - \frac{h}{2}\right) \partial_\mu \left[\left(1 + \frac{h}{2}\right) (\eta^{\mu\nu} - h^{\mu\nu}) \partial_\nu \vartheta \right] + \mathcal{O}(h^2 \vartheta) \\ &= \partial_\mu \left[\left(\eta^{\mu\nu} - h^{\mu\nu} + \frac{h}{2} \eta^{\mu\nu} \right) \partial_\nu \vartheta \right] - \frac{h}{2} \eta^{\mu\nu} \partial_{\mu\nu} \vartheta + \mathcal{O}(h^2 \vartheta) \\ &= \eta^{\mu\nu} \partial_{\mu\nu} \vartheta - (\partial_\mu h^{\mu\nu})(\partial_\nu \vartheta) - h^{\mu\nu} \partial_{\mu\nu} \vartheta + \frac{1}{2} \eta^{\mu\nu} (\partial_\mu h)(\partial_\nu \vartheta) + \mathcal{O}(h^2 \vartheta), \end{aligned} \tag{31}$$

one finds

$$\begin{aligned} \Box_\eta \, \vartheta_{(3)}^{\rm FZ} &= - \Box_h \, \vartheta \\ &= (\partial_\mu h^{\mu\nu})(\partial_\nu \vartheta) + h^{\mu\nu} \partial_{\mu\nu} \vartheta - \frac{1}{2} \eta^{\mu\nu} (\partial_\mu h)(\partial_\nu \vartheta). \end{aligned} \tag{32}$$

The leading contribution comes from spatial derivatives (not time derivatives). When solving the above wave equation by using the Green's function method and performing the volume integral of the source terms on the right-hand side, the contribution from the first and second terms to the FZ solution vanish upon integration by parts. Therefore, one is left with

$$\Box_\eta \, \vartheta_{(3)}^{\rm FZ} = -\frac{1}{2} (\partial_i h)(\partial_i \vartheta). \tag{33}$$

Let us first comment on the self-interaction contribution of the source term in Eq. (33) to the FZ scalar field solution. To leading PN order, one finds

$$\begin{aligned} \vartheta_{(3)}^{\rm FZ,self} &= -\frac{1}{4\pi} \frac{1}{r} \int \left(-\frac{1}{2} h_{1,i}^{\rm NZ} \vartheta_{1,i}^{\rm NZ} \right) d^3x + (1 \leftrightarrow 2) \\ &= -\frac{1}{2\pi} \frac{1}{r} m_1 \mu_1^j \int \partial_i \left(\frac{1}{r_1} \right) \partial_{ij} \left(\frac{1}{r_1} \right) d^3x + (1 \leftrightarrow 2). \end{aligned} \tag{34}$$

Such an integral diverges at the body's position. The effective source term in Eq. (23) captures the regular part of this contribution.

Let us then focus on the cross-interaction contribution of Eq. (33). The monopole radiation piece of $\vartheta^{\rm FZ}_{(3)}$ is given by

$$\begin{aligned}\vartheta^{\rm FZ}_{(3)} &= -\frac{1}{4\pi}\frac{1}{r}\int\left(-\frac{1}{2}h^{\rm NZ}_{1,i}\vartheta^{\rm NZ}_{2,i}\right)d^3x + (1\leftrightarrow 2)\\ &= -\frac{1}{2\pi}\frac{1}{r}m_1\mu_2^j\int\partial_i\left(\frac{1}{r_1}\right)\partial_{ij}\left(\frac{1}{r_2}\right)d^3x + (1\leftrightarrow 2)\\ &= \frac{1}{2\pi}\frac{1}{r}m_1\mu_2^j\partial_i^{(1)}\partial_{ij}^{(2)}\int\frac{1}{r_1}\frac{1}{r_2}d^3x + (1\leftrightarrow 2),\end{aligned} \tag{35}$$

where $\partial_i^{(A)}$ is the particle derivative with respect to the Ath body. Since the finite part of the remaining integral is given by $\int 1/(r_1 r_2)d^3x = -2\pi b$ [8], one finds

$$\vartheta^{\rm FZ}_{(3)} = -\frac{1}{r}m_1\mu_2^j\partial_i^{(1)}\partial_{ij}^{(2)}b + (1\leftrightarrow 2) = -\frac{1}{r}m_1\mu_2^j\partial_{jii}^{(1)}b + (1\leftrightarrow 2), \tag{36}$$

and since

$$\partial_{jii}^{(1)}b = \partial_{ji}^{(1)}n_{12i} = \partial_j^{(1)}\left(\frac{\delta_{ii}-n_{12ii}}{b}\right) = 2\partial_j^{(1)}\left(\frac{1}{b}\right) = -\frac{2}{b^2}n_{12j}, \tag{37}$$

one finally finds

$$\vartheta^{\rm FZ}_{(3)} = \frac{2}{r}m_1\mu_2^j\frac{n_{12}^j}{b^2} + (1\leftrightarrow 2) = \frac{2}{r}(m_1\mu_2^j - m_2\mu_1^j)\frac{n_{12}^j}{b^2} = \frac{2}{r}\ddot{\mu}_{ii}. \tag{38}$$

Combining Eqs. (25), (30) and (38), the new FZ scalar field solution is given by

$$\vartheta^{\rm FZ} = \sum_{k=1}^{3}\vartheta^{\rm FZ}_{(k)} = \frac{1}{r}\ddot{\mu}_{ij}(n^{ij}+\delta^{ij}). \tag{39}$$

We now derive the new scalar energy flux by substituting Eq. (39) into the formula

$$\delta\dot{E}^{(\vartheta)} = -\beta\lim_{r\to\infty}\int_{S_r^2}\langle(\dot{\vartheta}^{\rm FZ})^2\rangle_\omega r^2 d\Omega. \tag{40}$$

Doing so, one finds

$$\delta\dot{E}^{(\vartheta)} = -\beta\langle\ddot{\mu}_{ij}\ddot{\mu}_{kl}\rangle_\omega\int_{S_\infty^2}(n^{ij}+\delta^{ij})(n^{kl}+\delta^{kl})d\Omega. \tag{41}$$

Using the angle-averaging formulas

$$\int_{S_\infty^2} n^{ij}d\Omega = \frac{4\pi}{3}\delta^{ij}, \tag{42}$$

$$\int_{S_\infty^2} n^{ijkl}d\Omega = \frac{4\pi}{15}(\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk}), \tag{43}$$

we find

$$\begin{aligned}\delta\dot{E}^{(\vartheta)} &= -\frac{4\pi}{15}\beta\langle\ddot{\mu}_{ij}\ddot{\mu}_{kl}\rangle_\omega(\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk}+5\delta^{ij}\delta^{kl}+5\delta^{ij}\delta^{kl}+15\delta^{ij}\delta^{kl})\\ &= -\frac{4\pi}{15}\beta\langle\ddot{\mu}_{ij}\ddot{\mu}_{kl}\rangle_\omega(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk}+26\delta^{ij}\delta^{kl})\\ &= -\frac{4\pi}{15}\beta\langle[\ddot{\mu}_{ij}\ddot{\mu}^{ij}+27(\ddot{\mu}^i{}_i)^2]\rangle_\omega. \end{aligned} \tag{44}$$

Equation (135) of [1] gives

$$\ddot{\mu}^{ij} = \frac{1}{b^3}(m_1 v_{12}^{(i}\mu_2^{j)} - m_2 v_{12}^{(i}\mu_1^{j)}), \tag{45}$$

and substituting Eq. (10) into this equation, one finds

$$\ddot{\mu}^{ij} = -\frac{5}{2}\frac{\alpha_4}{\beta}\frac{m}{b^3}v_{12}^{(i}\bar{\Delta}^{j)}, \tag{46}$$

where

$$\bar{\Delta} \equiv \frac{m_2}{m}\chi_1\hat{S}_1^i - \frac{m_1}{m}\chi_2\hat{S}_2^i. \tag{47}$$

Substituting this equation into Eq. (44), we finally find the new scalar energy flux:

$$\delta\dot{E}^{(\vartheta)} = -\frac{5}{48}\zeta_4[\bar{\Delta}^2 + 27\langle(\bar{\Delta}\cdot\hat{v}_{12})^2\rangle_\omega]v^{14}. \tag{48}$$

This expression is very similar to Eq. (136) in [1]. The only difference is that the factor of "2" in front of $\langle(\bar{\Delta}\cdot\hat{v}_{12})^2\rangle_\omega$ has now become "27." We remind the reader that Eq. (27) does not imply that $\langle(\bar{\Delta}\cdot\hat{v}_{12})^2\rangle_\omega$ vanishes because the former is a condition on the four-velocity. Observe, however, that $\langle(\bar{\Delta}\cdot\hat{v}_{12})^2\rangle_\omega$ does vanish for spin-aligned binaries and, hence, this correction does not affect the main results in [6]

III OTHER CORRECTIONS

We list below minor typos that need to be corrected in Ref. [1]:

(i) In Eq. (68), d^3x' is missing in the integral.
(ii) In Eq. (119), $\boldsymbol{x}$ and d^3x should be $\boldsymbol{x}'$ and d^3x', respectively.
(iii) In the caption of Fig. 3, the relation between ζ_4 and $\zeta_{\rm CS}$ should be $\zeta_4 = \zeta_{\rm CS}/16$.
(iv) The first line of Eq. (145) should be $\Psi_{\rm GW} = -2\phi(t_0) + 2\pi f t_0$.
(v) $\mathfrak{h}_{ij}$ in Eqs. (94), (96)–(100), (103)–(105) and (108)–(117) should be $\bar{\mathfrak{h}}_{ij}$.

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