

Naive Play and the Process of Choice in Guessing Games*

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Abstract

We introduce a new experimental design to provide insight into strategic choice in one shot games. We incentivize and observe provisional choices in the 2/3 guessing game in the period after the structure of the game has been communicated. We define as naive those who play dominated strategies well after we have communicated the structure of the game. We identify a high proportion of such players (more than 40% of subjects). We find strong support for the standard assumption that naive types' choices average 50. This holds not only at the end of the game, but throughout the period of contemplation. Unlike their naive counterparts, strategic players show evidence of increasing sophistication as the period of contemplation increases.

1 Introduction

The predictions of equilibrium theory frequently fail when players interact in unfamiliar economic environments. For that reason, non-equilibrium theories focused on strategic sophistication are of growing interest. The game that more than any other sparked this interest is the 2/3 guessing game, in which players select an integer between 0 and 100,

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with the reward going to the individual closest to $2/3$ of the group average (Nagel (1995), Stahl and Wilson (1995), Stahl (1996), Duffy and Nagel (1997), Ho, Camerer, and Weigelt (1998), Camerer, Ho and Chong (2004)). While elimination arguments reveal the only equilibrium choices to be 0 and 1, in practice there is significant clustering around 33 and 22.

The simplest non-equilibrium theory aimed at explaining behavior in the $2/3$ guessing game is the “Level-K” theory (Nagel (1995), Stahl and Wilson (1995), Stahl (1996)). This theory is based on the assumption that naive “level zero” (L0) players make choices that are uniformly distributed over the range $[0,100]$, hence averaging 50 (see Nagel (1995), Stahl and Wilson (1995) and Camerer, Ho and Chong (2004)). Sophisticated type L2 players who pick numbers close to 22 are interpreted as best responding to type L1 players, who pick 33 in response to the assumed average of 50 associated with naive play.¹

One possible explanation for the fact that more select numbers close to 33 and 22 than close to 50 is that naive types are less prevalent than L1 and L2 types. If this interpretation is valid, most players must engage in strategic reasoning at least to some extent. Yet the idea that there are so few naive types is called into question by recent evidence showing that a substantial proportion of experimental subjects are all-but oblivious to strategic considerations.² Costa-Gomes and Weizsacker (2008) run 14 two-person normal form games in which subjects are both asked to choose an action and state a belief on their partner’s action. They find that subjects’ elicited beliefs and actions are inconsistent up to 40% of the time. In an auction context, Ivanov, Levin and Niederle (2010) show that many subjects are unable to best-respond to their own past play.

Given their foundational role in the theory and recent controversies concerning their prevalence, it is important to accurately identify naive players. It is hard to do this using standard choice data, since identification rests on group average play, not individual play.³ This has provoked interest in using non-standard data on choice procedures to gain additional insight into naive play. In a recent example of such non-choice based inference, Burchardi and Penczynski (2011) divide players into teams, allowing team members to pass to one another messages concerning their preferred choice and how best to play the game. They classify these arguments according to level of strategic sophistication. For those classified in this manner as being naive types, the mean action was 58, offering

¹More recent theories relax some of the stringent assumptions of the original Nagel (1995) model. For example, the Cognitive Hierarchy model of Camerer, Ho and Chong (2004) allows each individuals to have beliefs over the distribution of the less sophisticated types as opposed to imposing that distribution to be degenerate as in Nagel (1995). Recent work by Alaoui and Penta (2012) endogenizes the level of reasoning based on the tradeoff between thinking costs and the benefits of the extra steps of reasoning.

²For an up-to-date survey of the literature see Crawford, Costa-Gomes and Iriberry (2011).

³The theory asserts only that such players average 50, not that they all choose 50.

some support to the standard assumption.⁴

In this paper we use a novel strategic experiment to identify naive players. Rather than study only final choices, we incentivize and observe players' provisional choices in the three minute period after the structure of the guessing game is conveyed to them. The resulting "strategic choice process" (SCP) data capture how internal reflection on the structure of the game causes the perceived optimal decision to change.

In contrast with other non-standard data such as messages to team members, the non-standard SCP data that we gather are particularly easy to map to the theory. We identify as naive players who make dominated choices (above 67) after having had at least 30 seconds to consider the decision. Strategic choice process data is clearly essential to our identification strategy, since types are defined precisely from the pattern of their provisional choices. Further, the SCP allows us to identify naive subjects in the game itself, as opposed to using a different game that would not necessarily result in the same players being strategic or naive.

There are two basic findings. First, a high proportion of players (approximately 40%) are identified in this manner as naive. Second, these naive types make final choices that average close to 50, just as the theory suggests. Strikingly, they average close to 50 through much of the 3 minute observation period. Hence they fit the standard conception of naive players for most of the period of play.

Our choice process data also provide insight into the evolution of strategic thought for the remaining "strategic" players. They appear to be well described by hierarchical models of strategic reasoning, in that their final choices accord with those found in previous versions of the game. Moreover we identify a downward drift in their choices with the passage of time, as if further contemplation increased the extent of their strategic understanding. There is no such drift among those we identify as naive players. In addition, the players that we identify as strategic using the SCP data in the 2/3rds guessing game are more likely to be strategic in an unrelated Bayesian updating game (Monty Hall).⁵

⁴In a recent paper, Agranov et al. (2012) suggest what is perhaps the most direct method to estimate the proportion of naive players in the population. In that study, subjects are told that they are each playing against seven computers that are choosing numbers uniformly. Only about half of the subjects are capable of making one step of strategic reasoning and choose 33 in this experiment, which suggests that the remaining half of the subjects are naive players. While this method provides a way to estimate the proportion of naive players in the population, it can only be used when subjects play against computers. The SCP data and the method that we outline in this paper is broader and allows us to make this determination for subjects playing the game in its traditional form. Further, the SCP methodology allows us to track naive players' behavior over the course of the game.

⁵This game is modeled on the "Monty Hall" game of Nalebuff (1987), Friedman (1998) and Avishalom and Bazerman (2003). Our results are consistent with the results of Burchardi and Penczynski (2010), and Georganas, Healy and Weber (2010). They find that types based on final choice alone are not transferable across games.

We believe that the impact of contemplation time on strategic decisions is of broader interest. Our design can be used to explore strategic sophistication in a directly analogous manner whenever there are dominated strategies. Even when there are no such strategies, the data in our experiment displays intriguing patterns suggesting other identification strategies. Moreover, there are factors other than strategic sophistication that may fluctuate over time. Grimm and Mengel (2010) have recently shown that giving decision-makers additional time to decide in the ultimatum game greatly lowers the rate of rejection of small offers. One possible reason for this is that the emotional effects of a disappointing offer are felt less sharply once they are internalized. This is a case in which the passage of time may change the decision maker’s utility function. In addition, Dufwenberg, Sundaram and Butler (2010) have studied games in which a moment of “epiphany” is needed to identify a dominant strategy.

We believe that the SCP method may provide insight into many situations of this form, capturing as it does the evolution of perceived optimal decisions as time passes. It also offers one possible way around a key challenge facing those interested in how contemplation time interacts with game play. Indeed, the SCP experiment can be seen as eliciting from one individual an entire sequence of time-constrained choices in their very first play of an unfamiliar game.⁶ As noted above, naive players average close to 50 throughout the full three minute observation period. This suggests a possible connection between play in the SCP after a given period of contemplation and play in a corresponding standard game. To assess this analogy more broadly, we run standard guessing game experiments with 30 and 180 second time constraints. We find the resulting behavior to be very similar to that in the SCP experiment at the corresponding time. To a first approximation, this finding support the idea that play in the SCP is analogous to play in multiple guessing games with different time constraints. For that reason, the SCP methodology may be worth deploying in other situations in which contemplation time interacts with strategic choice.

The remainder of the paper is organized as follows: background material motivating our experimental design is in the remainder of this section. The experimental design can be found in Section 2. Results concerning the prevalence and patterns of naive and strategic play are in Section 3. Results suggesting the broader applicability of the SCP experiment are in Section 4. Concluding remarks are in Section 5.

⁶The standard procedure (between subject design) for examining the impact of contemplation time requires the use of separate pools of subjects for each time constraint. Hence very large subject pools are required to adequately control for individual differences. In addition to allowing one to economize on the number of plays, using the SCP treatment to explore the impact of time constraints removes the need to control for individual differences: the players are one and the same regardless of the time constraint.

1.1 Background

Guessing Games and Non-Standard Data

One question that has guided research on the guessing game is whether choice (say) of 33 is a sophisticated response to a fully reasoned belief that others will average 50 or a reflection of bounds on rationality.⁷ In trying to understand this, various researchers have begun to explore non-standard data that may aid in the interpretation of choices. For example, Costa-Gomes, Crawford and Broseta (2001) and Costa-Gomes and Crawford (2006) examined data on information search behavior recorded using MouseLab. Costa-Gomes and Crawford (2006) used MouseLab to study cognition via information search in a rich class of two-person guessing games. They provide compelling evidence that patterns of search as well as of choice can be well explained by Lk and cognitive hierarchy models.

A second line of research involves estimating subjects' levels of reasoning by analyzing verbal data associated with their choices (e.g. Sbriglia (2004), Bosch-Domenech, Montalvo, Nagel and Satorra (2002) and Arad (2009)). Burchardi and Penczynski (2010), for example, analyzed subjects' arguments while attempting to convince their "teammates" to follow their advice.

A separate line of work uses physiological and neurological measurements to gain insight into play in the guessing game. Dickinson and McElroy (2009) find that subjects apply higher levels of reasoning when well-rested rather than sleep-deprived, and when at their peak time of day rather than at their off-peak times. Coricelli and Nagel (2009) use fMRI techniques to explore levels of reasoning in a game in which subjects play against computers. They uncover systematic differences in the neurological responses of the players at different levels of strategic sophistication. Chen, Huang and Wang (2010) used eye-tracking data to complement choice data in a modified 2/3 guessing game played spatially on a two-dimensional plane. Taken together, the above research makes it clear that bounds on rationality play an important role in explaining the behavioral patterns in guessing games.

Strategic Choice Process Data

The recent expansion of the evidentiary base in studies of the guessing game mirrors a similar movement in decision theory. The desire to enrich standard choice data while retaining strong links to standard theory led Caplin and Dean (2011) to introduce "choice process" data in the search theoretic context. These data identify the evolution of perceived optimal choices during the period of search. Caplin, Dean, and Martin (2011)

⁷See Crawford (2008), Grosskopf and Nagel (2008) and Coricelli and Nagel (2009).

develop an experimental interface to capture these data, and use it to get new insights into the nature of the search process and the rules for stopping search.

The current paper is the first to use the choice process design to study strategic decision-making. Unlike in the search theoretic context, in the strategic case there is no external information to gather that would motivate changing one’s mind in the pre-decision period. Hence the question our SCP experiment addresses is the extent to which subjects learn to play the guessing game by turning it over in their mind. As they do this, they may reflect both on the structure of the game itself and on their own earlier thoughts (as in the Goeree and Holt (2004) model of “noisy introspection”). Do they gradually move to higher levels of strategic sophistication as they internalize the structure of the game? Do they use their own earlier thoughts to model the thoughts of others, as suggested by various models in social psychology (see Dawes (1990))? The SCP experiment gathers data on this process of learning while hewing closely to the decision theoretic tradition.

Learning by Thinking and Time To Decide

Economists have studied many forms of learning, such as Bayesian updating, learning by doing, and reinforcement learning. Our focus is on a quite distinct form of learning that involves no new external stimulus: “learning by thinking.” This raises the question of how long a period of time players have been given to contemplate their strategy in prior versions of the guessing game. Unfortunately there is little pattern in this respect. Ho, Camerer and Weigelt (1998) set an experimental time limit of 2 minutes. However this was the maximum allowed time and not the time it took for subjects to actually respond. Bosch-Domenech, Montalvo, Nagel and Satorra (2002) report results of a five minute laboratory experiment, and of other experiments conducted more remotely (e.g. via newspaper) with response times of up to two weeks.

We are aware of only one paper in which the time constraint in a 2/3 guessing game was manipulated. Kocher and Sutter (2006) examined the effects of time pressure and incentive schemes on choices in repeated plays of the guessing game. Surprisingly, they did not find much difference in first round play for different time constraints.⁸

Using a different design, Rubinstein (2007) explored the connection between contemplation time and choice in the context of an online version of the guessing game. There was no maximum time, but a server recorded the time a subject took to submit the answer. He found that the focal L1 and L2 responses of 33 and 22 respectively took longer

⁸This may be due to the fact that their subjects knew that they would repeat the game several times, and so would be able to change their decisions in later plays of the game. In our design, subjects play one and only one time, and may therefore more rapidly internalize the structure of the game.

on average than other choices.

In a particularly relevant precursor to the SCP design, Weber (2003) also explored the connection between contemplation time and choices in the guessing game. He had subjects play the guessing game ten times in a row, providing them with no feedback on their performance until all ten trials had been completed. What he found was that, while choice in the first round was entirely as in the standard game, by round ten the average choice had fallen significantly. This suggests that further reflection on the structure of the game led many to change their minds, without any feedback from outcomes and without new external information of any kind. However, in Weber's (2003) setting, subjects may change their decisions over time not only due to continued reflection on the structure of the game, but also because they expect others to change. In contrast, our SCP experiment is designed exclusively to focus on the impact of internal reflection.⁹

2 The Experimental Design

All of the experiments were run at the laboratory of the Center for Experimental Social Science (CESS) at New York University. Subjects were drawn from the general undergraduate population in the university by email solicitations. The guessing game experiments themselves lasted about 10 minutes. Subjects in the SCP treatment participated in additional tasks, as detailed below. Average payoffs were between \$10 and \$15.

In all treatments, subjects were first seated at their computer terminals, and then given the experimental instructions, face down. Once all subjects received their instructions, they were instructed that they could turn the sheets over and the instructions were read out loud. Subjects did not communicate with one another during the experiments. There was only a single play of the $2/3$ guessing game in each experiment. The precise experimental instructions differed across treatments as indicated below.

Given our interest in how learning takes place in a novel one shot game, we dropped subjects who reported being familiar with the game, whether in a lab, in a classroom or in any other context. This familiarity was assessed in a questionnaire at the end of each session. Some 25% of subjects had either played the game or heard of it. The remaining sample consists of 188 subjects.

⁹This is because the other payoff-relevant actions were settled in a prior interaction as we will describe in the next section.

2.1 Standard Guessing Games

Before running the SCP experiment, we conducted standard guessing games of 30 seconds and 180 seconds durations. In these games only the final choice of each subject mattered for payment. The longer time of 180 seconds was chosen since prior work suggests that it is enough time for most subjects to reason through the game, while the shorter time was chosen to cut short such reasoning. These Standard Experiments were included not only to gauge the importance of decision time in the outcome of the game, but also to provide benchmarks with which to compare the SCP treatment. In total, 66 subjects participated in the 30 second treatment, and 62 participated in the 180 second treatment.

The rules of the game and the task were described as follows:

RULES OF THE GAME:A few days ago 8 undergraduate students like yourselves played the following game. Each of the 8 students had 180 seconds to choose an integer between 1 and 100 inclusive, which they wrote on a piece of paper. After 180 seconds, we collected the papers. The winner was the person whose number was closest to two thirds of the average of everyone’s numbers. That is, the 8 students played among themselves and their goal was to guess two thirds of the average of everyone’s numbers. The winner won \$10 and in case of a tie the prize was split.

YOUR TASK:You will have 180 seconds to choose an integer between 1 and 100 inclusive. You win \$10 if you are “better than” those 8 students at determining two thirds of the average of their numbers. That is, you win \$10 if your number is the closest to two thirds of the average of the numbers in the past game.

OR

YOUR TASK:You will have 30 seconds to choose an integer between 1 and 100 inclusive. You win \$10 if you are “better than” those 8 students at determining two thirds of the average of their numbers. That is, you win \$10 if your number is the closest to two thirds of the average of the numbers in the past game.

The screen displayed 100 buttons, each representing an integer between 1 and 100 inclusive.¹⁰ Once the game started, subjects could select any number by clicking on the

¹⁰It is common to allow also the choice of zero. Having the minimum choice be 1 simplifies matters in that the unique Nash equilibrium, identifiable by iterated elimination of dominated strategies, is for all to select 1. In contrast, when zero is included as an option, there are multiple equilibria. It is also common to allow subjects to choose any real number, as opposed to integers. Our experimental apparatus - displaying all the possible choices on the screen - makes the restriction to integers a necessary one. The equilibrium is unchanged by this modification.

button displaying it. Subjects could change their selected number as many times as they wanted. Subjects could end the game earlier by clicking on a “Finish” button. There was no difference between choosing a number and staying with that number until the end of the game or instead clicking the Finish button. In the Standard Experiment, it was only their final choice (at 30 seconds or 180 seconds as specified in the instructions) that determined the participant’s payoff from the game.¹¹

Note that our experiment has the feature that a subject’s number is not included in the average. This ensures that the corresponding SCP treatment does not have additional equilibria. In technical terms, this makes the game analogous to a standard guessing game with a large number of participants (see Bosch-Domenech et al. (2002)).¹²

2.2 SCP Treatment

We had 60 subjects participating in the SCP treatment. While there was no change in the described rules of the game, what determined the subject’s payment in the SCP treatment was the subject’s choice at a random time.¹³ The experimental instructions were as follows.

When the game starts, you can select a number by clicking on the button displaying the number that you want. You may click when you want, however many times you want. The computer will record all the numbers you click on, as well as when you clicked on them. After 180 seconds, or when you click the finish button, the round will come to an end and you won’t be able to change your choice anymore. Just to make clear, if you choose a number and then stay with that number until the end, or instead decide to click on the “Finish” button, it will make no difference.

Only one of the numbers you selected will matter for payment. To determine which one, the computer will randomly choose a second between 0 and 180, with each second equally likely to be chosen. The number you selected at that time will be the one that matters. We will call this number **“Your Number.”**

We took measures to ensure that subjects participating in SCP treatments properly understood the incentive structure. Hence when they arrived in the lab we described the

¹¹There was no incentive to finish early, since the game lasted the same amount of time regardless.

¹²Formally: suppose the group size is n and a subject believes the average of the other participants is \bar{x} . If that subject’s number is counted in the average then that subject should choose $\frac{2(n-1)\bar{x}}{3n-2}$ so that as the group size gets larger and larger, this choice converges to $(2/3)\bar{x}$, which is what the subject should choose if his/her number were not counted in the average.

¹³In this treatment, it was the choice of a subject at a random second that was compared to the choices of the 8 subjects that had played the game previous to the experiment.

experimental methodology to them before introducing them to the guessing game. They were told that:

1. The game that they were about to play would last 180 seconds.
2. The computer would record their choice throughout the game.
3. After the 180 seconds were over, the computer would randomly select one of the 180 seconds.
4. Their choice at that random second would be the one that mattered for their payment.

Illustrative examples were provided to illuminate the nature of the final payoff.¹⁴ The examples illustrated that failure to pick an option would result in a certain payoff of zero. Hence subjects in the SCP treatment were incentivized to make a quick and intuitive first estimate of two-thirds of the average final number picked by the group that had played previously. Whenever further reflection causes this best estimate to change, they were incentivized immediately to make the corresponding change in their guess.

After completing the 2/3rds guessing game, all subjects in the SCP treatment played the Monty Hall Game. The Monty Hall Game is a classic problem in which intuition can diverge from Bayesian reasoning. We showed participants three closed doors on the screen, and let them know that there was \$5 behind one and only one randomly chosen door, with nothing behind the other two. They were then asked to choose one of the doors. At that point, the experimenter announced that he or she knew the location of the \$5, and opened one of two unselected doors to show that it contained nothing. The subject was then given the option either to stay with their initial choice or instead to switch to the other closed door.

3 Naive and Strategic Players

In this section we present three basic findings based on using play of dominated strategies to identify naive players. Our first finding relates to the proportion of such players in the population. The second relates to their pattern of behavior throughout the game. The third relates to the remaining subset of the population, the strategic players.

Finding 1: More than 40% of the population is naive.

¹⁴Appendix A contains the instructions for these SCP sessions.

Choosing a number above 67 is a dominated action, since $2/3$ of the highest possible number is 67 (66.7 rounded-up). We use this strong theory-driven cutoff to define naive players. Figure 1 presents the histogram of the last time each subject chose numbers above 67. For instance, someone who never chose numbers above 67 is counted in the 0 bin on this graph, while a person whose final choice is above 67 appears in the 180 bin.

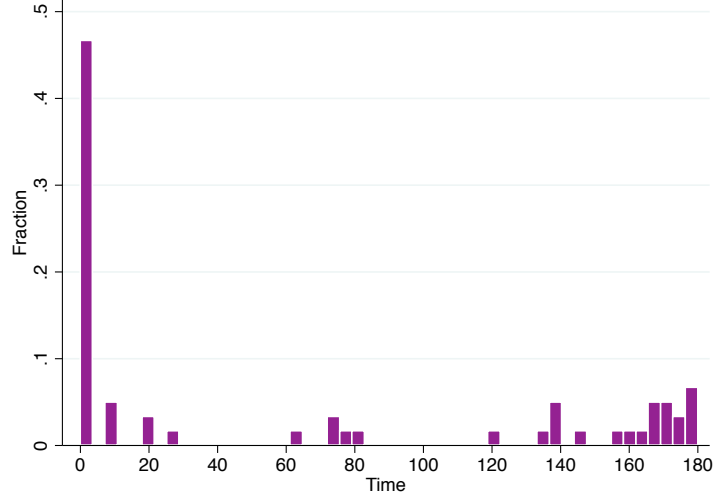


Figure 1: Last time each subject chose numbers above 67 in the SCP experiments.

As can be seen in Figure 1, more than 45% of subjects never chose a number above 67. However, it is plausible that some subjects make first choices that are instinctive, before even internalizing the structure of the game. For that reason, we define naive players as those who chose 67 at some point 30 seconds or more into the experiment.^{15,16}

According to our definition, 43.3% of subjects are naive players. This may underestimate the true number of naive players in the population since such players may never choose numbers above 67. Yet the proportion is consistent with other work that studies different games, such as Costa-Gomes and Weizsacker (2008), Ivanov, Levin and Niederle (2010) and Agranov et al. (2012).

It is instructive to ask at this point whether we really need SCP data to identify naive players. Could we instead have used the last choices of subjects, which is what is typically done in the literature? The answer is unequivocal: if one were to look only at the last choice made by subjects, all that is possible with standard choice data, only 5% (3 subjects) of subjects made choices above 67. Hence, without the SCP data, one is likely to dramatically underestimate the proportion of naive individuals in the population.

¹⁵Figure 1 shows that all of the subjects who chose numbers above 67 after 30 seconds also chose above 67 more than one minute into the experiment.

¹⁶All the results in this paper follow through if we instead choose 90 seconds as the cutoff for defining naive players.

Finding 2: Naive players average close to 50 (a) at the end of the game and (b) throughout the game.

Figure 2 shows the distribution of final choices of the naive players. Consistent with the standard assumption on naive play, 50 is within the 95% confidence interval for the mean final choice.¹⁷ We now consider the behavior of these players throughout the game.

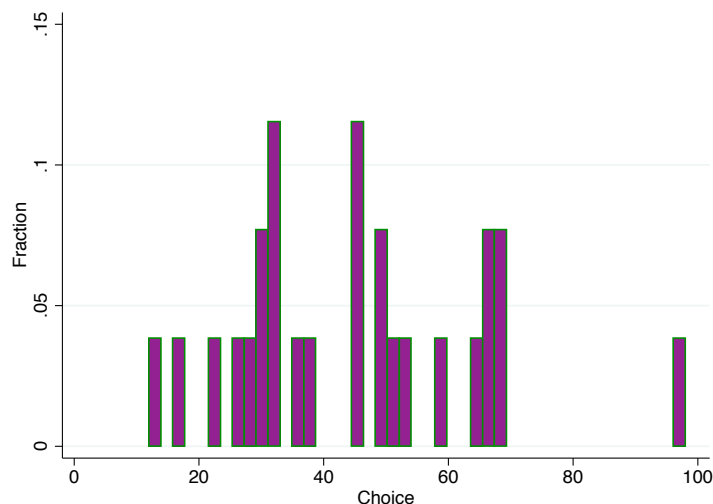


Figure 2: Distribution of choices of naive players at the end of the game.

	Naive Players	
	Mean Choice	Time Above 67
Seconds 1 to 60	53.4 (14)	30.9%
Seconds 61 to 120	51.06 (10.7)	25.9%
Seconds 121 to 180	52.4 (13.8)	27.6%

Table 1: Choices of naive players (standard deviations in parentheses).

Summary statistics on naive play are presented in Table 1. Over the course of the entire experiment, the average choice of naive players is 52.3. Further, as Table 1 indicates, the average choice of naive players remains fairly stable over the course of the experiment. Finally, the group of naive players spends close to a third of the time on choices above 67 throughout the whole experiment. These three observations provide further support

¹⁷The point estimate for the mean is just above 45, and the median is 45.5.

that the group we identify as naive players are indeed non-strategic (they choose numbers above 67 throughout the entire experiment, not only towards the beginning, or the middle, or the end). Further, our data provide support for the standard assumption in the literature concerning naive play.

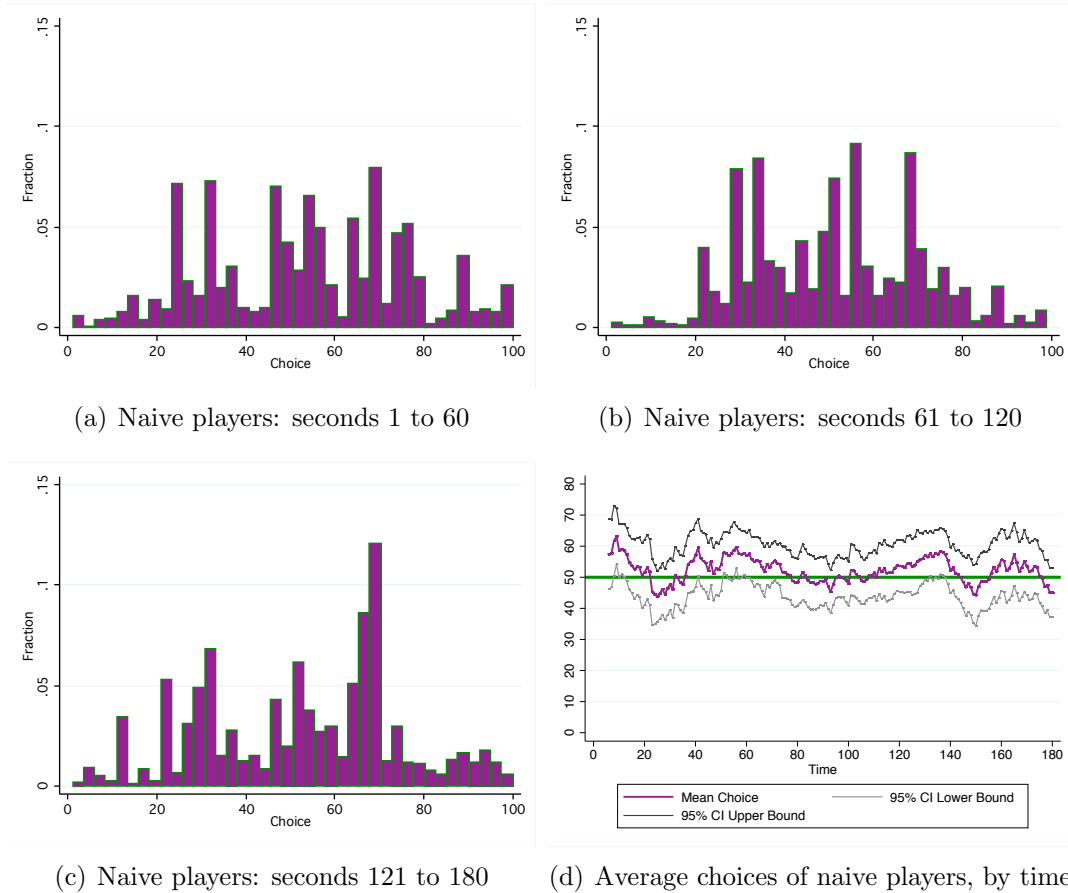


Figure 3: Choices of naive players

The first three panels of Figure 3 depict the histograms of all the choices made by naive players in the first 60, second 60 and third 60 seconds of the experiment, treating each choice for a given subject as an independent observation. Further to this, in the last panel of Figure 3, we present the average choice of naive players during the course of the experiment along with the 95% confidence interval for the mean choice.

It is quite striking that 50 is within the 95% confidence interval for average play of the naive types over 90% of the time. These players average close to 50 throughout the experiment. Hence they fit the intuitive conception of naive players through most of the period of play.

What makes this finding of such note is that our criterion to determine the naive subjects is the play of a dominated strategy. This does not in itself have strong implications

for the path of choice. Certainly, it does not imply that choices for the naive types should remain constant at close to 50 over the course of the experiment. We interpret the result as providing robust support for the hypothesis that naive players choices average 50 not only at the end of the game but throughout the game.

Finding 3: Strategic players’ choices decrease over time.

In this section we focus on those subjects who never or almost never choose dominated actions. We refer to these subjects as the strategic subjects.

Figure 4 displays the average choice as a function of consideration time in the SCP treatment for both the strategic and naive individuals. Figure 4 also displays the fitted regression lines using fractional-polynomial formulations. While Lk and Cognitive Hierarchy theories do not explicitly incorporate time, those with higher cognitive levels are treated as reasoning further through the game. One might expect their choices to have a decreasing trend over time, though this need not be the case if subjects adjust their beliefs concerning the play of others.

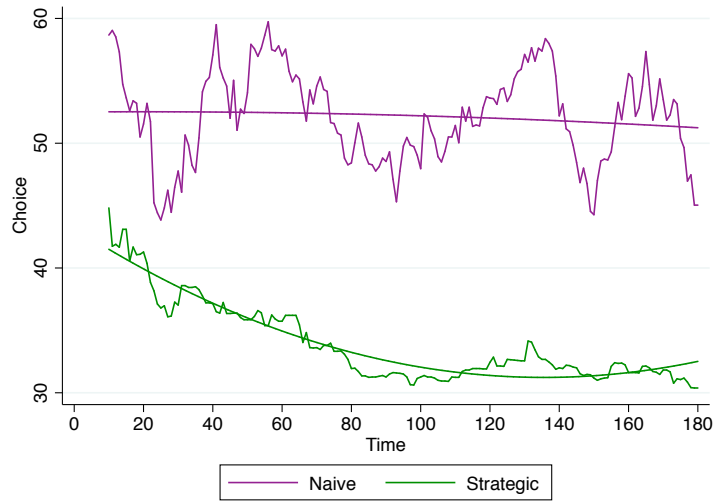


Figure 4: Choice over time: average data with fractional polynomial regressions superimposed.

Table 2 presents some statistics on the strategic and naive types. What is clear from Table 2 is that these types differ in aspects that are some way removed from the defining criterion of playing a dominated strategy. While time of first choice does not differ between these groups,¹⁸ naive subjects “change their minds” and switch numbers much more often than the strategic types.¹⁹ Further, the L0 types keep changing their minds

¹⁸A Ranksum as well as a Kolmogorov-Smirnov tests reject the null that both populations have identical medians or distributions, with p-values largely above 10%.

¹⁹Both a Ranksum and Kolmogorov-Smirnov test have a p-values strictly less than 1%.

	Naive Players	Strategic Players
Average Choice	52.3 (20.9)	34.1 (17)
Average Time of First Choice	6.5 (3.6)	7.8 (5.4)
Average Total Number of Switches	52.4 (57.5)	9.1 (18.4)
Average Time of Last Choice	161.8 (30.3)	102.1 (68.6)

Table 2: Some statistics on strategic and naive players.

almost to the end of the experiment, which is not the case for the strategic types.²⁰ In other words, our strategic types, while not defined that way, seem to converge faster to a final decision, making their decisions possibly more deliberate than those of the naive types.

The simplest form of introspective reasoning by which a subject may advance in type is based on best-responding to own past decision, which results in selecting precisely $2/3$ of the previous choice (a rapid learner might pick some power of $2/3$ by skipping levels of reasoning). One subject (subject 31) fit this pattern precisely, moving from 50 to 33 to 22, and retaining this choice for the remainder of the 180 seconds. In general, the fraction of switches that are very close to being in two thirds steps (plus or minus 5%) is higher for the strategic players than for the naive players. For example, more than 25% of strategic players make at least twenty percent of their switches in such steps. In contrast, only 8% of the naive group make this high a proportion of such steps. Even though the percentage of such adjustments is far from being the dominant feature of our data, it is far from negligible.

4 SCP and Other Games

In this section we present findings suggesting the value of the SCP data in other settings. We first explore behavior in the Monty Hall game to investigate the correlation between the sophistication of subjects in the two games. We then compare behavior in the SCP treatment with that in standard guessing games with distinct time constraints and show that the SCP protocol does not interfere with choice. Finally, we present results suggesting that the SCP protocol may be able to identify naive players in a broad class of games for which there is no dominated strategy.

²⁰Both a Ranksum and Kolmogorov-Smirnov test have a pvalues strictly less than 1%.

Finding 4: Strategic types defined through the SCP protocol are more likely to be strategic in the Monty Hall game.

The existing experimental literature on the guessing game suggests that there is little correlation in level classifications across games when classifications are based only on final choices.²¹

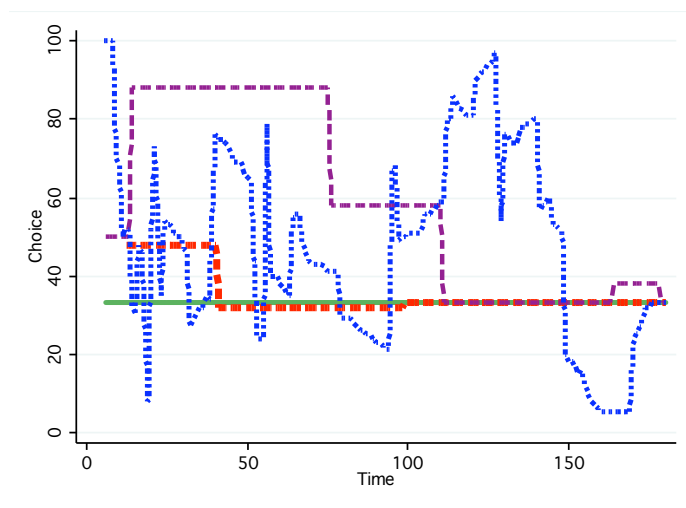


Figure 5: Is final choice enough? Paths of choice for four individuals with the same final choice.

One possible reason for the failure of types defined by level of reasoning to generalize is that they do not adequately summarize strategies, particularly when learning is taking place. If indeed a large fraction of the population is naive and making essentially random choices, final choice alone cannot be enough to identify those players. To drive home this point, Figure 5 presents four individuals whose final choice is the same (33), and who would be classified as L1 thinkers if only their final choice was observed. However, the manner in which they arrived at this final choice is dramatically different, and may contain information of value in understanding their behavior in various other environments.²²

We present simple results illustrating the potential value of understanding the path of play rather than just the final choice. Specifically, we consider the performance of naive and sophisticated players as identified in the guessing game in an entirely different game: the Monty Hall game. In this game, appropriate updating implies that the respondent

²¹There is some evidence that at the population level the distribution of types across games may be stable (see for example Camerer, Ho and Chong (2004)). At the individual level, Georganas, Healy and Weber (2010) find that though there is a correlation of levels within guessing games, choices in the guessing games fail to correlate with behavior outside the guessing game family. Burchardi and Penczynski (2010) reach similar conclusions.

²²These and all other complete paths of choice are in Appendix C.

should switch doors, yet it is intuitively plausible that it is equally good to remain with the initial choice.²³ Table 3 below presents the results of four Probit regressions from the Monty Hall game. The dependent variable y_i equals 1 if participant i switched from the initially chosen door and 0 otherwise. Regression 1 and 2 use final choice and Nagel's types as independent variables, respectively. Regression 2 uses dummy variables for the L1 and L2 types as defined by Nagel (1995) using final choice alone, leaving the naive type as the control group. In Regression 3, the independent variables are a dummy equal to 1 if a subject is classified as strategic using SCP methodology as well as an interaction term with the final choice. In Regression 4, we add final choice as an independent variable.

	Regression 1	Regression 2	Regression 3	Regression 4
Final choice	-0.013 (0.009)			0.012 (0.015)
Level 1 (Nagel)		-0.475 (0.682)		
Level 2 (Nagel)		0.698 (0.572)		
strategic SCP			1.531*** (0.588)	2.101** (0.910)
strategic SCP x Final Choice			-0.031* (0.016)	-0.043** (0.022)
Constant	-0.168 (0.365)	-0.908** (0.440)	-1.020*** (0.0.298)	-1.59** (0.757)
# of obs	60	35	60	60
Log Likelihood	-33.8026	-16.808	-31.249	-30.891
Pseudo R ²	0.0285	0.1066	0.102	.112

Coefficient (standard errors reported in the parenthesis)

** - significant at 5% *** - significant at 1%

Table 3: Predicting behavior in the Monty Hall Game.

Table 3 clearly shows that the regressions that uses our behavioral types as predictors have statistically significant coefficients. In other words, taken alone, the final choice of subjects in the guessing game does not correlate with behavior in the Monty Hall game. However, as is clear from Regressions 3 and 4, whether or not a subject was naive in the 2/3 game is a predictor of play in the Monty Hall game. Further, individuals who belong

²³The instructions are in Appendix B.

to the strategic category are more than twice as likely to switch doors after they receive new information on which door does not contain the prize money.²⁴ The positive sign of the coefficient of the interaction term shows that among those who are strategic in the 2/3 game, those who end up with a lower final choice are also more likely to switch door.

In intuitive terms, this suggests that the types that we identify as strategic in the guessing game are better than others at incorporating new information, whether this information results from internal reflection or a change in the information set on which to base a decision.

Finding 5: The SCP protocol is equivalent to a series of time constrained games.

By definition, a subject can play an unfamiliar game one and only one time. This poses a challenge for those seeking to understand how the contemplation period interacts with the final decision. For one so interested, the standard procedure (between subject design) requires the use of separate pools of subjects for each time constraint. Therefore, the standard procedure does not easily and cost-effectively allow the identification of individual learning differences.²⁵

The SCP experiment is designed to elicit from one individual an entire sequence of time-constrained choices in their very first play of an unfamiliar game. The extent to which this design provides information on how time constraints impact play depends on whether or not the choices it gives rise to are different than those in the corresponding sequence of time-constrained games. A first hint in this direction is provided by the finding that naive players average close to 50 throughout the game. Our experimental design allows us to investigate this issue in more detail based on the Standard Experiments that were conducted with 30 and 180 second time constraints.

Table 4 displays a powerful similarity between the SCP and Standard Experiments of equivalent horizon. A two-sample Wilcoxon rank sum (Mann-Whitney) test comparing the full distribution of 30 second choices in the SCP and in the Standard Experiment shows that we cannot reject the hypothesis that the two samples are from the same distribution ($p > 0.10$).²⁶ The same holds true when comparing the 180 second choices

²⁴In the data, over 35% of those who belong to the strategic category switch door, while only 15% of those who belong to the naive category group switch door. A test of proportions confirms that this difference is significant. The two-sided p-value for the test of proportion is 0.084, rejecting the null that the probability of switching door is the same for people in both groups.

²⁵At the very least, it requires a large sample to adequately control for individual differences.

²⁶Similarly, a two-sample Kolmogorov-Smirnov test for equality of distribution functions gives us the same results ($p > 0.10$).

	# of Obs	Mean Choice	Median Choice
30 seconds - Standard	66	42.83 (20.13)	42
30 seconds - SCP	60	41.68 (19.95)	42
180 seconds - Standard	62	36.35 (20.24)	33
180 seconds - SCP	60	36.73 (18.34)	33

Table 4: Summary Statistics of Choices in Standard and SCP Treatments (standard deviation in parenthesis).

in the SCP and Standard Experiments. Figure 6 presents the histograms of choices in Standard 30 second and 180 second treatments and SCP experiments at 30 and 180 seconds.

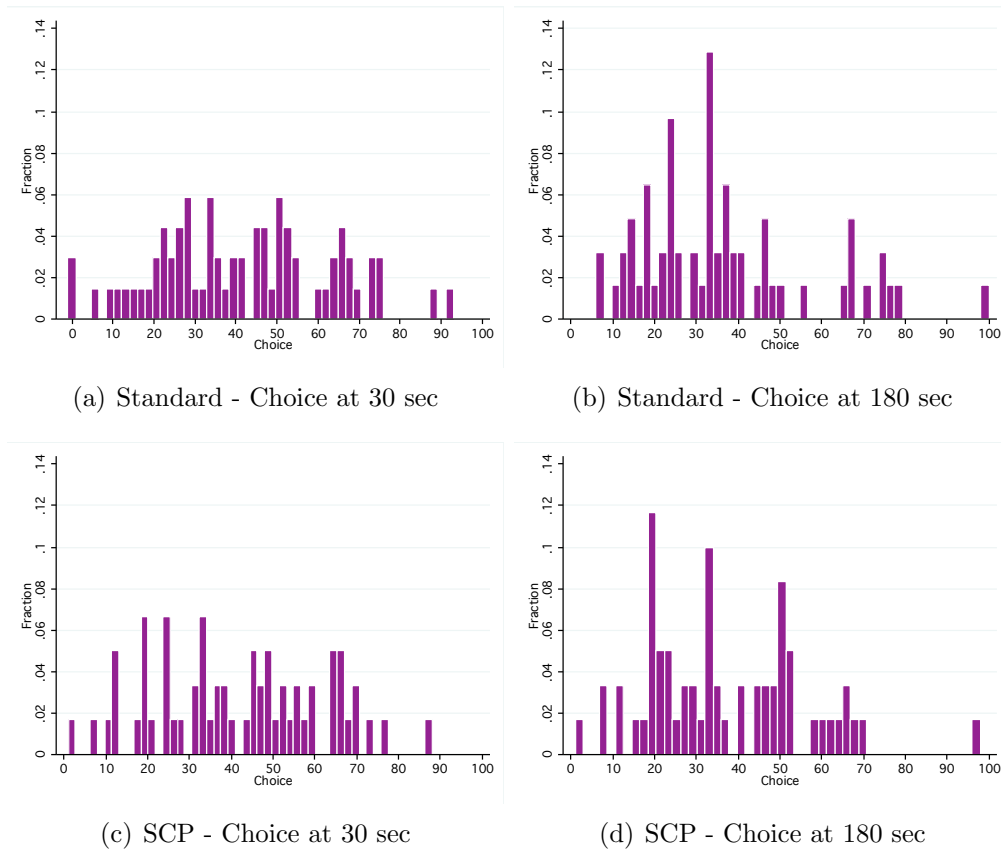


Figure 6: Histogram of Final Choices in the Standard and SCP Experiments

The results at 180 seconds in the SCP treatment and in the 180 second standard treatment are not only similar to one another, but also similar to those identified in the pioneering work of Nagel (1995), Ho, Camerer and Weigelt (1998), and Camerer, Ho and Chong (2004). In Table 5, we report the best fitting estimate of τ parameter (based

on the cognitive hierarchy model of Camerer, Ho and Chong (2004)²⁷) as well as the 90% confidence interval for τ from a randomized resampling with replacement bootstrap procedure. The table clearly indicates the similarity between play at 180 seconds in our SCP treatment, in our Standard Experiments, and in prior studies of guessing game.

	# obs.	Mean Choice	τ	Bootstrap 90% C.I.
Standard Experiment 30 Seconds	66	42.83	0.5	[0,0.25]
SCP Experiment 30 Seconds	60	41.68	0.6	[0,0.31]
Standard Experiment 180 Seconds	62	36.35	1.1	[0.41,1.33]
SCP Experiment 180 Seconds	60	36.73	1.06	[0.45,1.72]
Ho, Camerer, Weigelt (1998) $p = 0.7$	69	38.9	1	[0.5,1.6]
Nagel (1995) $p = 2/3$	66	37.2	1.1	[0.7,1.5]
Agranov et al. (2012) $p = 0.64$	91	35.1	1.13	[0.48,1.36]

Table 5: Estimating τ from Cognitive Hierarchy model of Camerer, Ho and Chong (2004).

We conclude that indeed there are strong similarities between SCP data and data on the corresponding sequence of time-constrained games. To a first approximation, the SCP treatment appears to be equivalent to multiple guessing games with different time constraints. In addition to allowing one to economize on the number of plays, using the SCP treatment to explore the impact of time constraints removes the need to control for individual differences: the players are one and the same regardless of the time constraint.

Finding 6: The SCP protocol can be used to identify naive players based on switching patterns

Table 2 showed that naive players that we identify with our dominated strategy methods tend to make many switches and to keep changing their choices until the end of the

²⁷The process begins with Level 0 players, who are assumed to play according to a uniform distribution. Level k thinkers assume that the other players are distributed according to a normalized POisson distribution (with parameter τ) from Level 0 to Level $k - 1$. Hence they correctly predict the relative frequencies of Levels 0 through $k - 1$, but may incorrectly believe that they are the only player of Level k and that there are no players more sophisticated than they are. The estimation of τ involves finding the value of τ that minimizes the difference between the observed sample mean and the mean implied by τ .

experiment. In games where there is no dominated strategy, this suggests the possibility of identifying naive players directly off the pattern of switching behavior.

	Correlation w/ strategic (Play above 67)	% who never play above 67
strategic (Std. Dev= 0)	0.38	87.5%
strategic (Std. Dev< 5)	0.64	89.7%
strategic (Std. Dev< 10)	0.76	86.5%
strategic (Std. Dev< 15)	0.55	71.7%

Table 6: Relationship between playing above 67 and standard deviation in second half of the experiment.

As an example, one can look at the standard deviation of choices in the last half of the experiment. Figure 7 shows the distribution of standard deviation of choices in the second half of the experiment. As Table 6 shows, those who are deemed strategic according to these measures are also largely deemed strategic according to Definition 1: strategic defined by either of these notions of convergence and strategic defined by play below 67 are highly correlated (with coefficients ranging from 0.38 to 0.76) and the great majority of those in the former group belong to the latter.

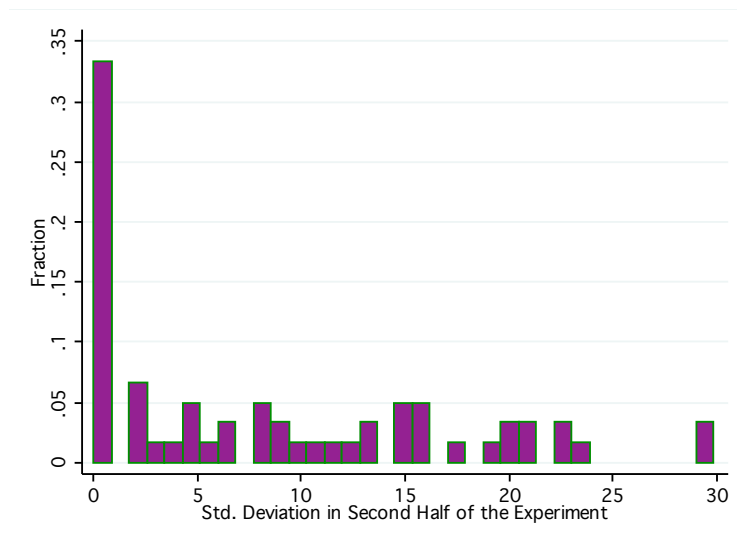


Figure 7: Standard deviation of choices in the second half of the experiment.

While our experiment was not designed with the “convergence” criterion in mind, defining strategic players using behavioral switches turns out to be as good a predictor for behavior in the Monty Hall game as the more theory-oriented measure we used in this

paper.²⁸ This suggests that it may be possible to use measures of convergence to identify naive players. Such measures are likely to be game-specific and to be guided by specific theories of strategic decision making.

5 Conclusions

We introduce a new experimental protocol to provide information on provisional choices in games, and hence the process of strategic decision-making. We implement our SCP treatment in the 2/3 guessing game and use it to identify naive players. We find that there is a high proportion of such players, that their choices average close to 50 for most of the three minute observation period, and that they differ in systematic fashion from the remaining strategic players. Additional findings suggest the broader value of the SCP methodology in understanding patterns of strategic play in a variety of other games.

²⁸In addition, the rest of our results are largely unchanged.

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A Instructions for the Choice Process Experiment

We will start with a brief instruction period. If you have any questions during this period, raise your hand. Experiment consists of two parts. You will be given instructions for the next part of the experiment once you finished this part. Anything you earn in the experiment will be added to your show-up fee of \$7.

PART I

We will start by describing what kinds of decisions you will be making in this game. We will then describe the rules of the game and the payments in this game.

Your task in this game is to choose a number from those presented on the screen.

The game lasts 180 seconds. At the top right corner of the screen you can see how many seconds are left. At the bottom right corner of the screen there is a “Finished” button. The rest of the screen is filled with buttons representing integer numbers between 1 and 100. They are arranged in decreasing order.

When the game starts, you can select the number by clicking on the button displaying the number that you want. You may click when you want, however many times you want.

The computer will record all the numbers you click on, as well as when you clicked on them.

After 180 seconds, or when you click the finish button, the round will come to an end and you won't be able to change your choice anymore. Just to make clear, if you choose a number and then stay with that number until the end, or instead decide to click on the “Finish” button, it will make no difference.

Only one of the numbers you selected will matter for payment. To determine which one, the computer will randomly choose a second between 0 and 180, each second is equally likely to be chosen. The number you selected at that time will be the one that matters. We will call this number “**Your Number.**” Below are two examples.

Example 1

Suppose you chose the button 100 for seconds 0 to 180. Suppose the computer randomly selects second 13 to be the random second.

Since at second 13 you were at button 100, 100 is “Your Number”.

Example 2

Suppose that after 10 seconds you selected the button 62. Suppose then that at second 55 you switched to button 40. Suppose that then at second 90 you switched to button 89 and then clicked on the Finish button.

In this case “Your Number” would be:

- if the computer randomly chooses a number between 0 and 9 seconds: none.
- if the computer randomly chooses a number between 10 and 54 seconds: 62
- if the computer randomly chooses a number between 55 and 89 seconds: 50
- if the computer randomly chooses a number between 90 and 180 seconds: 89

These examples are completely random and do not represent a hint at what you ought to do in this experiment. Note: once a button is clicked on, it becomes highlighted and you do not need to click on it again as it is already selected.

If you have not yet made a selection at the random second the computer chooses, then you cannot win this game.

Also, understand that if at any point you prefer a different number to the one you currently have selected, you should change the button you selected as this would reduce the chances of the less preferred number being recorded as “Your Number.”

The Structure of the Game

A few days ago 8 undergraduate students like yourselves played a game. Your payoff is tied to the choices made by those 8 students, so you need to understand the game they played. We will now distribute the rules of the game these 8 students played and the rules of the game you will be playing.

Your payoff will not depend on the choices made by the people in this room. It depends only on your choice and the choices these 8 students made a few days ago.

[Distribute the second set of instructions face down now. Wait for all to receive a copy. Read it out loud.]

The PAST game the 8 people played:

Each of the 8 students had 180 seconds to choose an integer between 1 and 100 inclusive, which they wrote on a piece of paper. After 180 seconds, we collected the papers. The winner was the person whose number was closest to two thirds of the average of everyone’s

numbers. That is, the 8 students played among themselves and their goal was to guess two thirds of the average of everyone's numbers.

The winner won \$10 and in case of a tie the prize was split.

The game YOU will be playing now:

You will have 180 seconds to choose an integer between 1 and 100 inclusive. You win \$10 if you are "better than" those 8 students at determining two thirds of the average of their numbers. That is, you win \$10 if **Your Number** is the **closest** to two thirds of the average of the numbers in the past game.

At any point, it is in your best interest to select the button corresponding to what you think is two thirds of the average of the numbers in the past game.

[Game starts right away.]

B Instructions for the Monty Hall Game

Screen 1

Behind one of these doors is \$5. Behind the other two is \$0. So, there is only one winning door.

Please choose one of the doors.

Screen 2

You have selected Door < their choice >.

We know which door contains \$5.

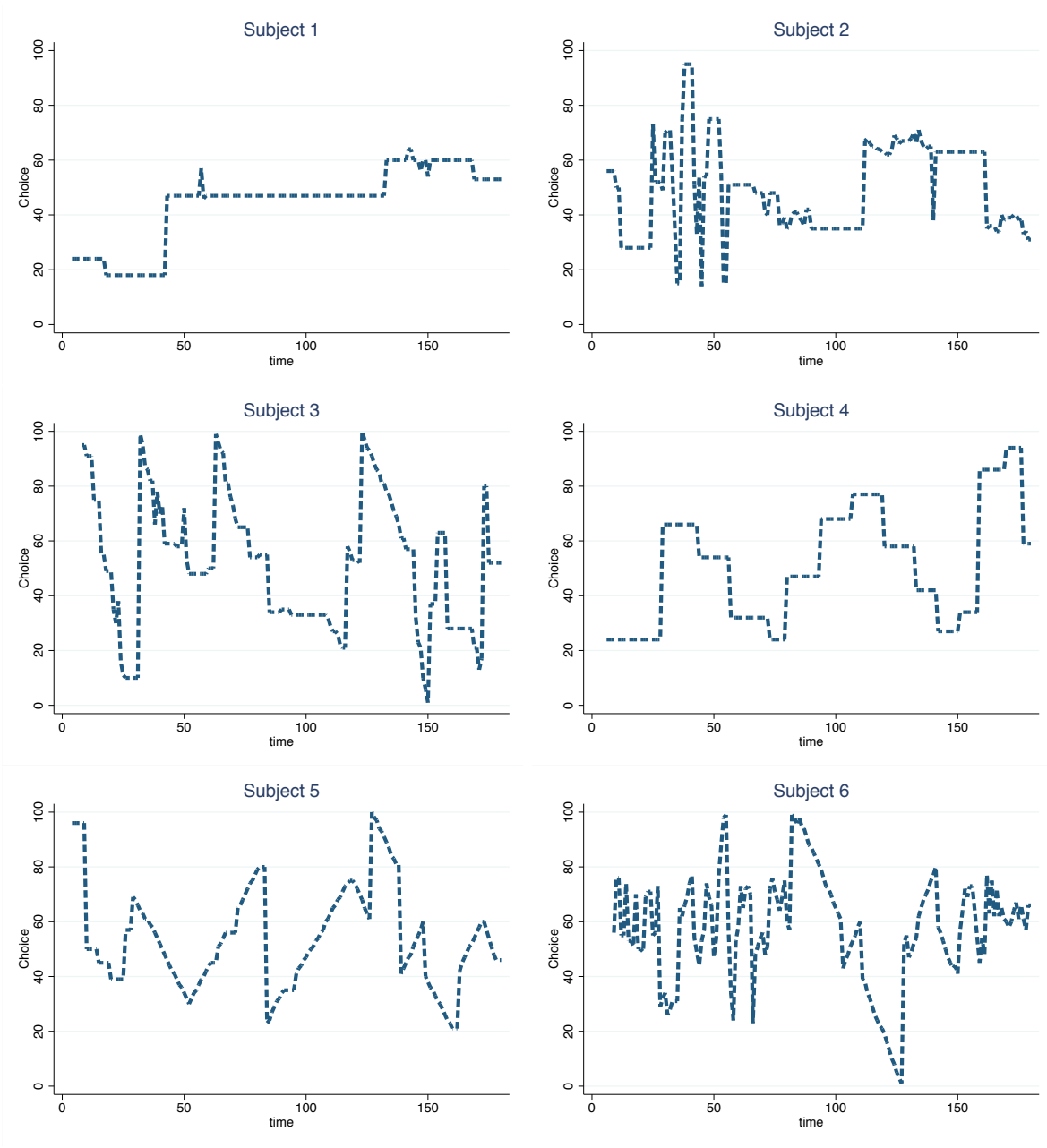
Before we open the door you selected, we are going to open one of the doors that contains \$0.

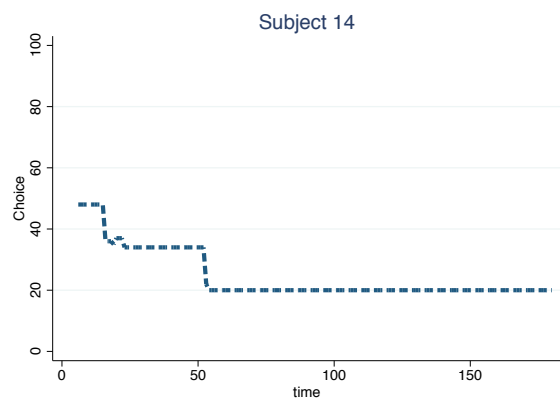
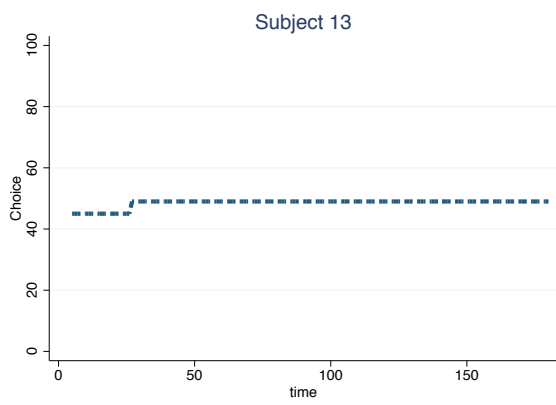
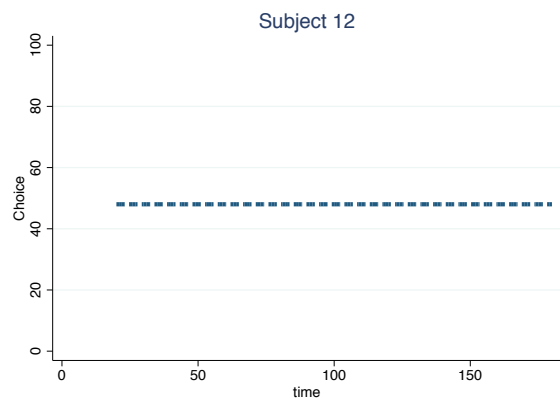
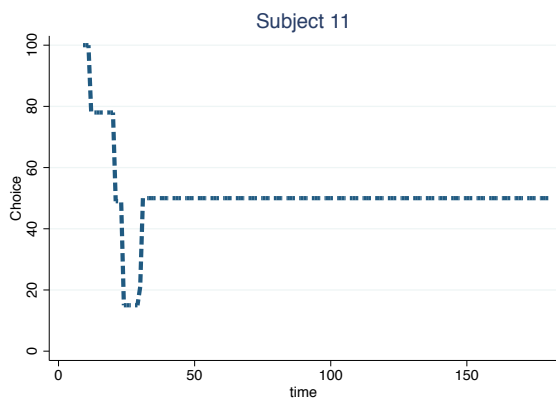
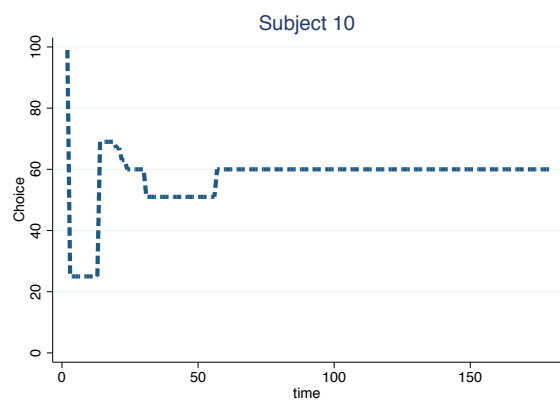
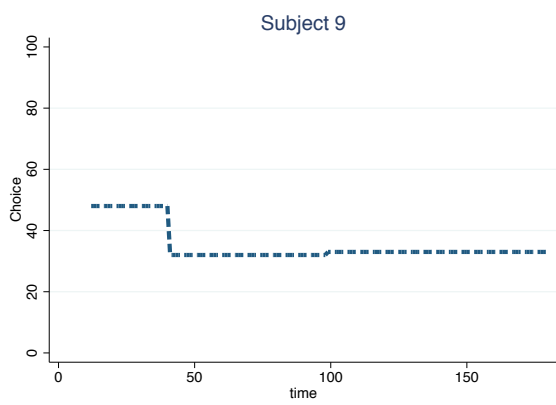
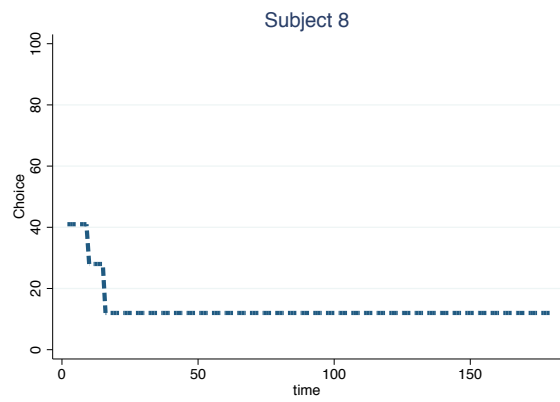
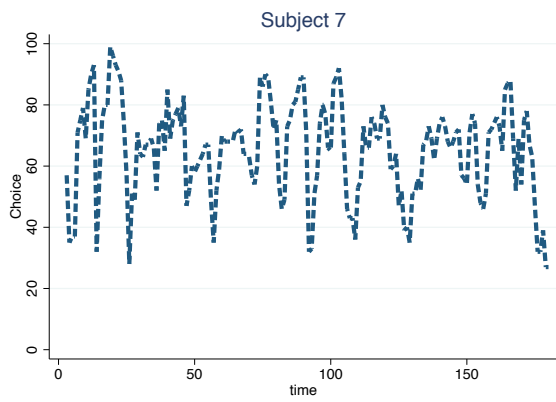
[We open one door that contains \$0.]

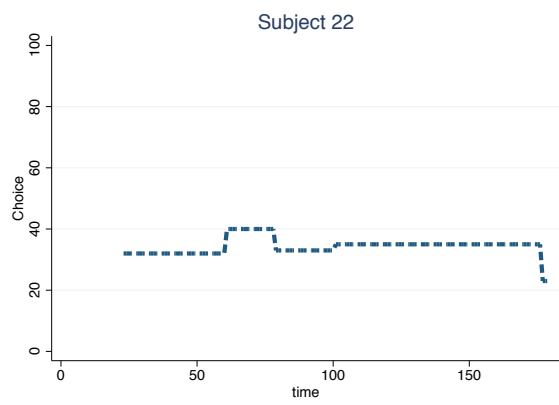
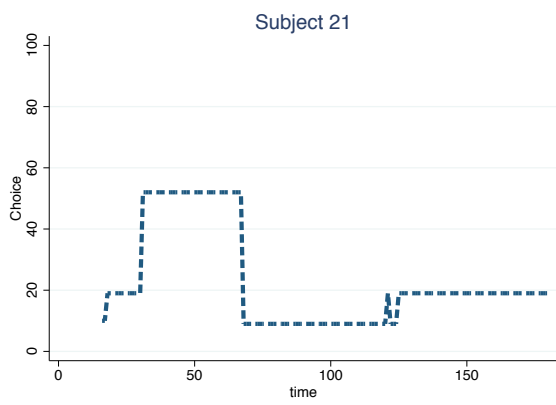
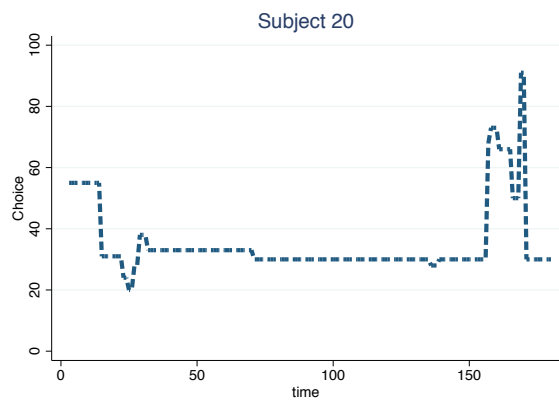
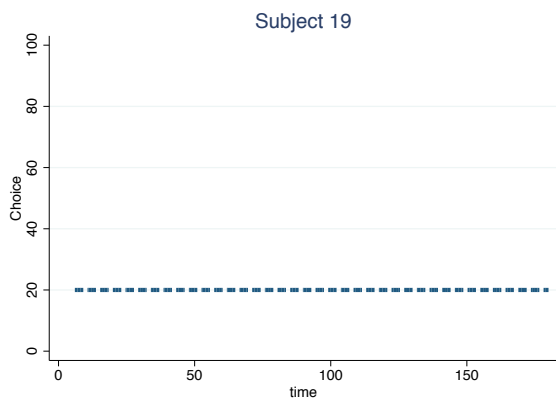
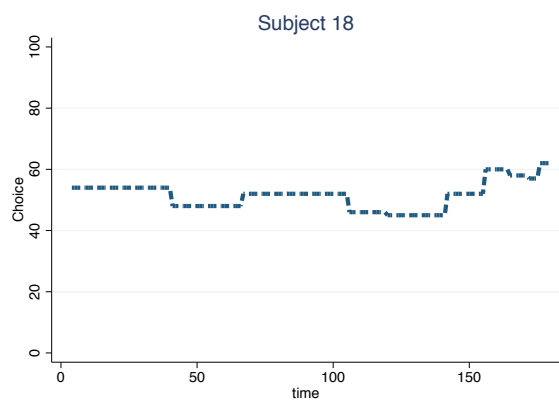
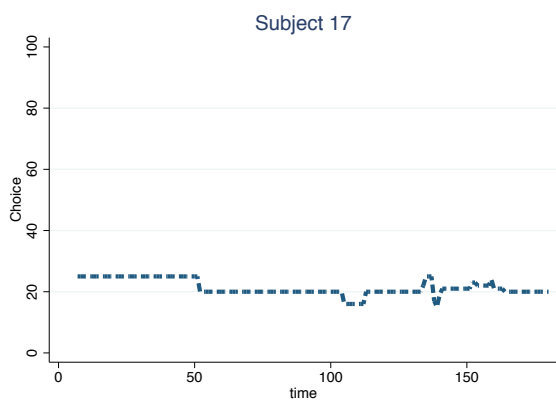
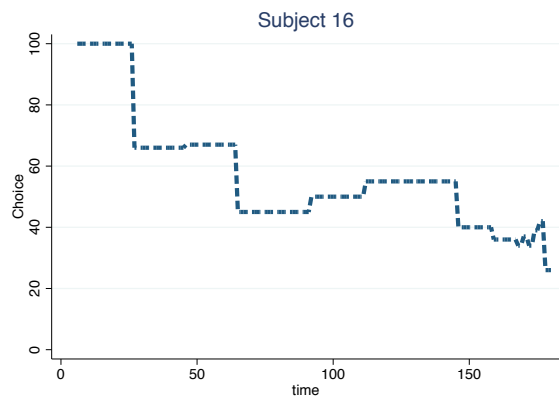
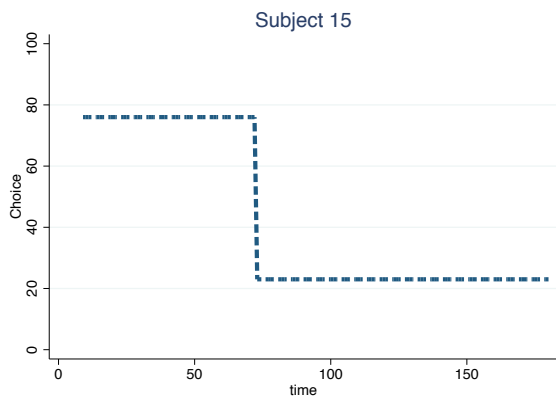
Screen 3

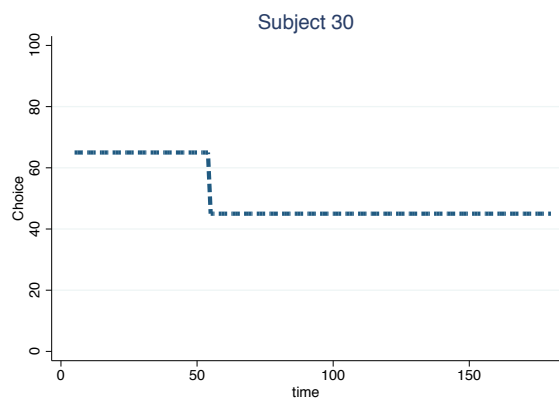
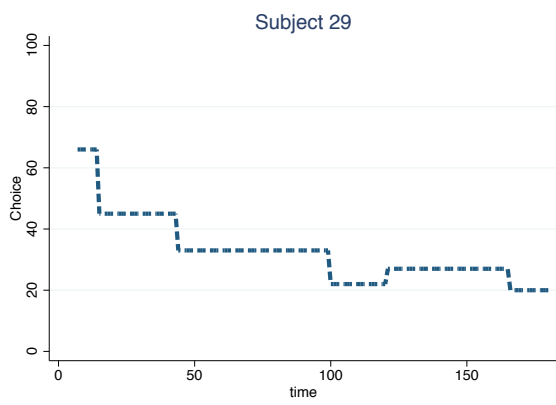
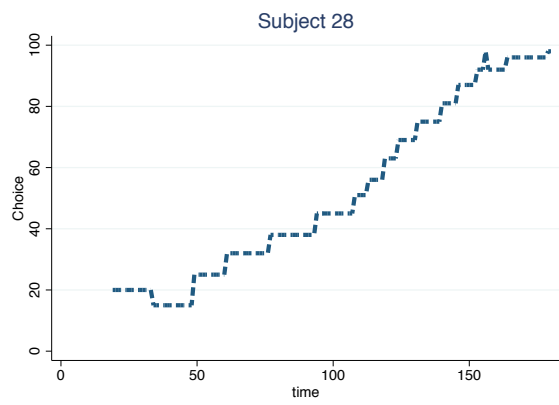
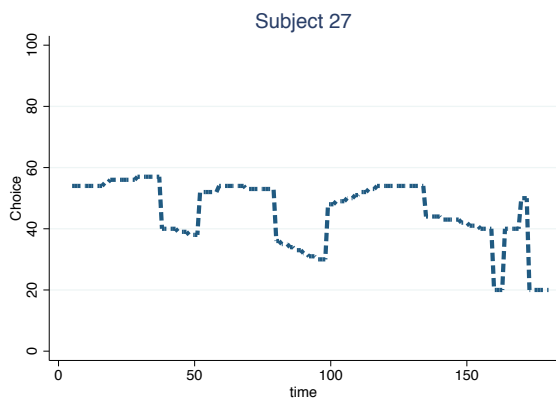
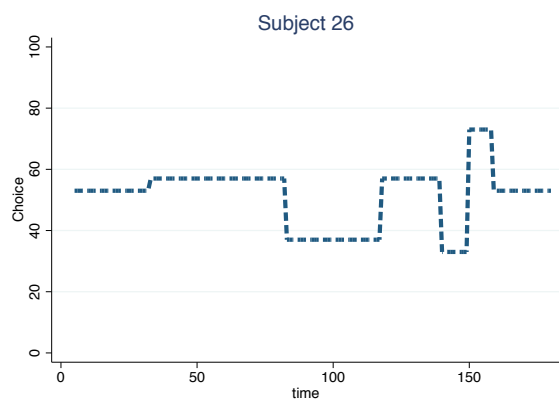
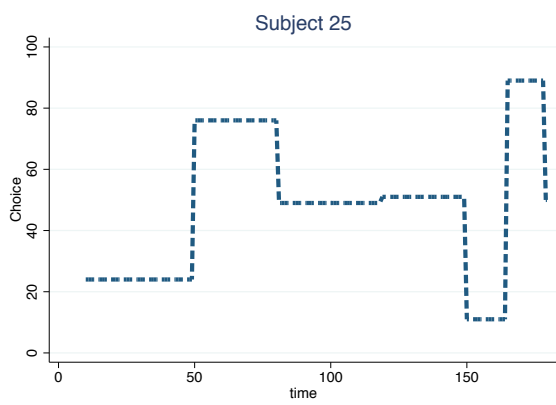
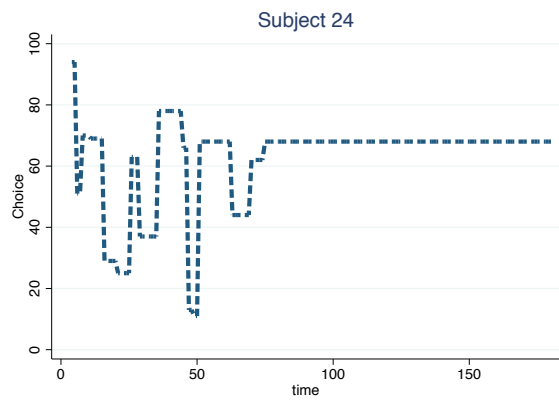
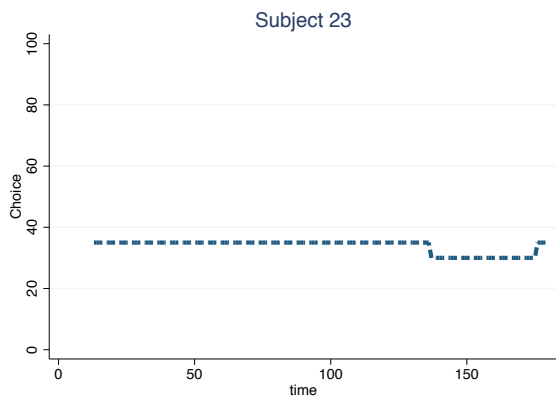
Do you want to keep Door < their choice > or switch to Door < other door >?

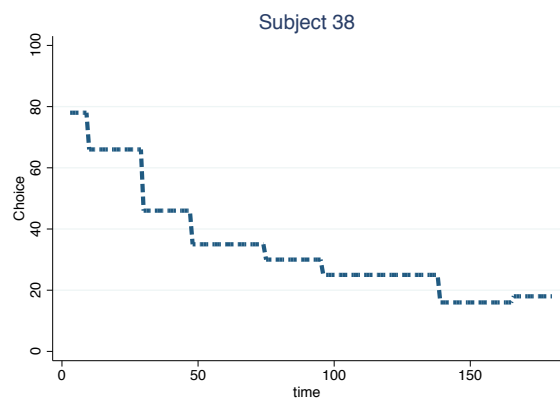
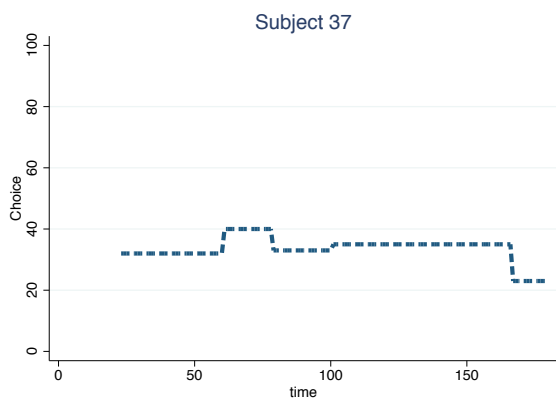
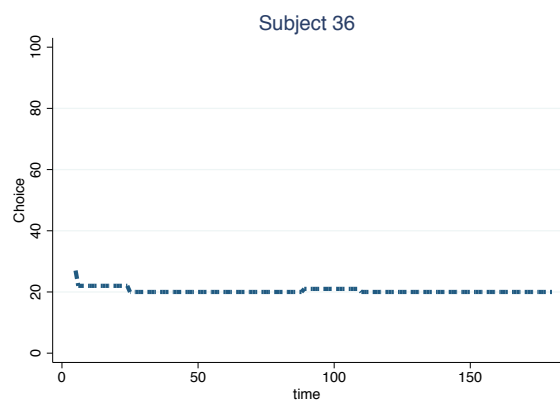
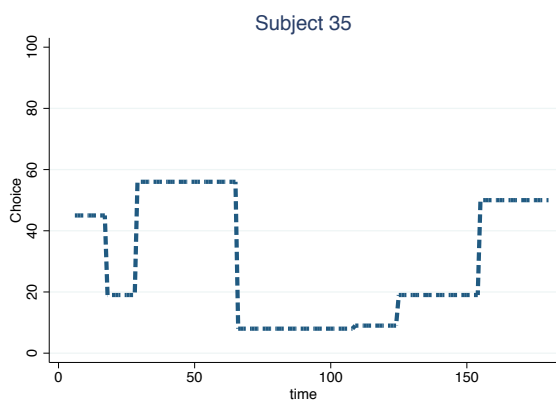
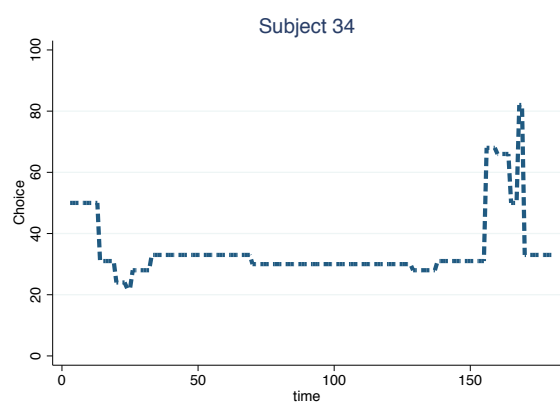
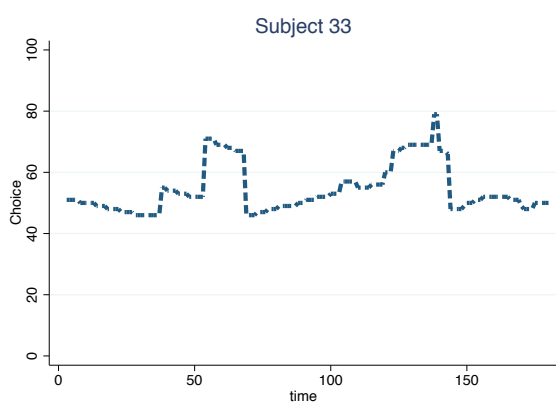
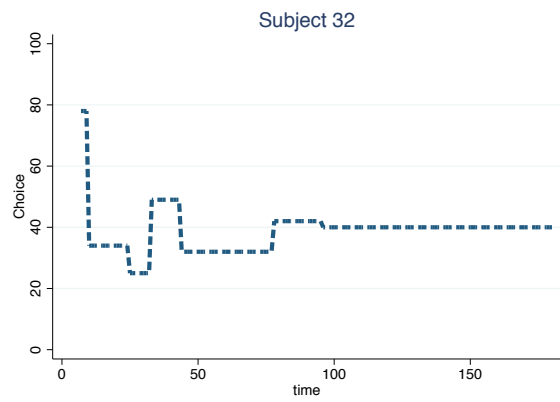
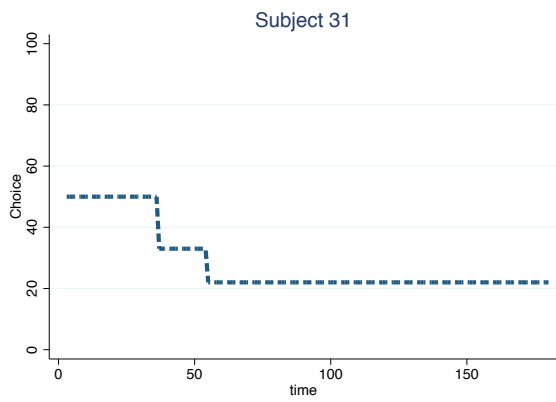
C Individual Paths

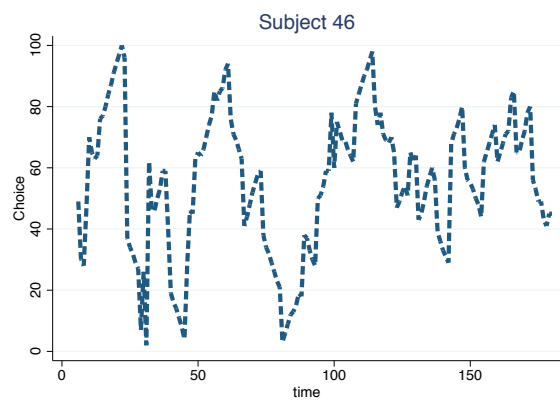
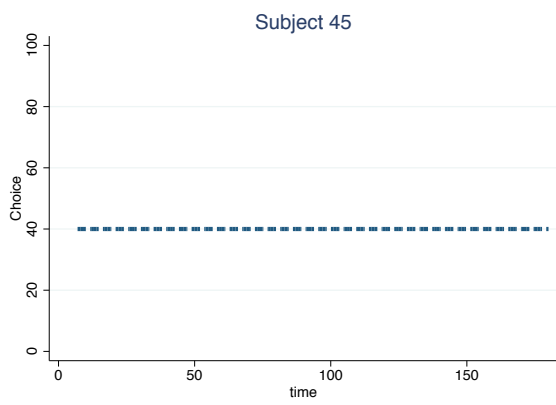
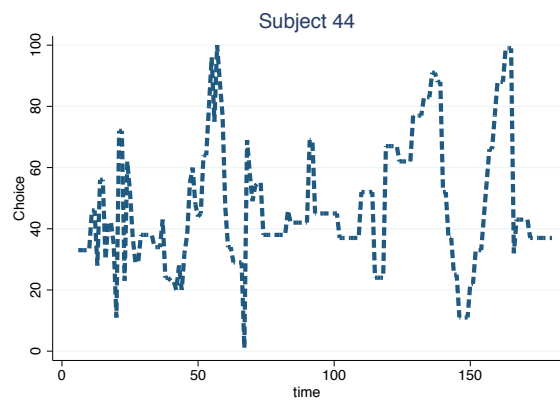
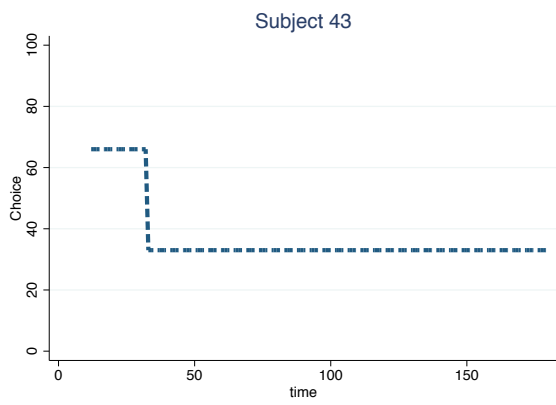
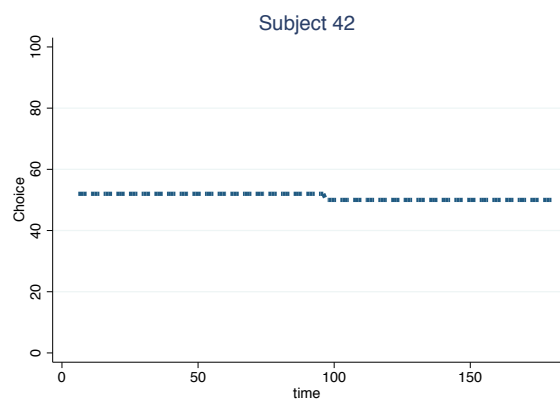
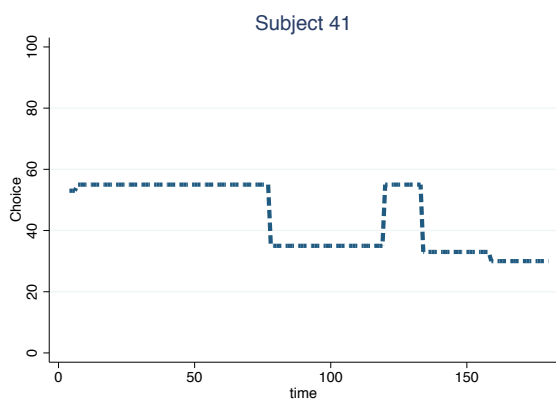
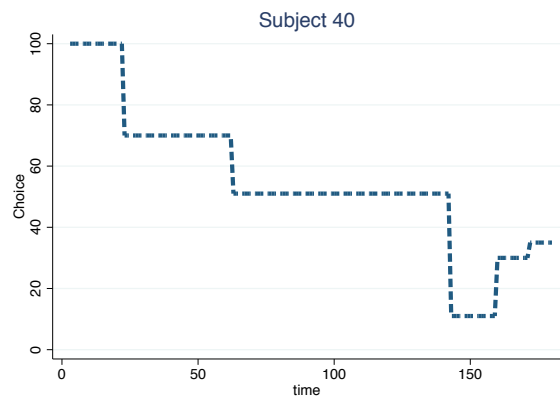
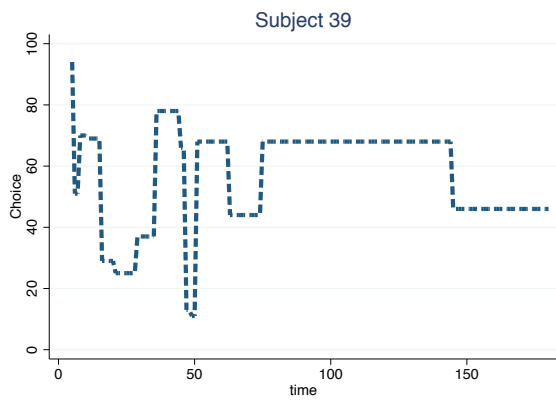


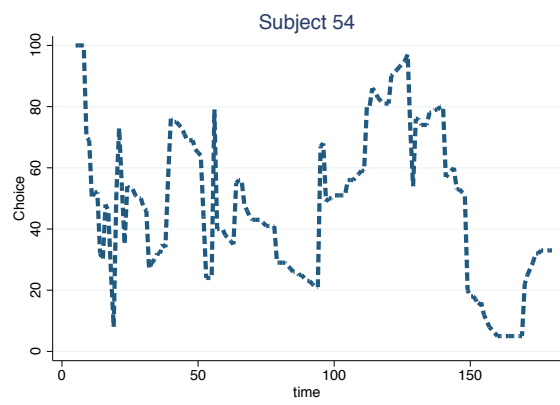
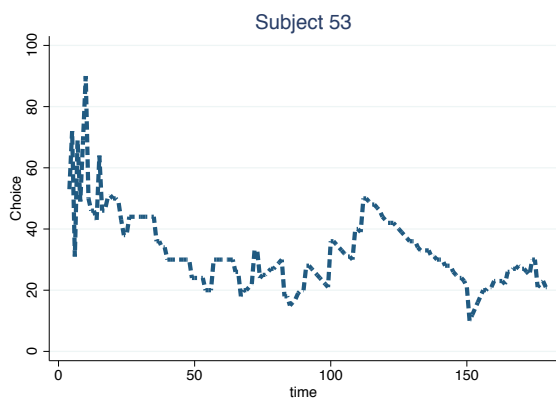
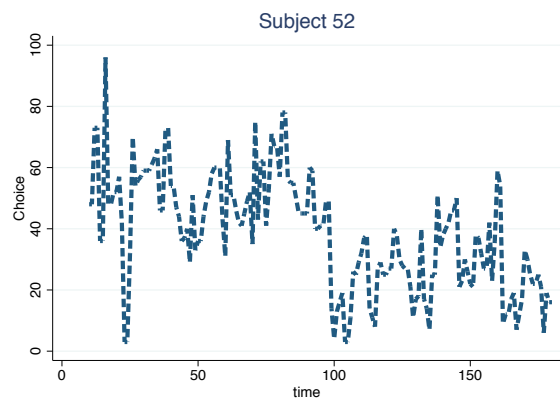
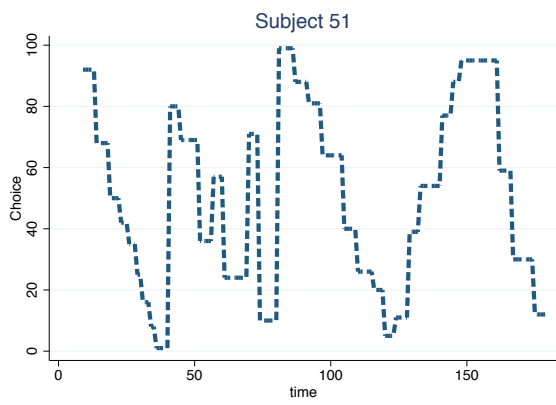
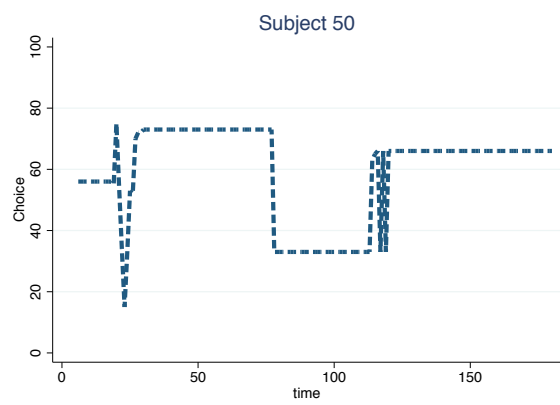
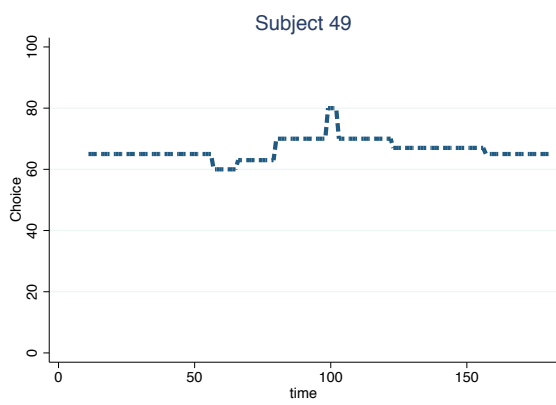
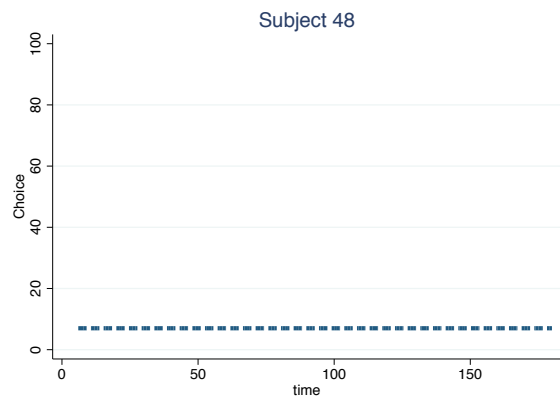
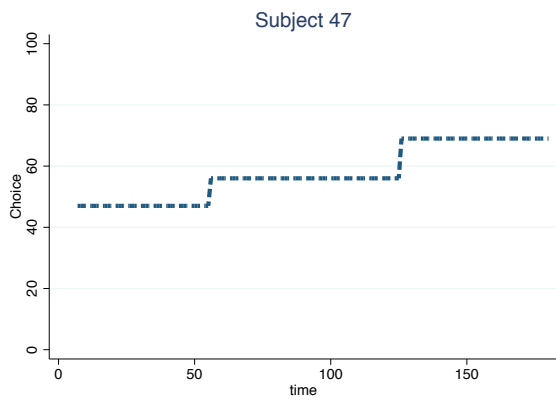












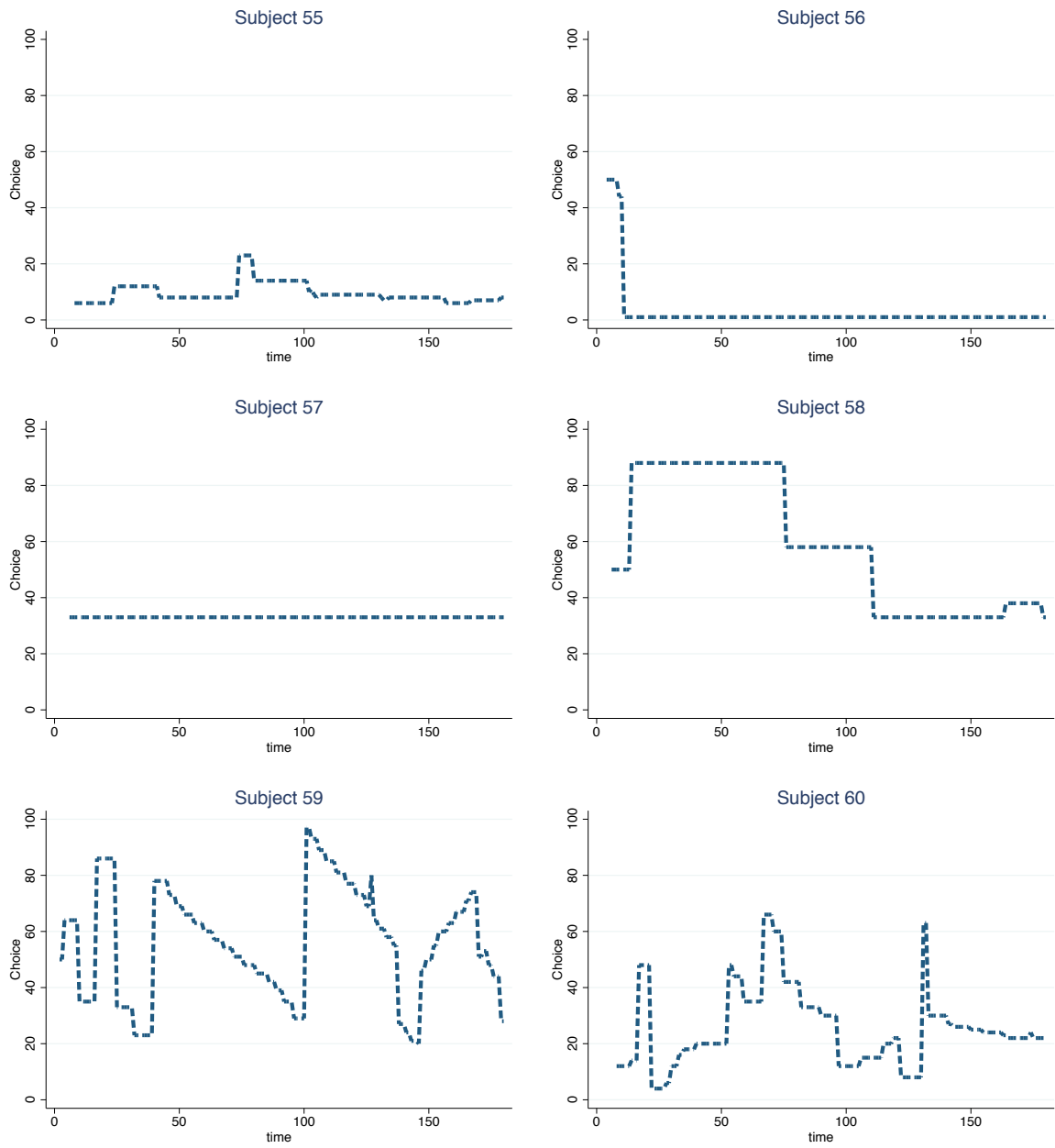


Figure 8: Individual time paths: choices over time.