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# Coulomb sum rule in the relativistic $\sigma\omega\rho$ model

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The Coulomb sum rule is studied using the random-phase approximation to the relativistic $\sigma\omega\rho$ model. The addition of the $\rho$ meson to the $\sigma\omega$ model has a large effect on the quenching of the sum rule because of its random-phase approximation correlation and its coupling with the isovector anomalous magnetic moment of the nucleon. The low-density surfaces of finite nuclei have a larger response than their interior so that the quenching of the Coulomb sum rule is not as significant as in the nuclear matter ground state. Calculations for finite nuclei using the local density approximation agree with existing experimental data on $^{12}C$, $^{40}Ca$, $^{48}Ca$, and $^{56}Fe$.

The Coulomb sum rule contains direct information of two-body correlations in a nuclear many-body system.[1] In fact, the *mathematical* Coulomb sum rule (MCSR) defined in Ref. 2 is proportional to the static two-body correlation function in the many-body ground state. However, the *physical* Coulomb sum rule (PCSR), in which the many-body final states with energy excitations $\omega_n < |\mathbf{q}|$ ($|\mathbf{q}|$ is the momentum transfer to the system) are summed over, has a much more complicated relation with the ground-state correlation because of the entanglements of the final states.[3] Nevertheless, the experimentally measurable PCSR is still an important quantity that constrains models of the many-body ground and excited states.

Actual measurements of the PCSR have been performed only recently when it became possible to separate the contributions to the electron scattering cross section from nuclear charge and current densities.[3-7] Many theoretical investigations have been made using the relativistic quantum field models of nuclei. The simple relativistic Fermi-gas model over predicts the PCSR by 20–30%.[1,2,8] The particle-hole random-phase approximation (RPA) correlations of the ground and excited states quench the independent-particle-model result by 15–20%.[9,10] The polarization of the dynamical vacuum introduces more quenching of the longitudinal response.[11-13] However, a calculation of the PCSR in finite nuclei with the $\rho$ meson contribution and the vacuum polarization is still missing. In this Rapid Communication, we carry out the calculation and compare the results with existing experimental data on $^{12}C$, $^{40}Ca$, $^{48}Ca$, and $^{56}Fe$.

The physical Coulomb sum rule is defined as the integral of the longitudinal response of a system

$$C(|\mathbf{q}|)=\int_0^{|\mathbf{q}|} S_L(|\mathbf{q}|,\omega)d\omega\,, \tag{1}$$

where $\omega$ and $|\mathbf{q}|$ are energy and three-momentum transfers to the system, respectively. In the local density approximation, $S_L(|\mathbf{q}|,\omega)$ is related to the integral of the imaginary part of the charge polarization tensor in local nuclear matter,

$$S_L(|\mathbf{q}|,\omega)=4\int_0^\infty r^2\,\mathrm{Im}\Pi(\rho,\rho)dr\,, \tag{2}$$

where $\rho$ specifies the charge density operator of the system and the dependence of the polarization on the energy-momentum transfer has been omitted. The polarization tensor $\Pi(\rho,\rho)$ can be calculated in the RPA as

$$\begin{aligned}\Pi(\rho,\rho)=\Pi^H(\rho,\rho)+\{&\chi_s[1-\chi_v\Pi^H(\gamma_0,\gamma_0)]\Pi^H(1,\rho)^2+\chi_v[1-\chi_s\Pi^H(1,1)]\Pi^H(\gamma_0,\rho)^2\\&+2\chi_s\chi_v\Pi^H(\rho,1)\Pi(1,\gamma_0)\Pi^H(\gamma_0,\rho)\}/\{[1-\chi_v\Pi^H(\gamma_0,\gamma_0)][1-\chi_s\Pi^H(1,1)]-\chi_s\chi_v\Pi^H(1,\gamma_0)^2\}\\&+\chi_\rho\Pi^H(\gamma_0\tau_3,\rho)^2/\{4-\chi_\rho\Pi^H(\gamma_0\tau_3,\gamma_0\tau_3)\}\,.\end{aligned} \tag{3}$$

The Hartree polarization tensors $\Pi^H(A,B)$ are defined as

$$\Pi^H(A,B)=-i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[AG^H(p)BG^H(p+k)]\,, \tag{4}$$

where $G^H(k)$ is the Hartree nucleon propagator; and $\chi_s$, $\chi_v$, and $\chi_\rho$ are the $\sigma$, $\omega$, and $\rho$ meson propagators, respectively.[10]

The polarization tensor $\Pi^H(A,B)$ can be separated into two parts. The first part is density dependent and vanishes when the nucleon density is zero. This part contains a term which cancels out the antinucleon propagation in the Fermi sea. The second part is the normal Feynman polarization of vacuum, and thus is infinite in magnitude and must be properly renormalized. The analytical expression for the density-dependent polarization was calculated in Ref. 9; the renormalization of the Feynman part can be found in Ref. 11. We should point out that the photon-vector-meson polarizations are nonrenormalizable because of the anomalous magnetic moment of the nucleon. However, if we assume the nucleon mass $M$ that appears in the nuclear magneton is changed to the effective nucleon mass $M^*$ in the medium, the polarizations become renormalizable; the result is[11]

$$\frac{1}{8\pi^2}(\kappa_p \pm \kappa_n)(k_\mu k_\nu - g_{\mu\nu}k^2) \times \int_0^1 dx \ln\left[\frac{M^{*2}-k^2x(1-x)}{M^2}\right], \quad (5)$$

where $\kappa_p$ and $\kappa_n$ are the anomalous magnetic moments of proton and neutron, respectively. The plus (minus) sign in Eq. (5) is used for isoscalar (isovector) vector mesons. Because the isovector anomalous magnetic moment is large, Eq. (5) constitutes a large contribution to the $\gamma-\rho$ polarization. However, the increase of the nuclear magneton by the effective nucleon mass contradicts with the experimental data on the transverse response function. To be consistent, Eq. (5) is empirically reduced by a factor of $M^*/M$ in the following calculations. Fortunately, the longitudinal response has a small sensitivity to this modification (less than 3% in the present calculation).

The Coulomb sum rule for the nuclear matter ground state ($k_F=1.30$ fm, $E_B=15.7$ MeV) is calculated in various approximations, and the results are presented in Fig. 1. The calculation is done with equal numbers of protons and neutrons ($Z=N=20$) and can be compared with existing $^{40}$Ca data,[5] also shown in the figure. The parameters used here are same as those used in Ref. 11. The heavy dotted line shows the result of the Hartree approximation, which essentially is the relativistic Fermi-gas result with corrections from the effective nucleon mass and the nucleon form factors. The Hartree calculation overestimates the Coulomb sum rule by a large amount compared with the $^{40}$Ca data. The dashed curve shows the RPA calculation without the Feynman part in the polarization tensors (the unrenormalized RPA). The reduction in the Coulomb sum rule resulting from the RPA correlation is about 20%. The further inclusion of the Feynman part (the vacuum polarization) in the RPA correlation (the renormalized RPA) reduces the PCSR by another 15–20%, as the solid curves shows. If the anomalous magnetic moment contribution to the Feynman part of the polarizations, expressed in Eq. (5), is not included, as in Refs. 12 and 13, the reduction of PCSR from the vacuum polarization is about 10–15%, shown in the dash-dotted curve. The extra 5–10% reduction from the anomalous magnetic moment is mainly isovector in character and is contributed from the $\rho$ meson.

We also calculated the PCSR in the renormalized RPA of the $\sigma\omega$ model, shown by the light dotted curve in Fig. 1. Our result is slightly bigger than that of the same calculation in Refs. 12 and 13 because of the inclusion of the isoscalar anomalous magnetic moment, whose sign is opposite to the isovector one. Comparing the light dotted curve to the solid one, the $\rho$ meson has a large effect on the PCSR although it affects little the properties of nuclear matter. Both the $\sigma\omega$ and the $\sigma\omega\rho$ seem to agree with the data on $^{40}$Ca because of the large experimental error bars. Therefore, one cannot draw a definite conclusion about the necessity of the $\rho$ meson from Fig. 1.

However, for a finite nuclear system, it is important to include the $\rho$ meson as well as its isovector coupling with the anomalous magnetic moment of the nucleon. In Fig. 2, we show the results of the local density calculation for $^{40}$Ca with similar approximation steps as used in the nuclear-matter calculations. The radial baryon density and effective nucleon mass distribution in $^{40}$Ca are calculated with the self-consistent Hartree-approximation.[14] At the Hartree level, the PCSR calculated in the finite nucleus is already larger than that in nuclear matter, especially at small momentum transfers. This is due to the effects of the large effective nucleon mass and the low baryon density on the nuclear surface where the Pauli principle is not very effective and the response to the virtual photon probe is generally strong. The unrenormalized RPA correlation introduces about a 10% reduction of the

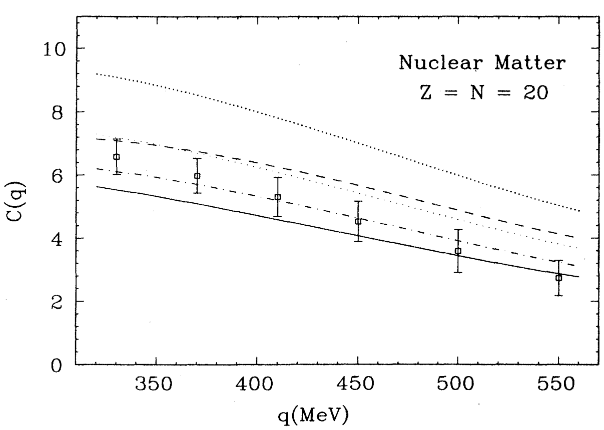


FIG. 1. The Coulomb sum rule for nuclear matter with $Z=N=20$. The heavy dotted curve shows the Hartree approximation; the dashed curve shows unrenormalized $\sigma\omega\rho$ RPA; the solid curve shows the renormalized RPA; the dash-dotted curve shows the renormalized RPA without the anomalous magnetic form factor. The light dotted curve shows the renormalized RPA in the $\sigma\omega$ model. The experimental data for $^{40}$Ca are taken from Ref. 6.

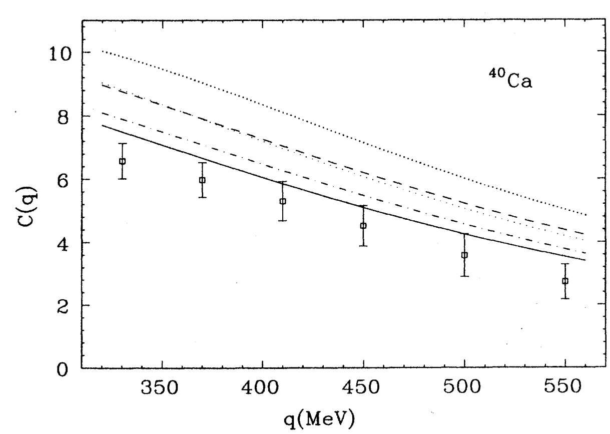


FIG. 2. The Coulomb sum rule for $^{40}$Ca calculated in the local density approximation. The different curves represent the same approximations used in Fig. 1.

PCSR. The renormalized RPA calculation reduces the PCSR by another 15% and brings the theory into good agreement with experimental data. The $\rho$ meson coupling with the anomalous magnetic moment of the nucleon seems to be necessary to quench the sum rule sufficiently. The $\sigma\omega$ model alone clearly over predicts the PSCR in the $^{40}$Ca nucleus, as shown by the light dotted curve. The degree of the agreement between the solid curve and the experimental data is similar to that shown in Fig. 2 of Ref. 13, where the calculation was performed in nuclear matter using the $\sigma\omega$ model.

The same model calculations for three other nuclei, $^{12}$C, $^{48}$Ca, and $^{58}$Fe, are presented in Fig. 3 by the solid, the dashed, and the dotted curves, respectively. The experimental data taken from Refs. 3 and 4 are also shown in the figure for comparison. A few comments about the calculation are necessary. First, for the $^{12}$C nucleus the Hartree PCSR (not shown) is only slightly bigger than the experimental data. Therefore, the quenching of the sum rule resulting from the RPA correlations in the $\sigma\omega\rho$ model should not be as strong as in $^{40}$Ca. This expectation is observed from the comparison of the solid curve and the data (shown in squares) in Fig. 3. The small strength reduction by the correlations is again due to the fact that most of the nucleons in $^{12}$C are on the surface. Second, the theoretical predictions of the PSCR for both $^{40}$Ca and $^{48}$Ca nuclei are almost identical. This is in contrast with the experimental finding that the total response in $^{48}$Ca is stronger than that in $^{40}$Ca for large momentum transfers. For instance, the isotopic increase of the PCSR is about 10% at momentum transfer 550 MeV. This discrepancy between the model prediction and the data is probably due to the inaccurate proton distribution generated in $^{48}$Ca by the renormalized Hartree calculation. Finally, the $^{56}$Fe nucleus is not doubly magic; a more sophisticated method should be used to calculate its ground-state baryon density. In the present study, we used a spherical Hartree solution in which the two valence neutrons are restricted to the $1p_{3/2}$ shell and the two proton holes to the $0f_{7/2}$ shell. The comparison between experiment and theory in this case should be taken less seriously.

Further refinement of the present calculation is needed. First of all, the quality of the local density approximation used in this work deserves more study. The unrenormalized RPA equation has been solved exactly in coordinate space in Ref. 15; however, inclusion of the $\rho$ meson and the dynamical vacuum, as well as the nucleon Pauli form factor, complicates the problem enormously. Second, the parameters used in this study are taken from the fit of the finite-nucleus renormalized Hartree calculation to experimental ground-state properties.[14] In doing the RPA approximation, we effectively treat the ground state of a nucleus with correlations. In principle, parameters such as the mass of the scalar meson, have to be refitted to the properties of the ground state in the RPA model. In that sense, our calculation is still inconsistent. Finally, the assessment of high-order effects, such as two-particle two-hole excitations, is necessary in the present type of model calculations.

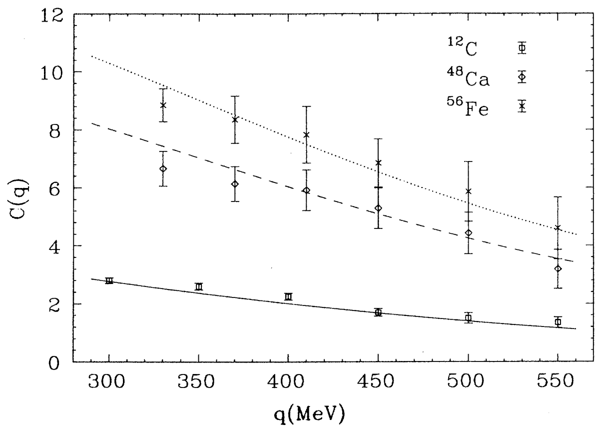


FIG. 3. The Coulomb sum rules for $^{12}$C, $^{40}$Ca, and $^{56}$Fe. The experimental data is taken from Refs. 5 and 6. The $\sigma\omega\rho$ RPA calculation is shown by the solid, the dashed, and the heavy-dotted curves for $^{12}$C, $^{40}$Ca, and $^{56}$Fe, respectively.

It is clear that there is no unique way to calculate the contribution to responses from the anomalous moment term of the $\gamma-\rho$ coupling. Any self-consistent field theoretical model of hadrons does not permit the appearance of the anomalous magnetic moment in bare Lagrangians. Introducing such a term in calculations like the present one inevitably causes the problems of renormalizability and double counting. Therefore the phenomenological studies cannot be fully consistent and without ambiguities. Nevertheless, its effect on the vacuum polarization can still be estimated in the lowest order by a reasonable way. This study shows at least qualitatively that the Coulomb sum rule is changed significantly by such a phenomenological term.

In conclusion, the total longitudinal response in finite nuclei is found to be larger than that in nuclear matter due to the low-density nuclear surface. The resulting enhancement in the PCSR is largely canceled by the RPA correlation induced by the $\rho$ meson and its coupling with the nucleon isovector anomalous magnetic moment. Therefore, the nuclear matter PCSR calculated in the $\sigma\omega$ model[13] is similar in magnitude to the present, more complete, calculation. Neglecting other uncertain factors, the $\sigma\omega\rho$ RPA calculation with the vacuum polarization reproduces the existing experimental data.

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