results of the present experiment). Combining this result with the relative measurements published previously[2] seems to indicate that the cloud-chamber results are higher than the delayed coincidence measurements. More exact determination of the reason for this difference is difficult because of the uncertainty in zenith angle distribution (as well as the variation in scattering among the different experiments). Moreover, the cloud-chamber results may still contain some small contributions from other particles such as electrons and π^- mesons, whereas the present experiment records only the $\mu^\pm$ and π^+ mesons which stop in the absorber.

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Comparison of Spin-Flip Dispersion Relations with Pion-Nucleon Scattering Data*

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The dispersion relations for the spin-flip, forward-scattering amplitude have been tested against pion-nucleon scattering data for energies up to 300 Mev. The Fermi set of phase shifts satisfy these relations while the Yang set do not. An approximate value for the renormalized coupling constant, $f^2=(g\mu/2M)^2$, of 0.1 is obtained from the P-wave phase shifts.

1. INTRODUCTION

THERE are four noninterfering scattering amplitudes for pion-nucleon scattering, corresponding to the independent possibilities of flipping the spin or isotopic spin of the nucleon. The squared magnitude of each gives its contribution to the differential cross section. Independent dispersion relations have been derived[1] for each of these amplitudes, which relate their real part to integrals over energy of their imaginary part.

We will consider here only the amplitude for spin flip, and examine the phase shift interpolations made by Anderson and Metropolis[2] in the light of these dispersion relations. We are thus imposing some new constraints on the phase shift determination problem. Since we are discussing the spin-flip amplitudes, we are in effect performing a theoretical polarization experiment and will in fact be able to differentiate between the Fermi and Yang phase shifts.

2. THE DISPERSION RELATIONS

Though the spin-flip amplitude vanishes in the forward direction, we can determine its derivative with respect to $\sin\theta$, where θ is the angle of scattering in the center-of-mass system, evaluated at $\theta=0$. This derivative (in the center-of-mass system) can be written as $(1/\mu)\bar{\eta}^2a$, where $\bar{\eta}$ is the center-of-mass momentum in units of μc and a is a dimensionless quantity which in general approaches a finite nonzero limit as $\bar{\eta}\to 0$. We will work with four a's, $a^{1,2}$ corresponding to isotopic spin nonflip and flip, and $a_{3,1}$ corresponding to total isotopic spin of 3/2 and 1/2. These are related by

$$\begin{aligned} a^1&=\tfrac{1}{3}(2a_3+a_1),\\ a^2&=\tfrac{1}{3}(a_1-a_3). \end{aligned} \tag{2.1}$$

The quantities a_3 and a_1 can be expressed in terms of the corresponding phase shifts by

$$a_{3,1}=\sum_{l=1}^{\infty}\frac{l(l+1)}{2i}\frac{1}{\bar{\eta}^3}(e^{2i\delta_{l+}}-e^{2i\delta_{l-}})_{3,1}, \tag{2.2}$$

where $\delta_{l\pm}$ is the phase shift for the state of orbital angular momentum l, total angular momentum $l\pm 1/2$, and total isotopic spin 3/2 or 1/2 as indicated outside the parenthesis.

To terms of order $(\mu/\varphi)^2$, the dispersion relations

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[1] This work has been done by several people. See, for example, M. L. Goldberger, *Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, to be published); M. L. Goldberger, Midwest Conference on Theoretical Physics, State University of Iowa, Iowa City, Iowa, 1956 (unpublished); R. Oehme, Phys. Rev. **100**, 1503 (1955); **102**, 1174 (1956); A. Salam, Nuovo cimento **3**, 424 (1956).

[2] H. L. Anderson, *Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, to be published).

for $a_\pm$ are

$$\mathrm{Re}(a^1)=2\frac{f^2}{\gamma}+\frac{2\gamma}{\pi}P\int_1^\infty d\gamma'\frac{\mathrm{Im}(a^1)}{\gamma'^2-\gamma^2},$$
$$\mathrm{Re}(a^2)=-2\frac{f^2}{\gamma}\left(\frac{\gamma_B}{\gamma}\right)+\frac{2}{\pi}P\int_1^\infty d\gamma'\frac{\gamma'\,\mathrm{Im}(a^2)}{\gamma'^2-\gamma^2},\tag{2.3}$$

where f^2 is the renormalized coupling constant ($f=g\mu/2M$, where g is the renormalized coupling constant of the symmetric pseudoscalar theory), γ is the total laboratory energy in units of μc^2, and $\gamma_B=\mu/2M$.

Our knowledge of δ_{33} is considerably better than that that of the other phase shifts, and by taking the difference of these two equations, we obtain one in which only a_3 appears on the left and in that part of the integral containing a singularity. Specifically,

$$\mathrm{Re}(a_3)=2\frac{f^2}{\gamma}\left(1+\frac{\gamma_B}{\gamma}\right)+\frac{1}{\pi}P\int_1^\infty d\gamma'\,\mathrm{Im}\left[\frac{a_3}{\gamma'-\gamma}-\frac{2a_1+a_3}{3(\gamma'+\gamma)}\right].\tag{2.4}$$

To within the accuracy of the experimental data, the contribution to the integral from the a_1 term is negligible and we will drop it. The contribution to the integral from large values of γ' may be separated out into a term independent of γ to give

$$\mathrm{Re}(a_3)=2\frac{f^2}{\gamma}\left(1+\frac{\gamma_B}{\gamma}\right)+2C+\gamma[I_3(\gamma)+\tfrac{1}{3}I_3(-\gamma)],\tag{2.5}$$

where

$$C=\frac{1}{3\pi}\int_1^\infty d\gamma'\frac{\mathrm{Im}(a_3)}{\gamma'},$$

and

$$I_3(\gamma)=\frac{1}{\pi}P\int_1^\infty d\gamma'\frac{1}{\gamma'-\gamma}\frac{\mathrm{Im}(a_3)}{\gamma'}.$$

If the energy dependence of a_3 is known up to sufficiently high energy to determine the integrals $I(\gamma)$ but not C, then we can consider f^2 and C as parameters and adjust them to fit the data. This can be conveniently done by plotting the experimental points so that Eq. (2.5) is represented by a straight line whose y intercept depends on f^2 and whose slope depends on C. We make the substitutions

$$x=\gamma\Big/\left(1+\frac{\gamma_B}{\gamma}\right),$$
$$y=\tfrac{1}{2}x\{\mathrm{Re}(a_3)-\gamma[I_3(\gamma)+\tfrac{1}{3}I_3(-\gamma)]\},\tag{2.6}$$

which satisfy

$$y=f^2+Cx.\tag{2.7}$$

3. COMPARISON WITH EXPERIMENT

The values of a_3 were obtained from the energy dependence of the phase shifts found by Anderson[2] to give the best fit to the present pion-proton data for energies up to 300 Mev. He has set D waves equal to zero. Since our dispersion relation does not contain S waves, only the P-wave phase shifts are involved. The energy dependence which Anderson obtained is

$$\bar{\eta}^3\cot\delta_{33}=\frac{1+0.77\bar{\eta}^2}{0.248}\left(\frac{1.9427-\bar{\gamma}}{0.9427}\right),$$
$$\bar{\eta}^3\cot\delta_{31}=-\frac{1}{0.0415-0.00775\bar{\eta}^2}.\tag{3.1}$$

Since we have only P waves, Eq. (2.2) reduces to

$$a_3=(e^{2i\delta_{33}}-e^{2i\delta_{31}})/i\bar{\eta}^3.\tag{3.2}$$

In Eq. (3.1), $\bar{\gamma}$ is the total pion energy in the center-of-mass system in units of μc^2.

The integrals, $I_3(\gamma)$, were evaluated numerically. In the neighborhood of the pole, the integrand varies rapidly, and the more direct methods introduce considerable error. However, the integral can be written as

$$I(\gamma)=\frac{1}{\pi}P\int_1^\infty d\gamma'\frac{1}{\gamma'-\gamma}f(\gamma'),\tag{3.3}$$

where

$$f(\gamma')=\mathrm{Im}(a_3)/\gamma'$$

is a smoothly varying function and can be well approximated by a sequence of straight lines. The integrals over each region in which $f(\gamma)$ varies linearly can be evaluated exactly. We choose equal intervals defined by $\gamma'=\gamma+n\Delta$, where n is a positive or negative integer and Δ is the length of each interval. At these points, we denote the values of the function by $f_n=f(\gamma')=f(\gamma+n\Delta)$. Then with this straight-line interpolation,

$$P\int d\gamma'\frac{1}{\gamma'-\gamma}f(\gamma')=\sum_n w_nf_n,\tag{3.4}$$

where

$$w_n=(n+1)\ln|n+1|+(n-1)\ln|n-1|-2n\ln|n|.$$

The w_n are constants independent of f and Δ and have the properties

$$w_{-n}=-w_n,$$
$$w_n=2\left(\frac{1}{1\times 2n}+\frac{1}{3\times 4n^3}+\frac{1}{5\times 6n^5}+\cdots\right),\tag{3.5}$$
$$w_0=0.$$

The values of x and y corresponding to laboratory kinetic energies of 0, 60, 100, 160, 200, 240, and 300 Mev are plotted in Fig. 1. The line drawn through the points has an intercept of $f^2=0.10$ and a slope of $C=0.03$. The contribution to C from the integral up to 300 Mev is 0.025, which is in reasonable agreement. This value of f^2 is somewhat higher than the value of 0.082 ± 0.015 which has been obtained from other extrapolation procedures,[3,4] but the difference does not appear beyond the range of experimental uncertainties, and these have not yet been evaluated quantitatively. Anderson's value for a_1-a_3 is $0.282\bar{\eta}$ as compared to the value of $0.27\bar{\eta}$ used by Chew and Haber-Schaim. This would raise their value slightly.

It is perhaps worth pointing out that our equations used for the determination of f^2 have the following advantages over those used previously: (1) They are more accurate than those of the Chew-Low theory in that they are rigorously correct to order $(\mu/M)^2$, and (2) they do not involve the rather poorly known S-wave phase shifts.

4. CHOICE BETWEEN FERMI AND YANG SOLUTIONS

The Yang set of phase shifts can be obtained from the Fermi set by changing the sign of the spin-flip amplitude. Specifically, if δ_{33} and δ_{31} are the Fermi phase shifts, then, defining θ by

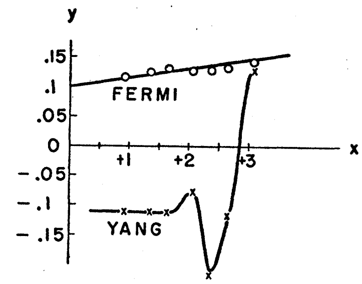
$$\tan\theta=\frac{2\sin2\delta_{33}+\sin2\delta_{31}}{2\cos2\delta_{33}+\cos2\delta_{31}},\tag{4.1}$$

(and the sign of $\sin\theta$ the same as that of $2\sin2\delta_{33}+\sin2\delta_{31}$), we obtain the Yang phase shifts from

$$\delta_{33'}=\theta-\delta_{33},\qquad \delta_{31'}=\theta-\delta_{31}.\tag{4.2}$$

[3] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[4] G. F. Chew, Midwest Conference on Theoretical Physics, State University of Iowa, Iowa City, 1956 (unpublished); U. Haber-Schaim, Phys. Rev. **104**, 1113 (1956), this issue.

FIG. 1. Plot of $y=\frac{1}{2}x\{\mathrm{Re}(a_3)-\gamma[I_3(\gamma)+\frac{1}{3}I_3(-\gamma)]\}$ *vs* $x=\gamma/(1+\gamma_B/\gamma)$ to test dispersion relations and obtain a value of the coupling constant by extrapolation.



Since this dispersion relation involves the spin-flip amplitude, it determines the sign of this amplitude and hence can exclude the Yang solutions, much as the nonflip relations determined the sign of the forward-scattering amplitude.[5] The values for a_3 corresponding to the Yang set were evaluated using Eqs. (4.1) and (4.2) and from these, the values of x and y were calculated for the same energies as in the Fermi case. This set of phase shifts extrapolates to a negative value for f^2, and at the higher energies deviates grossly from a straight line behavior. In lieu of an experiment to measure directly the polarization of the recoil nucleus, this represents a stronger argument for the Fermi solutions than that based on photoproduction data.[6]

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[5] Anderson, Davidon, and Kruse, Phys. Rev. **100**, 339 (1955); R. Karplus and M. Ruderman, Phys. Rev. **98**, 771 (1955).
[6] *Note added in proof.*—Gilbert and Screton have also used this dispersion relation to exclude the Yang phase shifts. A. Salam, CERN Symposium on High Energy Physics, Geneva 15, Switzerland (June, 1956).