Clipping Noise Cancellation in OFDM Systems Using Oversampled Signal Reconstruction

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***Abstract*—Clipping the OFDM signals in the digital part of the transmitter is one of the simplest methods to reduce the peak factor. However, it suffers from additional clipping distortion, peak regrowth after digital to analog conversion, and out-of-band radiation in the case of oversampled sequence clipping. In this letter, we use oversampled sequence clipping to combat the effect of peak regrowth and propose a method to reconstruct the clipped samples and mitigate the clipping distortion in the presence of channel noise at the expense of bandwidth expansion. We show through extensive simulations that by slightly increasing the bandwidth of the system, we can significantly improve the performance while limiting the maximum amplitude of the analog signal.**

***Index Terms*—Clipping, OFDM, oversampled signal reconstruction, peak to mean envelope power ratio (PMEPR).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive multicarrier modulation for high-speed wireless access in multipath fading environments. However, OFDM signals exhibit a large peak to mean envelope power ratio (PMEPR) which requires a highly linear amplifier; otherwise, the signal may suffer from significant spectral spreading and in-band distortion [1], [2].

Several alternative solutions have been proposed to reduce the PMEPR of OFDM signals. Clipping the OFDM signal in the digital part of the transmitter seems to be the simplest method. However, digital clipping suffers from three problems: in-band distortion, which degrades the bit error rate (BER) performance [1], out-of-band radiation, which reduces the spectral efficiency [2], and peak regrowth after digital to analog conversion, which results in an increase of PMEPR [1], [4].

Addressing the aforementioned issues, Li and Cimini [1] showed that oversampled sequence clipping results in PMEPR improvement for sufficiently high values of clipping threshold (PMEPR of greater than 4) with negligible performance degradation. By further decreasing the threshold value, considerable power penalty must be paid. Moreover, bandpass filtering can be used to remove the out-of-band radiation. Oversampled sequence clipping has been also proposed to reduce the peak regrowth after digital to analog conversion. The peak regrowth problem after digital to analog conversion has been analyzed analytically in [3], [5] and also through simulations in [1], [4] and oversampling rate of 4 is shown to be sufficient to bound the peak of the continuous signal.

One way to compensate the performance degradation is to reconstruct the clipped samples based on the other samples in oversampled systems. In this case, clipped samples are considered to be the lost ones. The reconstruction of lost samples of an oversampled signal is fully discussed in [6], [7]. Meanwhile, by completely removing the out-of-band components of the signal, these methods fail to reconstruct the lost samples [6], [8]. Moreover, reconstruction algorithms, such as Reed–Solomon (RS) decoding, are very sensitive to additive noise, which may result in instability of the reconstruction algorithm [6]. In this letter, we use oversampled signal reconstruction to compensate for the SNR degradation due to digital clipping for low values of clipping threshold. Clipping the oversampled sequence, we make sure that the peak regrowth after digital-to-analog conversion is not a severe problem [4], [5]. Besides, we suggest the least square method to reconstruct the clipped samples which is a robust method against additive channel noise. Finally, considering the out-of-band radiation of the clipped signal, we will show through extensive simulations that it is not necessary to preserve all the out-of-band radiation caused by clipping. It will be shown that by increasing the system bandwidth by 1.25 to at most 2 times, we can improve the BER performance while lowering the clipping threshold value. The improvement becomes more significant for higher signal to noise ratio (SNR) values when the major part of BER is due to clipping noise rather than the channel noise.

While we were working on this letter, we found out that Henkel [9], [11] has suggested the RS decoding method to reconstruct the clipped samples in multicarrier systems for high values of SNR of 50 dB, and also without removing the out-of-band radiation of the signal. However, we propose the least square decoding instead of RS decoding to have a more robust method against channel noise. We also investigate the trade off between bandwidth expansion and BER improvement.

The letter is organized as follows: Section II introduces the OFDM signals and PMEPR factor to measure the amplitude fluctuation and then describes a typical system of interests. In Section III, the reconstruction method is briefly discussed. Simulation results are presented in Section IV, and finally, Section V concludes the letter.

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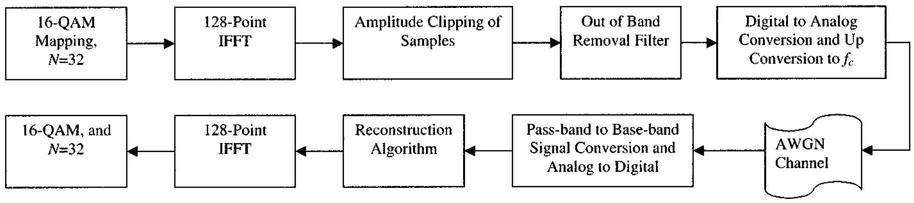

Fig. 1. OFDM transmitter and receiver architecture with digital clipping and reconstruction.

II. Definitions and System Description

The low pass equivalent of an OFDM signal at time t with N subcarriers may be written

$$S(t) = \sum_{n=0}^{N-1} c_n e^{j2\pi n f_0 t}, \qquad 0 \le t \le T \tag{1}$$

where f_0 is the subchannel spacing, T is the symbol period, and c_i is a complex number from a given constellation. Consequently, the OFDM signal will be the real part of that signal after up conversion to the carrier frequency f_c

$$G(t) = \mathrm{Re}\left\{ \sum_{n=0}^{N-1} c_n e^{j2\pi(f_c+nf_0)t} \right\}, \qquad 0 \le t \le T. \tag{2}$$

The amount of amplitude fluctuation of OFDM signals may be measured in terms of the ratio of the peak power of the envelope signal, to the average envelope power (PMEPR) of the signal. More specifically, the PMEPR is defined as [2]:

$$\mathrm{PMEPR} \equiv \max_{(c_0,\ldots,c_{N-1}),\, 0\le t<T} \frac{|S(t)|^2}{P_{av}} \tag{3}$$

where P_{av} is equal to $E\{\|(c_0, c_1, \ldots, c_{N-1})\|_2^2\}$ which is a constant that depends on the constellation size and N for uncoded systems.

In this letter we consider a typical OFDM system with 32 subcarriers ($N = 32$) and 16 QAM constellation in which oversampled sequence clipping is used to reduce PMEPR for oversampling rate of 4. As shown in Fig. 1, oversampling is performed in the transmitter by padding the modulating vector, $C = (c_0, c_1, \ldots, c_{N-1})$, with $3N$ zeros, and then taking $4N$ point IFFT. The amplitude of base-band oversampled signal is then clipped by a hard-limiter with characteristics as,

$$g(|x|e^{j\phi}) = \begin{cases} |x|e^{j\phi}, & |x| \le A \\ Ae^{j\phi}, & |x| > A. \end{cases} \tag{4}$$

In order to reconstruct the clipped samples, we should not omit all the out-of-band components of the signal. Therefore, we use an ideal filter to remove a portion of out-of-band components. The major part of out-of-band energy can be preserved with just 50% bandwidth expansion. Therefore, we consider a system with variable bandwidth expansions and we will discuss the effect of bandwidth expansion on the performance of our method.

III. A Brief Review of Reconstructing the Lost Samples of an Oversampled Signal

Sampling a band-limited signal above the Nyquist rate is equivalent to error correcting codes [6], [11]. It means that as long as the average rate of the samples of a signal is above the Nyquist rate, lost samples can be recovered based on the other samples. Below we briefly discuss the method we employ to reconstruct the lost samples of a signal in the presence of channel noise.

Taking FFT of an arbitrary vector X with length N is equivalent to multiplying the vector by a square matrix A whose elements are $a_{mn} = \exp(-2jmn\pi/N)$. Thus we have

$$\mathrm{FFT}(X) = AX. \tag{5}$$

To briefly describe the reconstruction method, assume that arbitrary number of elements of FFT(X), say K, are zero. If K samples of X are lost, we can recover them by solving a linear system of K equations and K unknowns. This idea is a basic hypothesis for various methods of recovering the lost samples of a signal discussed in [6], [7]. In these methods, such as RS decoding, sensitivity to noise is a major issue.

Now suppose that the number of lost samples say P is less than the number of padded zeros ($P < K$). Methods discussed in [8], [9] use a linear system with P equations and P unknowns for lost sample recovery, and the remaining $K - P$ zeros do not play a role in recovery. In this case, the stability of decoding process will not increase by adding more zeros and the reconstruction process remains unstable for low SNR's. To overcome such a limitation, we try to incorporate all the padded zeros in calculating the lost samples by using the least square method. In this case, we have a system with K equations but P unknowns where $K > P$. In order to achieve the optimum solution for this system, we consider the least square method (LS) for over-determined systems. In this method, instead of finding an exact solution for the equation system $TX = b$, we try to find a $P \times 1$ vector X such that $\|TX - b\|$ is minimized over all possible values of X, where b and T are $K \times 1$ and $K \times P$ matrices, respectively. The LS method is implemented by pseudo-inverse algorithm in the MATLAB implementation.

Intuitively and according to our simulation results using the LS method, the decoding process is much more stable. In this case, the more the number of padded zeros with respect to the number of lost samples is, the more stable the reconstruction process will become.

In our proposed method, we consider the clipped samples as lost ones to be reconstructed. Since we oversample the signal with the rate of 4, up to 75% of sample loss can be tolerated.

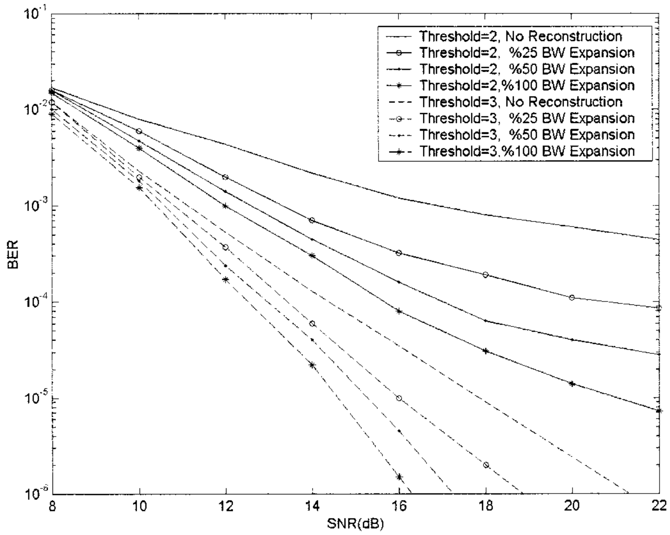

Fig. 2. Plot of BER versus SNR for PMEPR clipping thresholds of 2 and 3 for various BW expansions.

Moreover, as the PMEPR clipping threshold is at least 2, approximately 10% of the samples are clipped. Therefore, the number of remaining samples is sufficiently above the Nyquist rate and the decoding process will be robust against the additive noise. For example, we are able to reconstruct the clipped samples successfully even for the SNR values around 10 dB.

As mentioned in Section II, in order to reconstruct the clipped samples, we should not omit all the out-of-band components of the clipped signal. In fact, if all the out-of-band part of the signal is removed, we will have only a low-pass version of the clipped signal and there will not be accurate nonclipped samples for recovering the clipped samples.

IV. Simulation Results

In this section, we present the simulation results for the system shown in Fig. 1. In Fig. 2, the BER performance of the system is shown for various SNR values when the clipping threshold is set to 2 and 3. In order to study the effect of bandwidth expansion on BER improvement, the simulation is done for three bandwidth expansions of 25%, 50%, 100%, and also for the case without any reconstruction. By increasing the SNR, we have more performance improvement. The reason is that in high SNR values, the major part of BER is due to clipping distortion rather than the channel noise. To gain a better insight, consider the system with clipping at PMEPR threshold of 2 at BER of 10^{-3} for SNR of 16 dB. By using the reconstruction method for bandwidth expansion of 25%, 50%, and 100%, we have 3.5, 4 and 5 dB improvement, respectively. For higher SNR's, this may increases to 7 dB and more by only 25% bandwidth expansion. This improvement is also valid for clipping at PMEPR threshold of 3. As it is shown, for BER of 10^{-5} (or SNR of 18 dB) by using the reconstruction method for bandwidth expansion of 25%, 50%, and 100%, we will have 2, 2.5 and 3 dB improvement, respectively.

By further decreasing the threshold value, we will have more out-of-band radiation and in order to save its major components, we need to have more bandwidth expansion, which is not attractive. Besides, as the number of clipped samples increases, the reconstruction process becomes more unstable.

V. Conclusions

In this letter, oversampled sequence clipping of OFDM signals is considered. There are three issues to be addressed: clipping distortion, peak regrowth of the signal after digital-to-analog conversion, and out-of-band radiation. We have considered a system with four times oversampling to limit the peak regrowth [1], [4].

By using the LS method for oversampled signal reconstruction, we can remove the clipping distortion. Moreover, by simulations, we have studied the trade off between bandwidth expansion and clipping distortion mitigation. It was shown that by only 25% extra bandwidth, we can significantly improve the BER performance of the system for a clipping threshold of two.

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