PHYSICAL REVIEW D VOLUME 40, NUMBER 8 15 OCTOBER 1989

Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication in **Physical Review D** *should be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but because of the accelerated schedule, publication is not delayed for receipt of corrections unless requested by the author or noted by the editor.*

Effect of a finite quark mass on the deconfinement temperature as calculated in dual QCD

M. Baker
University of Washington, Seattle, Washington 98105

James S. Ball
University of Utah, Salt Lake City, Utah 84112

F. Zachariasen
California Institute of Technology, Pasadena, California 91125
(Received 5 May 1989)

We calculate, to lowest order, the contribution of quark loops to the finite-temperature vacuum energy density. From this, in the high-temperature approximation, we obtain the dual QCD deconfinement transition temperature as a function of the input quark mass.

I. INTRODUCTION

It is becoming more and more clear that dual QCD provides a convenient method for analyzing many features of QCD in regimes which are difficult to study in the more conventional formulation of the theory.[1] In this description of QCD, color confinement is the result of a magnetic gluon condensate, which is the nonperturbative vacuum state. Among the recent applications of dual QCD has been the analytic calculation of the QCD deconfinement temperature in the high-temperature approximation.[2] This calculation, however, did not include the effects of quarks (in other words, quarks were assumed to have infinite mass) because of the difficulties inherent in writing down how quarks couple to dual gluons. Thus far, the couplings of quarks are understood only to lowest order.[3] (The problem is of course just the same as the problem of introducing monopoles as well as electric charges into QED, complicated by the non-Abelian nature of QCD.)

In this paper we wish to study how the introduction of quarks of finite mass affects the QCD deconfinement temperature. As indicated above, quark couplings will be dealt with only to lowest order, but this is appropriate to a high-temperature expansion. Even so, the fact that two quark vertices appear in this order requires some delicacy in handling the strings associated with these vertices; how this is done is explained in Sec. II.

We have described elsewhere[4] how quarks are introduced into the dual QCD Lagrangian, and given there the appropriate Feynman rules for the various quark vertices. The lowest-order quark loop diagram contributing to the finite-temperature vacuum energy density Ω/V is shown in Fig. 1. The vertices marked × represent the coupling term $(M/2)\tilde{F}^a_{\mu\nu}G^a_{s\mu\nu}$ in the Lagrangian, where $\tilde{F}^a_{\mu\nu}$ is replaced by its value for the gluon condensate, $\tilde{F}^{a(0)}_{\mu\nu}$, which is a constant in spacetime, and so contributes no momentum transfer at the vertex, and where G_s is the "string field" associated with the quark sources. The momenta of both quark lines in the diagram are therefore the same, and are called k. The quark propagators $S(k)$ are the usual ones: $S(k)=(i\not{k}-m_Q)^{-1}$. We must, however, address the issue of what m_Q is. Even in the absence of an input, or bare, quark mass, quarks develop a finite mass due to chiral-symmetry breaking. We have shown in II, for input quark mass $m=0$, how chiral-symmetry breaking is understood in the language of dual QCD, and estimated m_Q when $m=0$. For finite m, we need an analogous estimate of $m_Q(m)$, which we may derive in much

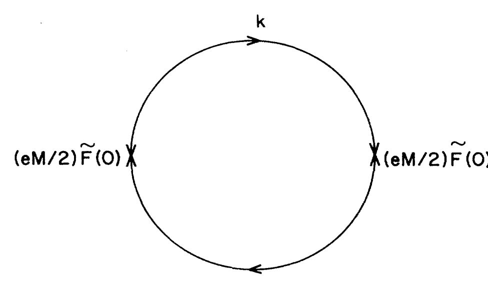

FIG. 1. Graph of lowest-order quark contribution to the finite-temperature vacuum energy density. The × indicates the $M\tilde{F}(0)G_s/2$ vertex.

the same way as $m_Q(0)$. Evidently $m_Q(m)$ is the mass we want to use in the quark propagator $S(k)$ of Fig. 1; yet the final result of interest to us is how Ω/V, and therefore the deconfinement temperature T_C, depends on m, not on m_Q.

II. THE QUARK LOOP

We begin this section with an outline of how the strings must be handled when two quark vertices are present in a diagram, such as those indicated by the × in Fig. 1. First suppose we look at two opposite static point charges, located on the z axis as z_1 and z_2, with $z_1 < z_2$. Gauss's law reads

$$\nabla\cdot\mathbf{E}_s = e\hat{\mathbf{e}}_z[\delta(z-z_2)-\delta(z-z_1)]\delta^2(\mathbf{x}_\perp) \tag{2.1}$$

for the string field $\mathbf{E}_s$. The solution having the string joining the two charges is

$$\mathbf{E}_s(\mathbf{x}) = e\hat{\mathbf{e}}_z\delta^2(\mathbf{x}_\perp)\theta(z_2-z)\theta(z-z_1)\,. \tag{2.2}$$

If $\mathbf{E}_s$ is Fourier transformed on the field point $\mathbf{x}$, we obtain

$$\mathbf{E}_s(\mathbf{q}) = e\hat{\mathbf{e}}_z\int_{z_1}^{z_2} e^{iq_z z}dz = e\hat{\mathbf{e}}_z\frac{e^{iq_z z_2}-e^{iq_z z_1}}{iq_z}\,. \tag{2.3}$$

Letting $\hat{\mathbf{n}} = \hat{\mathbf{e}}_z = (\mathbf{x}_2-\mathbf{x}_1)/|\mathbf{x}_2-\mathbf{x}_1|$ be the string direction, (2.3) can be rewritten as

$$\mathbf{E}_s(\mathbf{q};\mathbf{x}_1,\mathbf{x}_2) = e\frac{\hat{\mathbf{n}}}{i\mathbf{q}\cdot\hat{\mathbf{n}}}(e^{i\mathbf{q}\cdot\mathbf{x}_2}-e^{i\mathbf{q}\cdot\mathbf{x}_1})\,, \tag{2.4}$$

where $\mathbf{x}_{1,2}$ are the source positions. When $\mathbf{q}\to 0$ (which will be the relevant limit for our application), then

$$\mathbf{E}_s(\mathbf{q};\mathbf{x}_1,\mathbf{x}_2)\to e\frac{\hat{\mathbf{n}}i\mathbf{q}\cdot\hat{\mathbf{n}}\,|\mathbf{x}_2-\mathbf{x}_1|}{i\mathbf{q}\cdot\hat{\mathbf{n}}} = e(\mathbf{x}_2-\mathbf{x}_1)\,. \tag{2.5}$$

Generalizing this argument to $G_{s\mu\nu}$, and allowing $x_{1,2}$ to be four-vectors, we find, as $q\to 0$, that

$$G_{s\mu\nu}\to -e\epsilon_{\mu\nu\lambda\sigma}(x_2-x_1)_\lambda\bar{\psi}(x_1)\gamma_\sigma\frac{\lambda^a}{2}\psi(x_1)\,, \tag{2.6}$$

where we have put in the appropriate Dirac γ matrix, color matrix λ, and quark wave function $\psi(x_1)$, for the vertex at x_1. A similar expression with $\psi(x_1)\to\psi(x_2)$ holds for the vertex at x_2.

Combining (2.6) with the rest of the vertex $(M/2)\times\tilde{F}^{a(0)}_{\mu\nu}(0)$, the entire loop of Fig. 1 is simply

$$\left(\frac{eM}{2}\right)^2\tilde{F}^{a(0)}_{\mu\nu}(0)\tilde{F}^{a'(0)}_{\mu'\nu'}(0)\int d^4x_1\int d^4x_2\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\mu'\nu'\lambda'\sigma'}(x_2-x_1)_\lambda(x_2-x_1)_{\lambda'}$$
$$\times\mathrm{Tr}\left(\frac{\lambda^a}{2}\frac{\lambda^{a'}}{2}\right)\mathrm{Tr}[\bar{\psi}(x_1)\gamma_\sigma\psi(x_1)\bar{\psi}(x_2)\gamma_{\sigma'}\psi(x_2)]\,. \tag{2.7}$$

We now switch to momentum space, so that

$$\psi(x_1) = \int u(k_1)e^{ik_1x_1}d^4k_1/(2\pi)^4 \tag{2.8}$$

and note that we may replace $(x_2-x_1)_\lambda$ by $i\partial/\partial k_\lambda$. Finally, therefore, we obtain for the contribution of Fig. 1 to the vacuum energy density

$$\frac{\Omega}{V} = \frac{1}{2}\left(\frac{eM}{2}\right)^2\tilde{F}^{a(0)}_{\mu\nu}(0)\tilde{F}^{a(0)}_{\mu'\nu'}(0)C_F\int dk\,\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\mu'\nu'\lambda'\sigma'}\mathrm{Tr}\left[\left(\frac{\partial}{\partial k_\lambda}\gamma_\sigma\frac{1}{\not{k}-m_Q}\right)\left(\frac{\partial}{\partial k_{\lambda'}}\gamma_{\sigma'}\frac{1}{\not{k}-m_Q}\right)\right], \tag{2.9}$$

where we have used the fact that $\mathrm{Tr}(\lambda^a/2)(\lambda^{a'}/2) = C_F\delta^{aa'}$. Here $\int dk$, at finite temperature, stands, as usual, for $(1/\beta)\sum_{n=-\infty}^{\infty}\int d^3k/(2\pi)^3$, and k_0 is to be replaced, as appropriate for a fermion, by $(2\pi i/\beta)(n+\frac{1}{2})$.

III. THE QUARK CONTRIBUTION TO THE DECONFINEMENT TEMPERATURE

The evaluation of the traces in (2.9) is straightforward, and we find (taking $C_F = \frac{4}{3}$)

$$\frac{\Omega}{V} = 2e^2M^2\tilde{F}_0^2\int dk\frac{1}{k^2-m_Q^2}\left(-1+\frac{m_Q^2}{k^2-m_Q^2}\right), \tag{3.1}$$

where $\tilde{F}_0^2\equiv\tilde{F}^{a(0)}_{\mu\nu}(0)\tilde{F}^{a(0)}_{\mu\nu}(0)$ is the magnetic condensate in the nonperturbative confining vacuum.[1,2,4] Evaluation of the sum over n implicit in $\int dk$ gives

$$\frac{\Omega}{V} = \frac{e^2M^2\tilde{F}_0^2}{2\pi^2}\int_0^\infty k^2dk\frac{1}{2\pi i}\int_C\frac{dz}{z}\tanh\beta z\frac{1}{(z^2-E^2)^2}\left(-1+\frac{m_Q^2}{z^2-E^2}\right), \tag{3.2}$$

where $E^2 = k^2+m_Q^2$. The contour C encloses both poles at $z = \pm E$ in the clockwise direction, so that, after removing the $T\to 0$, or $\beta\to\infty$, limit we are left with

$$\frac{\Omega}{V} = \frac{e^2M^2\tilde{F}_0^2}{2\pi^2}\int_0^\infty\frac{dx}{e^{\bar{m}\sqrt{x^2+1}}}\left(\frac{1}{\sqrt{x^2+1}}+\frac{\bar{m}}{2}\frac{e^{\bar{m}\sqrt{x^2+1}}}{e^{\bar{m}\sqrt{x^2+1}}+1}\right), \tag{3.3}$$

where $\bar{m}\equiv\beta m_Q = m_Q/T$. (Note that the combination $-e^2M^2\tilde{F}_0^2/4\pi^2$ appearing in these equations is simply G_2, the

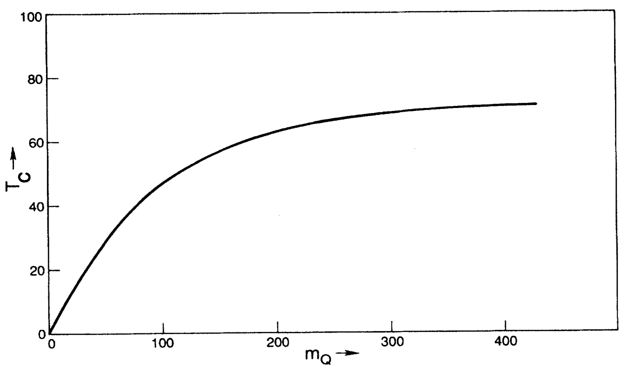

FIG. 2. Graph of the deconfinement temperature T_C as a function of the mass m_Q appearing in the quark loop. (All quantities are measured in MeV.)

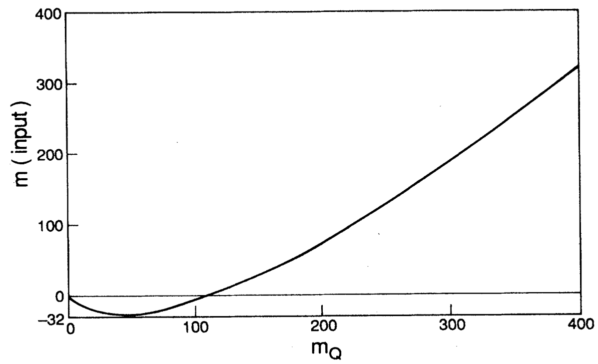

FIG. 3. The physical quark mass m_Q as a function of the input quark mass m.

value of the magnetic condensate.)

To find the transition temperature, this contribution to the vacuum energy density is now to be added to that obtained in I for the $m_Q = \infty$ case. This produces an additional temperature-dependent contribution to the $\tilde{F}_{\mu\nu}$ mass. The deconfinement temperature T_C [for SU(N)] is then given as the solution of the equation

$$0 = \frac{N^2}{12(N^2-1)}\left[-\frac{11}{3}+\frac{T_C^2}{-\tilde{F}_0^2}\left(\frac{g^2(\tilde{F}_0^2)^2}{G_2}+\frac{11}{12}(16+15N^2)\right)\right]+I\left(\frac{m_Q}{T_C}\right). \qquad (3.4)$$

The first term in Eq. (3.4) arises from the original $\tilde{F}_{\mu\nu}$ mass term in $\mathcal{L}(C)$; the second (proportional to T_C^2) arises from the dual gluon and $\tilde{F}_{\mu\nu}$ loops as calculated in I; the third term, $I(m_Q/T)$, which arises from the quark loop of Fig. 1, is the integral of Eq. (3.3) without the multiplying factor. There is consequently no solution to Eq. (3.4) if $I > 11N^2/[36(N^2-1)] = \frac{11}{32}$ for SU(3). Taking $G_2 = (330\text{ MeV})^4$, and $g^2 = 6.3$ as determined from the heavy-quark–antiquark potential, and $\tilde{F}_0^2 \sim (500\text{ MeV})^2$, as estimated from the string tension, we obtain $g^2(\tilde{F}_0^2)^2/G_2 \approx 35$. This is small compared to $\frac{11}{12}(16+15N^2) \approx 138$ for SU(3) and hence the value of the coefficient of $T_C^2/(-\tilde{F}_0^2)$ in Eq. (3.4) is not sensitive to the value of $\tilde{F}_0^2$. For the case of SU(3) Eq. (3.4) then takes the form

$$-\frac{11}{32}+16.1\left(\frac{T_C^2}{-\tilde{F}_0^2}\right)+I\left(\frac{m_Q}{T_C}\right)=0. \qquad (3.5)$$

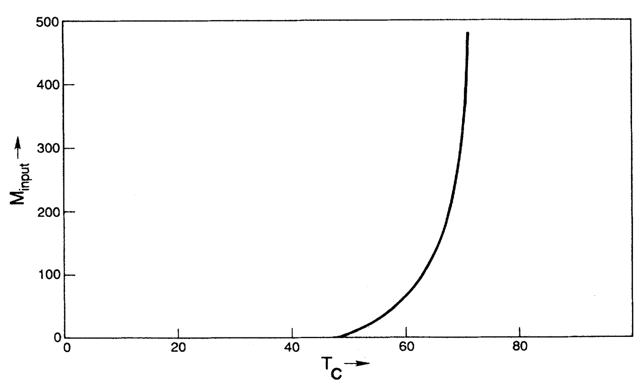

FIG. 4. The result of combining Figs. 2 and 3: T_C vs the input mass m.

Equation (3.5) determines $T_C^2/\tilde{F}_0^2$ as a function of $-\tilde{F}_0^2/m_Q^2$. Numerical evaluation of I and the condition $I < \frac{11}{32}$ requires $m_Q/T_C > 1.5$. Obviously, as $m_Q/T \to \infty$, $I \to 0$ and T_C reduces to the value given in Ref. 2: namely, about 72 MeV. A graph of T_C vs m_Q is shown in Fig. 2.

We are less interested, however, in how T_C behaves as a function of m_Q than in how it behaves as a function of the input quark mass m. Our last task is therefore to relate m_Q to m and T.

In Ref. 4 we have calculated m_Q when $m = 0$. As in superconductivity, m_Q has a chirally spontaneously broken value $m_Q(0)$ at $T = 0$, which is relatively insensitive to T until near $T = T_C'$, at which point it falls rapidly to zero.[4,5] Since we expect[4] $T_C \ll T_C'$, it suffices for our purposes to calculate m_Q at $T = 0$, as in Eq. (4.4) of II. When the input mass m is not zero, this equation is replaced simply by

$$m_Q = m + m_Q\frac{3e^2C_F}{8\pi^2}\ln\left(-\frac{\tilde{F}_0^2}{m_Q^2}\right). \qquad (3.6)$$

A graph of m_Q vs m based on (3.6) is shown in Fig. 3. The chiral-symmetry-breaking mass $m_Q^2(m=0) = (-\tilde{F}_0^2)e^{-8\pi^2/3e^2C_F}$ is indicated there. Combining Figs. 2 and 3 [i.e., Eqs. (3.5) and (3.6)] yields our final result, shown in Fig. 4, for the deconfinement temperature as a function of the input quark mass m.

To summarize, we have calculated to lowest order, in the dual coupling constant, the effect of a quark loop on the deconfinement transition temperature, previously calculated[2] for infinite quark mass. We find that as the quark mass is reduced, the deconfinement transition temperature is lowered, in qualitative agreement with the results of lattice calculations.[6]

ACKNOWLEDGMENTS

One of us (J.S.B.) would like to thank the École Polytechnique Theoretical Physics Group and the CNRS for their hospitality and partial support during the course of this work. The work of M.B. was supported in part by the U.S. Department of Energy under Contract No. DE-AC06-81ER40008. The work of F.Z. was supported in part by the U.S. Department of Energy under Contract No. DE-AC0381-ER40050.

[1]M. Baker, J. S. Ball, and F. Zachariasen, Phys. Rev. D **37**, 1036 (1988).

[2]M. Baker, J. S. Ball, and F. Zachariasen, Phys. Rev. Lett. **61**, 521 (1988), hereafter referred to as I.

[3]One should keep in mind that the introduction of dynamical quarks destroys G. t' Hooft's definition [Nucl. Phys. **B153**, 141 (1979)] of dual Wilson loops, and therefore also destroys the definition of the dual potential given by S. Mandelstam [Phys. Rev. D **19**, 2391 (1979)]. Therefore, not only do we not understand how to couple quarks to dual gluons beyond the lowest (Abelian) order, we do not even know how to define the dual potential. In fact, strictly speaking, the presence of dynamical quarks even prevents us from giving a precise definition of confinement, in that the Wilson loop no longer needs to obey an area law.

[4]M. Baker, J. S. Ball, and F. Zachariasen, Phys. Rev. D **38**, 1926 (1988), hereafter referred to as II.

[5]See, for example, A. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971), p. 449.

[6]See, for example, M. Fukugita, in *Field Theory on the Lattice,* proceedings of the International Symposium, Fermilab, Batavia, Illinois, 1988, edited by A. Kronfeld *et al.* [Nucl. Phys. B (Proc. Suppl.) (in press)]; or A. Ukawa, CERN-TH-5245/88, review at GIFT Seminar, 1988 (unpublished).