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THE PERCEPTION-ADJUSTED LUCE MODEL

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Abstract

We develop an axiomatic model that builds on Luce's (1959) model to incorporate a role for perception. We identify agents' "perception priorities" from their violations of Luce's axiom of independence from irrelevant alternatives. Using such perception priorities, we adjust choice probabilities to account for the effects of perception. Our axiomatization requires that the agents' adjusted random choice conforms to Luce's model. Our model can explain the attraction, compromise, and similarity effects, which are very well-documented behavioral phenomena in individual choice.

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Key words: Stochastic choice; Luce model; Independence of irrelevant alternatives; Compromise effect; Similarity effect; Attraction effect.

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1 Introduction

We study the role of perception in individual random choice. The main idea is to identify a *perception priority order* from an agent's violations of independence from irrelevant alternatives (IIA), the rationality axiom that lies behind Luce's (1959) model of choice. In other words, we attribute any violation of Luce's model to the role of perception. Our model, a *perception-adjusted Luce model (PALM)*, can explain the attraction, compromise, and similarity effects: these are the best known deviations from Luce's model in experiments.

The IIA axiom says that the probability of choosing x relative to that of choosing y is not affected by the presence of alternatives other than x or y. We use an agent's violations of IIA to identify his perception priority order; a kind of revealed preference relation that captures the agent's perception. The priority order means that some alternatives are perceived before others.

We adjust the agent's random choice using his perception priority order. The priority order defines a *hazard rate*: the probability of choosing an object, conditional on not choosing any of the objects with higher perception priority. So hazard rates account for the effects of perception.

The resulting model of choice is termed the *perception-adjusted Luce model* (PALM). We show that PALM is characterized by three axioms on choice behavior. The first axiom requires that the agent's perception priority order be well-behaved: it must be complete and transitive. The other two axioms are imposed on hazard rates. The effect of perception on IIA has been accounted for in the hazard rates, so we require that hazard rates satisfy the two standard properties of Luce's (1959) model; namely IIA as well as the regularity axiom. The regularity axiom says that the probability of choosing x from a set A cannot be larger than the probability of choosing x from a subset of A.

PALM can explain some of the best known violations of Luce's model in experiments. It can explain attraction effects, compromise effects, and similarity effects: Section 4.2 has all the details. Some of these effects stem from violations of the regularity axiom. PALM can violate the regularity axiom.

There are models within the economic axiomatic literature that explain some of these violations of Luce's model. We are not aware of any existing model that can explain them all. Section 5 discusses the related literature.

2 Primitives and Luce's model

Let X be a nonempty set of *alternatives.* Let $\mathscr{A}$ be the set of finite and nonempty subsets of X.[1] We model an agent who makes a probabilistic choice from $A \cup \{x_0\}$. The element $x_0 \notin X$ represents an outside option that is always available to the agent. Choosing the outside option can simply mean that the agent does not make a choice. We shall allow the agent never to choose the outside option.

Definition: A function $\rho : X \cup \{x_0\} \times \mathscr{A} \to [0, 1]$ is called *a stochastic choice function* if

$$\sum_{a \in A \cup \{x_0\}} \rho(a, A) = 1$$

for all $A \in \mathscr{A}$. A stochastic choice function ρ is *nondegenerate* if $\rho(a, A) \in (0, 1)$ for all $A \in \mathscr{A}$ with $|A| \geq 2$ and $a \in A$.

[1]Our results hold for more restrictive definitions of $\mathscr{A}$.

We write $\rho(B, A)$ for $\sum_{a \in B} \rho(b, A)$, and say that $\rho(\emptyset, A) = 0$.

Note that we allow for $\rho(x_0, A) = 0$. So it is possible that the outside option is never chosen with positive probability, even when ρ is nondegenerate.

Definition: A stochastic choice function ρ *satisfies Luce's independence of irrelevant alternatives (IIA) axiom at* $a, b \in X$ if, for any $A \in \mathscr{A}$,

$$\frac{\rho(a, \{a, b\})}{\rho(b, \{a, b\})} = \frac{\rho(a, A)}{\rho(b, A)}.$$

Moreover, ρ *satisfies IIA* if ρ satisfies IIA at a, b for all $a, b \in X$.

Luce (1959) proves that, if a non-degenerate stochastic choice function satisfies IIA, then it can be represented by the following model:

Definition: ρ satisfies Luce's model if there exists a real-valued function u on $X \cup \mathscr{A}$ such that

$$\rho(a, A) = \frac{u(a)}{\sum_{a' \in A} u(a') + u(A)}. \tag{1}$$

Luce presented his model in the absence of an outside option. When $\rho(x_0, A) = 0$, then $u(A) = 0$. Here we allow for an outside option, and present the Luce model in which not choosing in A is possible. The number $u(A)$ can be interpreted as the utility of abstaining from choosing an element from A. For later reference it is important to note that

$$u(A) = \sum_{a \in A} u(a) \left(\frac{1}{\sum_{a \in A} \rho(a, A)} - 1 \right). \tag{2}$$

When $u(A) = 0$ for all A, we obtain Luce's model without an outside option.

Luce's model without an outside option satisfies a monotonicity property: $\rho(x, A) \geq \rho(x, B)$, if $A \subset B$. This property is called *regularity*. In general, when an outside option is present, we need an assumption on u in order for Luce's model to satisfy regularity.

3 Axioms

We introduce the perception priority order derived from ρ, and the resulting "perception adjusted" random choice function: the hazard rate function.

<u>Perception priority.</u> We capture the role of perception through a weak order $\succsim$. The idea is that when $a \succ b$, then a tends to be perceived sooner than b. In particular, we denote by $\succsim^*$ the (revealed) priority relation that we obtain from the data in ρ. To define $\succsim^*$, first we identify the directly revealed priority relation $\succsim^0$ from ρ. Then $\succsim^*$ is defined as the transitive closure of $\succsim^0$. In our framework, $\succsim^*$ is analogous to (observed) revealed preference in the standard choice framework.

We shall attribute all violations of IIA to the role of perception. That is, we require that $a \sim^0 b$ when IIA holds at a and b. In other words, when two alternatives a and b do not exhibit a violation of IIA then we impose that they are equivalent from the view point of perception: they have the same perception priority.

In contrast, if a and b are such that IIA fails at a and b, then we shall require that a and b are strictly ordered by $\succ^0$: we shall require that either $a \succ^0 b$ or that $b \succ^0 a$. Which possibility of the two, $a \succ^0 b$ or $b \succ^0 a$, is determined by the nature of the violation of IIA.

Suppose that IIA fails at a and b because there is some c such that

$$\frac{\rho(a,\{a,b\})}{\rho(b,\{a,b\})} > \frac{\rho(a,\{a,b,c\})}{\rho(b,\{a,b,c\})}. \tag{3}$$

In words, the presence of c lowers the probability of choosing a relative to that of b. When does adding an option hurt one alternative more than another? We claim that *high priority alternatives are hurt by adding options.* The reason is that, by adding c we are "muddying the waters." We are making the choice between a and b less clear than before, and thus diluting the advantage held by a over b. Formally,

Definition: Let a and b be arbitrary elements in X.

(i)

$$a \sim^0 b \quad \text{if} \quad \frac{\rho(a,\{a,b\})}{\rho(b,\{a,b\})} = \frac{\rho(a,\{a,b,c\})}{\rho(b,\{a,b,c\})},$$

for all $c \in X$;

(ii)

$$a \succ^0 b \quad \text{if} \quad \frac{\rho(a,\{a,b\})}{\rho(b,\{a,b\})} > \frac{\rho(a,\{a,b,c\})}{\rho(b,\{a,b,c\})},$$

for all $c \in X$ such that $c \nsim^0 a$ and $c \nsim^0 b$, and if there is at least one such c. We write $a \succsim^0 b$ if $a \sim^0 b$ or $a \succ^0 b$.

(iii) Define $\succsim^*$ as the *transitive closure* of $\succsim^0$: that is, $a \succsim^* b$ if there exist $c_1, \dots, c_k \in X$ such that

$$a \succsim^0 c_1 \succsim^0 \cdots c_k \succsim^0 b.$$

The binary relation $\succsim^*$ is called the *revealed perception priority* derived from ρ.

We shall impose the following condition on $\succsim^*$:

Axiom (Weak Order) $\succsim^*$ is a weak order.

<u>Hazard rate.</u> The second important component of our analysis is the hazard rate function. The hazard rate is the probability of choosing an object, conditional on not choosing any of the objects with higher perception priority.

Definition (Hazard Rate): For all $a \in X$ and $A \in \mathscr{A}$, define

$$q(a, A) = \frac{\rho(a, A)}{1 - \rho(A_a, A)},$$

where $A_a = \{b \in A | b \succ^* a\}$, $A \in \mathscr{A}$ and $a \in A$. q is called ρ's *hazard rate function.*

We ascribe all violations of IIA to the role of perception, and consider the hazard rate function to be a sort of adjusted choice probability. Hazard rates are adjusted for the role of perception, and hence for the phenomena that lie behind the violations of IIA. We shall therefore impose IIA and regularity as conditions on hazard rates.

Axiom (Hazard Rate IIA) The hazard rate function q satisfies Luce's IIA.

Axiom (Hazard Rate Regularity) $q(a, ab) \geq q(a, abc)$, for all $a, b, c \in X$; and $q(a, ab) > q(a, abc)$ when $b \nsim^0 c$.

In the regularity axiom above, the first condition is standard. In the second condition, $q(a, ab) > q(a, abc)$ is only required when $b \nsim^0 c$. This is because if $b \sim^0 c$, then adding c does not affect the decision maker's perception priority so that it does not affect the corrected choice probability.

4 Theorem

A PALM decision maker is described by two parameters: a weak order $\succsim$ and a utility function u. She perceives each element of a set A sequentially according to the perception priority $\succsim$. Each perceived alternative is chosen with probability described by μ, a function that depends on utility u according to Luce's formula (1). Formally, the representation is as follows.

Definition: A perception-adjusted Luce model (PALM) is a pair $(u, \succsim)$ of a weak order $\succsim$ on X, and a function $u : X \times \mathscr{A} \to \mathbf{R}$ such that

$$\rho(a, A) = \mu(a, A) \prod_{\alpha \in A/\succsim :\, \alpha \succ a} (1 - \sum_{c \in A:\, c \in \alpha} \mu(c, A)), \tag{4}$$

where

$$\mu(a, A) = \frac{u(a)}{\sum_{b \in A} u(b) + u(A)},$$

and $u(X) \subseteq \mathbf{R}_+$.

The notation $A/\succsim$ is standard: $A/\succsim$ is the set of equivalence classes in which $\succsim$

partitions A. That is, (i) if $A/\succsim = \{\alpha_i\}_{i\in I}$, then $\cup_{i\in I}\alpha_i = A$; and (ii) $x \sim y$ if and only if $x, y \in \alpha_i$ for some $i \in I$. The notation $\alpha \succ a$ means that $x \succ a$ for all $x \in \alpha$.

For any PALM $(u, \succsim)$, we denote by $\rho_{(u,\succsim)}$ the stochastic choice defined through (4). (When there is no risk of confusion, we write ρ instead of $\rho_{(u,\succsim)}$.)

Before stating the theorem, we discuss two properties of a PALM model. Firstly, we are interested in stochastic choice for which μ satisfies regularity. As with Luce's model with an outside option, this requires an assumption on utility u.

Definition: A PALM $(u, \succsim)$ is *regular* if for all $a, b, c \in X$, $u(c) \geq u(\{a, b\}) - u(\{a, b, c\})$ and, moreover, $u(c) > u(\{a, b\}) - u(\{a, b, c\})$ if $b \not\succ c$.

Regularity means that, given two alternatives a and b, the impact of adding c *on the utility of not choosing*, cannot be greater than the utility of c.

The second property is a technical "richness" axiom. We do not need this assumption to prove the sufficiency of the axioms for the representation. We need it to prove the necessity of the axioms, in particular, the result that $\succsim = \succsim^*$

Axiom (Richness) For any pair (a, b), there is $c \in X$ with $c \succ a$ or $b \succ c$.

Theorem 1 *If a nondegenerate stochastic choice function ρ satisfies Weak Order, Hazard Rate IIA, and Hazard Rate regularity, then there is a regular PALM $(u, \succsim)$ such that $\succsim^* = \succsim$ and $\rho = \rho_{(u,\succsim)}$.*

Conversely, if $(u, \succsim)$ is a regular PALM, and $\succsim$ satisfies Richness, then $\rho_{(u,\succsim)}$ satisfies Weak Order, Hazard Rate IIA, and Hazard Rate regularity, and $\succsim = \succsim^*$.

The proof of the theorem is in Section 6. The sufficiency of the axioms for the representation is straightforward. The converse of Theorem 1 states, not only that PALM satisfies the axioms, but that $\succsim$ must coincide with $\succsim^*$. The perception priority is therefore identified, given data on stochastic choice. The bulk of the proof consists of establishing that $\succeq = \succeq^*$.

4.1 Discussion of PALM

Luce's model is a special case of PALM, in which $a \sim b$ for all $a, b \in X$. It is useful to compare how Luce and PALM treat the outside option, the probability of not making a choice from a set A.

The utility $u(A)$ for $A \in \mathscr{A}$ has a similar expression to Equation (2), obtained for Luce's model. Indeed,

$$u(A) = \sum_{a \in A} u(a) \left(\frac{1}{\sum_{a \in A} q(a, A)} - 1 \right), \tag{5}$$

with the hazard rates q in place of ρ.

It is interesting to contrast the value of $u(A)$ with the utility one would obtain from Equation (2). Given a PALM model $(u, \succsim)$ we can calculate $\hat{u}(A)$ from $\rho_{(u,\succsim)}$ and the utility u by application of Equation (2). If we do that, we obtain

1. $\hat{u}(A) \geq u(A)$,
2. and $\hat{u}(A) = u(A)$ when $a \sim b$ for all $a, b \in A$.

The inequality $\hat{u}(A) \geq u(A)$ reflects that there are two sources behind choosing the outside option in PALM. One source is the utility $u(A)$ of not making a choice; this is the same as in Luce's model with an outside option. The second source is due to the sequential nature of choice in PALM. When we consider sequentially choosing an option following the priority order $\succsim$, then it is possible that we exhaust the choices in A without making a choice. When that happens, it would seem to inflate (or bias) the value of the outside option; as a result we get that $\hat{u}(A) \geq u(A)$.

4.2 Consistency with Violation of IIA–Similarity Effect and Compromise Effect

The similarity and compromise effects are well-known deviations from Luce's model. See Rieskamp et al. (2006) for a survey. In this section, we demonstrate how PALM can capture each of these phenomena.

The similarity and compromise effects are defined in the same kind of experimental setup. An agent makes choice from the sets $\{x, y\}$ and $\{x, y, z\}$. The "effects" relate to the consequences of adding the alternative z.

4.2.1 Similarity Effect

Suppose that our three alternatives are such that x and z are somehow very similar to each other, and clearly distinct from y. This setup is discussed by Tversky (1972a), building on an example of Debreu (1960). In Debreu's example, x and z are two different recordings of the same Beethoven symphony while y is a suite by Debussy. A stochastic choice ρ is said to exhibit the *similarity effect* if the addition of z to the set $\{x, y\}$ will reduce $\rho(x, \{x, y, z\})$ proportionally more than $\rho(y, \{x, y, z\})$. Formally,

Definition: ρ exhibits the *similarity effect* at $\{x, y\}$ with respect to z, if

$$\frac{\rho(x, \{x, y, z\})}{\rho(y, \{x, y, z\})} < \frac{\rho(x, \{x, y\})}{\rho(y, \{x, y\})}.$$

When $x \succ y \succ z$, the regular PALM always exhibits the similarity effect at $\{x, y\}$ with respect to z.

Proposition 1: *If $x \succ y \succ z$ and $\rho_{(u,\succsim)}$ satisfies regularity, then $\rho_{(u,\succsim)}$ exhibits the similarity effect at $\{x, y\}$ with respect to z.*

Proof of Proposition 1: Since $x \succ y \succ z$, then

$$\frac{\rho_{(u,\succsim)}(x,\{x,y,z\})}{\rho_{(u,\succsim)}(y,\{x,y,z\})} = \frac{\mu(x,xyz)}{\mu(y,xyz)(1-\mu(x,xyz))} = B,$$

and

$$\frac{\rho_{(u,\succsim)}(x,xy)}{\rho_{(u,\succsim)}(y,xy)} = \frac{\mu(x,xy)}{\mu(y,xy)(1-\mu(x,xy))} = A.$$

Then

$$\frac{A}{B} = \left[\frac{\mu(x,xy)/\mu(y,xy)}{\mu(x,xyz)/\mu(y,xyz)}\right]\frac{(1-\mu(x,xy))}{(1-\mu(x,xyz))}$$

The term in brackets equals 1 by hazard rate IIA. The result follows because hazard rate regularity gives $\mu(x,xy) > \mu(x,xyz)$. ■

PALM can capture the similarity effect when we assume that y is "in between" x and z with respect to priority. This is not the only possibility: other assumptions on $\succsim$ also allow us to capture the similarity effect. For example, in Appendix B.1, we show that PALM can capture the similarity effect with $x \sim z \succ y$, which means that the two Beethoven recordings are similar and more salient from the viewpoint of perception than Debussy.

4.2.2 Compromise Effect

Consider again three alternatives, x, y and z. Suppose that x and z are "extreme" alternatives, while y represents a moderate middle ground, a compromise. In the experiment studied by Simonson and Tversky (1992), x is X-370, a very basic model of Minolta camera; y is MAXXUM 3000i, a more advanced model of the same brand; and z is MAXXUM 7000i, the top of the line offered by Minolta in this class of cameras.

The agent's choice set is $\{x,y\}$ in Experiment 1 and $\{x,y,z\}$ in Experiment 2. The experimental data shows that the probability of choosing y increases when going from Experiment 1 to 2. Simonson and Tversky (1992) call this phenomenon the *compromise*

Model	Price ($)	Choices Exp. 1	Choices Exp. 2
x (X-370)	169.99	50%	22%
y (MAXXUM 3000i)	239.99	50 %	57%
z (MAXXUM 7000i)	469.99	N/A	21%

Figure 1: Compromise effect in Simonson and Tversky (1992)

effect. We follow Rieskamp et al. (2006) and define the compromise effect as follows.[2]

Definition: ρ exhibits the *compromise effect* at $\{x, y\}$ with respect to z, if

$$\frac{\rho(x, \{x, y, z\})}{\rho(y, \{x, y, z\})} < 1 < \frac{\rho(x, \{x, y\})}{\rho(y, \{x, y\})}.$$

Proposition 2: *When $x \succ y \succ z$, $\rho_{(u,\succsim)}$ exhibits the compromise effect at $\{x, z\}$ with respect to y if and only if $u(y) > u(x)$ and*

$$u(z) + u(\{x, y, z\}) > \frac{u^2(x) - u^2(y) + u(x)u(y)}{u(y) - u(x)} > u(\{x, y\}). \tag{6}$$

Proposition 2 results from a straightforward calculation so the proof is omitted. Note that the condition (6) is consistent with the regularity: $u(z) + u(\{x, y, z\}) > u(\{x, y\})$.

Simonson and Tversky (1992)'s explanation for the compromise effect is that subjects are averse to extremes, which helps the "compromise" option y when facing the problem $\{x, y, z\}$. PALM can capture the compromise effect when we assume that y is "in between" x and z with respect to priority. This is not the only possibility: other assumptions on $\succsim$ also allow us to capture the compromise effect.

A recent marketing study by Mochon and Frederick (2012) attributes the compromise effect to how the options are presented to subjects in experiments. They critique the explanation in Simonson and Tversky, arguing that the effect is not driven by the interpretation of some options as being a compromise. Instead, they find that which options

[2]Simonson and Tversky (1992) use a different definition. In Appendix B.2, we show that PALM can capture the definition by Simonson and Tversky (1992).

are more salient could lie behind the effect. We note that their point is consistent with PALM as a description for the compromise effect. Mochon and Frederick's explanation could correspond to the assumption of $x \sim z \succ y$, where the extremes are perceived more prominently than the compromise option. (In Appendix B.3, we show that PALM can capture the compromise effect with $x \sim z \succ y$.)

4.3 Consistency with Violation of Regularity–Attraction Effect

PALM can also accommodate violations of regularity. We focus on the well-known attraction effect, documented by Simonson and Tversky (1992) using the following experiment. Consider our three alternatives again, x, y and z. Suppose now that y and z are different variants of the same good: y is a Cross pen (meaning a higher quality pen), while z is a pen of regular quality: z is clearly dominated by y. The alternative x is some given monetary quantity.

Option	Choices Exp. 1	Choices Exp. 2
x (\$6)	64 %	52 %
y (Cross pen)	36 %	46 %
z (Other pen)	N/A	2 %

Figure 2: Attraction effect in Simonson and Tversky (1992)

Simonson and Tversky (1992) (p. 287) asked subjects to choose between x and y in Experiment 1 and to choose among x, y, and z in Experiment 2. They found that in Experiment 2, the share of subjects who chose y becomes higher than that in Experiment 1. This effect is called the *attraction effect*, formally defined as a violation of regularity as follows:

Definition: ρ exhibits the *attraction effect* at $\{x, y\}$ with respect to z, if

$$\rho(y, xyz) > \rho(y, xy).$$

Proposition 3: *If* $x \succ y \succ z$ *and* $u(x)$ *is large enough, then* $\rho_{(u,\succsim)}$ *exhibits the attraction effect at* $\{x, y\}$ *with respect to* z*.*

Proof of Proposition 3: We have

$$\begin{aligned} \rho(y, xyz) > \rho(y, xy) \quad &\Leftrightarrow \quad q(y, xyz)(1 - q(x, xyz)) > q(y, xy)(1 - q(x, xy)) \\ &\Leftrightarrow \quad u(x) > \sqrt{(u(y) + u(z) + u(xyz))(u(y) + u(xy))} \end{aligned}$$

■

The assumption $x \succ y \succ z$ means that x is more salient than the higher quality pen y. The Cross pen y, in turn, could be more salient than z because it dominates z in terms of quality.

In fact, we can generate very similar numbers to the one in the table above by suitably choosing the parameters in a PALM model. It is important that we can generate the numbers in the table because Luce with an outside option can also explain violations of regularity, but it cannot generate the numbers in the table as there was no outside option in Simonson and Tversky's experiment. The choices $\rho(x, \{x, y\}) = 0.59$, $\rho(x, \{x, y, z\}) = 0.5$, $\rho(y, \{x, y\}) = 0.4$, $\rho(y, \{x, y, z\}) = 0.415$, and $\rho(y, \{x, y, z\}) = 0.02$ result from a PALM model with $x \succ y \succ z$, $u(x) = 0.6$, $u(y) = 1$, $u(z) = 0.2$, $u(xy) = -0.587$, and $u(xyz) = -0.675$. Since $u(z) = 0.2 > (xy) - u(xyz) = 0.088$, the PALM $(u, \succeq)$ is regular. The calculations are omitted.

4.4 Correlation between Utility u and Perception Priority $\succsim$

Perception and utility are two independent parameters in PALM. Therefore, PALM allows us to model scenarios where perception is positively correlated with utility, negatively correlated, or simply unrelated.

Consider, for example, an agent making choices in the supermarket, a setting investigated experimentally by Reutskaja et al. (2011). Reutskaja and coauthors find no intrinsic correlation between utility and perception (a similar finding is reported in Krajbich and Rangel (2011)). High-utility items are not *per se* more likely to be perceived more prominently than others. It is therefore important that PALM not force a particular relation between perception and utility.

One can imagine settings, however, where perception is correlated with utility because the setting is manipulated by someone with knowledge of the agents' preferences. A supermarket or other vendor may want to display items so that an agent perceives better the alternatives that she is more likely to want to buy. Below we investigate conditions on PALM that reflect a positive correlation between perception and utility. In Section 4.4.2 below we investigate a situation in which there is negative correlation.

4.4.1 Positive Correlation

We give some conditions under which $u(a) > u(b)$ if and only if then $a \succ b$. Since we look at choice from the set $\{a, b\}$, such a condition must have the agent choosing the outside option with strictly positive probability: this is explained below, when we consider negative correlation.

Proposition 4: *Suppose $a \not\sim b$ and $u(a) \neq u(b)$. If $\rho_{(u,\succsim)}(x_0, ab) \geq \min\{\rho_{(u,\succsim)}(a, ab), \rho_{(u,\succsim)}(b, ab)\}$ and $u(ab) \leq 0$, then $u(a) > u(b)$ if and only if $a \succ b$.*

Proof of Proposition 4: First, we show that if $u(a) > u(b)$ then $a \succ b$. By way of contradiction, suppose $b \succ a$. By calculation, $\rho(a, ab) = \frac{u(a)(u(a)+u(ab))}{(u(a)+u(b)+u(ab))^2}$, $\rho(b, ab) = q(b, ab) = \frac{u(b)}{u(a)+u(b)+u(ab)}$, and $\rho(x_0, ab) = 1 - \rho(b, ab) - \rho(a, ab) = \frac{(u(a)+u(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2}$.

First consider the case when $\rho(a, ab) = \min(\rho(a, ab), \rho(b, ab))$. Then $\rho(x_0, ab) \geq \rho(a, ab)$ if and only if $\frac{(u(a)+e(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2} \geq \frac{u(a)(u(a)+u(ab))}{(u(a)+u(b)+u(ab))^2}$ if and only if $u(b) + u(ab) \geq u(a)$. Therefore, since $u(ab) \leq 0$, $\rho(x_0, ab) \geq \rho(a, ab)$ implies $u(b) \geq u(a)$. Contradiction.

Second consider the case when $\rho(b, ab) = \min(\rho(a, ab), \rho(b, ab))$. Then $\rho(x_0, ab) \geq \rho(b, ab)$ if and only if $\frac{(u(a)+u(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2} \geq \frac{u(b)}{u(a)+u(b)+u(ab)}$ if and only if $(u(a)+u(ab))(u(b)+$

$u(ab)) \geq u(b)(u(a)+u(b)+u(ab))$. Therefore, since $u(ab) \leq 0$, $\rho(x_0, ab) \geq \rho(b, ab)$ implies $(u(a) + e(ab))(u(b) + u(ab)) \geq (u(b) + u(ab))(u(a) + u(b) + u(ab))$, i.e., $u(a) + u(ab) \geq u(a) + u(b) + u(ab)$. Contradiction. Therefore, we proved that $a \succ b$.

Finally, we show that if $a \succ b$ then $u(a) > u(b)$. Suppose $u(b) > u(a)$. Then by the previous part, $u(b) > u(a)$ implies $b \succ a$. Contradiction. ■

The condition that $\rho(x_0, ab) \geq \min\{\rho(a, ab), \rho(b, ab)\}$ means that the probability of choosing the outside option must be the large enough. It is necessary to achieve positive correlation, as evidenced in the following result.

Proposition 5: *If* $a \succ b$, $u(a) > u(b)$, *and* $u(b) - u(a) \leq u(ab)$, *then* $\rho_{(u,\succsim)}(a, ab) > \rho_{(u,\succsim)}(x_0, ab) \geq \rho_{(u,\succsim)}(b, ab) = \min\{\rho_{(u,\succsim)}(a, ab), \rho_{(u,\succsim)}(b, ab)\}$.

Proof of Proposition 5: By calculation, we obtain $\rho(a, ab) = q(a, ab) = \frac{u(a)}{u(a)+u(b)+u(ab)}$, $\rho(b, ab) = q(b, ab)(1 - q(a, ab)) = \frac{u(b)(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2}$, and $\rho(x_0, ab) = 1 - \rho(a, ab) - \rho(b, ab) = \frac{(u(a)+u(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2}$.

First, $\rho(a, ab) > \rho(x_0, ab)$ if and only if $\frac{u(a)}{u(a)+u(b)+u(ab)} > \frac{(u(a)+u(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2}$ if and only if $u(a)(u(a) + u(b) + u(ab)) > (u(a) + u(ab))(u(b) + u(ab))$. Since $u(a) + u(b) + u(ab) > u(b) + u(ab)$, we obtain $(u(a) + u(ab))(u(b) + u(ab)) \geq (u(a) + u(ab))(u(a) + u(b) + u(ab))$. Therefore, $\rho(a, ab) > \rho(x_0, ab)$. Second, $\rho(x_0, ab) \geq \rho(b, ab)$ if and only if $\frac{(u(a)+u(ab))(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2} \geq \frac{u(b)(u(b)+u(ab))}{(u(a)+u(b)+u(ab))^2}$ if and only if $u(a) + u \geq u(b)$. Therefore, $\rho(x_0, ab) \geq \rho(b, ab) = \min(\rho(a, ab), \rho(b, ab))$. ■

4.4.2 Negative Correlation

We now consider a scenario that induces a negative correlation. Suppose that the supermarket tries to maximize the probability of a purchase (in our model this is to minimize the probability of choosing the outside option x_0). It will then place in hard-to-find, non-salient places, the goods that customers most need to buy, while prominently displaying goods that consumers do not need that much. Imagine placing rice on the bottom shelves, and new flavors of salad dressing at eye-level.

PALM gives a negative correlation between perception and utility under a very simple condition. When $\succ$ strictly orders a set A, and the outside option x_0 has probability 0 in A, then the *last* element according to $\succ$ in A must have the highest utility. Formally,

Proposition 6: *Suppose that $\rho_{(u,\succsim)}(x_0, A) = 0$. If $x \in A$ is an alternative such that $y \succ x$ for any $y \in A \setminus \{x\}$, then $u(x) > u(y)$ for any $y \in A \setminus \{x\}$.*

Proof of Proposition 6: Consider a menu $A = \{a_1, a_2, \ldots, a_k\}$ with $a_1 \succ a_2 \succ \ldots \succ a_k$. Note that

$$1 - \sum_i \rho_{(u,\succsim)}(a_i, A) = \prod_i (1 - q(a_i, A)).$$

Therefore, $\sum_i \rho_{(u,\succsim)}(a_i, A) = 1$ holds only if there exists j such that $q(a_j, A) = 1$. Since $\rho_{(u,\succsim)}(a_k, A) = q(a_k, A) \prod_{j<k}(1 - q(a_j, A)) > 0$ and $q(a_j, A) < 1$ for all $j < k$. So, $\sum_i \rho_{(u,\succsim)}(a_i, A) = 1$ holds only if $q(a_k, A) = 1$. Then,

$$q(a_k, A) = 1 = \frac{u(a_k)}{\sum_j u(a_j) + u(A)}$$

implies that $u(A) = -\sum_{j<k} u(a_j)$. Then $q(a_i, A) = \frac{u(a_i)}{\sum_j u(a_j)+u(A)} = \frac{u(a_i)}{u(a_k)}$.Since $q(a_i, A) < 1$ for all $i < k$, $u(a_i) < u(a_k)$. ■

5 Related Literature

Section 4.2 explains how PALM relates to the relevant empirical findings, namely the similarity effects, the compromise effects, and the attraction effects. We now proceed to discuss the relation between PALM and some of the most important theoretical models of stochastic choice.

There is a literature in psychology that proposes several models which can explain similarity, compromise and attraction. Rieskamp et al. (2006) is an excellent survey. Examples are Tversky (1972b), Roe et al. (2001) and Usher and McClelland (2004). The latter two papers propose *decision field theory*, which allows for violations of Luce's regularity axiom. We shall not discuss these papers here, and focus instead on the more narrowly related axiomatic literature in economics.

The benchmark economic model of rational behavior for stochastic choice is the random utility model. Luce's model is a special case of both PALM and random utility. So PALM and random utility are not mutually exclusive; PALM is, however, not always a random utility model. Below we discuss the recent papers of Gul, Natenzon and Pesendorfer (which is distinct from, but compatible with PALM), and of Manzini and Mariotti (which turns out to be incompatible with PALM).

5.1 Random Utility Models

The random utility model is described by a probability measure over preferences over X; $\rho(x, A)$ is the probability of drawing a utility that ranks x above any other alternative in A. The random utility model is famously difficult to characterize behaviorally: see the papers by Falmagne (1978), McFadden and Richter (1990), and Barberá and Pattanaik (1986).

There are instances of PALM which violate the regularity axiom. A random utility model must always satisfy regularity. Thus PALM is not a special case of random utility. Moreover, when there is no outside option, Luce's is a random utility model and a special case of PALM. So PALM and random utility intersect, but they are distinct.

5.2 Gul, Natenzon and Pesendorfer

The recent paper by Gul et al. (2010) presents a model of random choice in which object attributes play a key role. Object attributes are obtained endogenously from the observed stochastic choices. Their model has the Luce form, but it applies sequentially, first for choosing an attribute and then for choosing an object. In terms of its empirical motivation, the model seeks to address the similarity effect.

Gul, Natenzon and Pesendorfer's model is a random utility model (in fact they show that any random utility model can be approximated by their model). There are therefore instances of PALM that cannot coincide with the model in Gul et al. (2010). (Importantly, PALM can explain violations of the regularity axiom.) On the other hand, Luce's model is a special case of their model and of PALM. So the two models obviously intersect.

5.3 Manzini and Mariotti

Manzini and Mariotti (2013) study a stochastic choice model where *attention* is the source of randomness in choice. In their model, preferences are deterministic, but choice is random because attention is random. Manzini and Mariotti's model takes as parameters a probability measure g on X, and a linear order $\succ_M$. Their representation is then

$$\rho(a, A) = g(a) \prod_{a' \succ_M a} (1 - g(a')).$$

In PALM, perception is described by the (non-stochastic) perception priority relation $\succsim$. Choice is stochastic because it is dictated by utility intensities, similarly to Luce's model. In Manzini and Mariotti, in contrast, attention is stochastic, but preference is deterministic.

Manzini and Mariotti's representation looks superficially similar to ours, but the models are in fact different to the point of not being compatible, and seek to capture totally different phenomena. Manzini and Mariotti's model implies that IIA is violated for any pair x and y, so their model is incompatible with Luce's model. PALM, in contrast, has Luce as a special case. Appendix A shows that the two models are disjoint. Any instance of their model must violate the PALM axioms, and no instance of PALM can be represented using their model. So their model and ours seek to capture completely different phenomena.

5.4 Non-stochastic choice

Some related studies use the model of non-stochastic choice to explain some of the experimental results we describe in Section 4.2. This makes them quite different, as the primitives are different. The paper by De Clippel and Eliaz (2012) is important to mention; it gives an axiomatic foundation for models of non-stochastic choice that can capture the compromise effect. PALM gives a different explanation for the compromise effect, in the context of stochastic choice.

Another related paper is Lleras et al. (2010). (See also Masatlioglu et al. (2012) for a different model of attention and choice.) They attribute violations of IIA to the role of attention. They elicit revealed preference (not perception priority, but preference) in a similar way to ours. When the choice from $\{x, y, z\}$ is x and from $\{x, z\}$ is z, then they conclude that x is revealed preferred to z (this is in some sense, the opposite of the inference we make).

6 Proof of Theorem 1

6.1 Necessity

We start by proving the converse statement. Let $(u, \succsim)$ be a regular PALM in which $\succsim$ satisfies Richness. Let $\succsim^*$ be derived revealed perception priority from $\rho_{(u,\succsim)}$. We shall first prove that $\succsim^* = \succsim$. The next lemma is useful throughout this section.

Lemma 1 *If $c \succ a \succ b$, or $a \succ b \succ c$, then* $\dfrac{\rho(a, abc)}{\rho(b, abc)} < \dfrac{\rho(a, ab)}{\rho(b, ab)}$.

Proof: Let $a \succ b$.

Case 1: $c \succ a \succ b$. Since $b \not\sim c$,

$$\frac{\rho(a,abc)}{\rho(b,abc)}\Big/\frac{\rho(a,ab)}{\rho(b,ab)} = \frac{\mu(a,abc)(1-\mu(c,abc))}{\mu(b,abc)(1-\mu(c,abc))(1-\mu(a,abc))}\Big/\frac{\mu(a,ab)}{\mu(b,ab)(1-\mu(a,ab))}$$
$$= \frac{(1-\mu(a,ab))}{(1-\mu(a,abc))}\Big[\frac{u(a)}{u(b)}/\frac{u(a)}{u(b)}\Big] < 1,$$

where the last strict inequality is by Regularity.

Case 2: $a \succ b \succ c$. Since $b \not\sim c$,

$$\frac{\rho(a,abc)}{\rho(b,abc)}\Big/\frac{\rho(a,ab)}{\rho(b,ab)} = \frac{\mu(a,abc)}{\mu(b,abc)(1-\mu(a,abc))}\Big/\frac{\mu(a,ab)}{\mu(b,ab)(1-\mu(a,ab))}$$
$$= \frac{1-\mu(a,ab)}{1-\mu(a,abc)} < 1,$$

where the last strict inequality is by Regularity. ■

First, we prove $a \sim b$ if and only if $a \sim^* b$. Then, we prove $a \succ b$ if and only if $a \succ^* b$.

Lemma 2 *$a \sim b$ if and only if $a \sim^* b$.*

Proof of Lemma 2:

Step 1: If $a \sim b$ then $a \sim^* b$.

Proof of Step 1: Fix $c \in X$ to show $\frac{\rho(a,abc)}{\rho(b,abc)}/\frac{\rho(a,ab)}{\rho(b,ab)} = 1$.

Case 1: $a \sim b \succsim c$.

$$\frac{\rho(a,abc)}{\rho(b,abc)}\Big/\frac{\rho(a,ab)}{\rho(b,ab)} = \frac{\mu(a,abc)}{\mu(b,abc)}\Big/\frac{\mu(b,ab)}{\mu(b,ab)} = \frac{u(a)}{u(b)}\Big/\frac{u(a)}{u(b)} = 1.$$

Case 2: $c \succ a \sim b$.

$$\frac{\rho(a,abc)}{\rho(b,abc)}\Big/\frac{\rho(a,ab)}{\rho(b,ab)} = \frac{\mu(a,abc)(1-\mu(c,abc))}{\mu(b,abc)(1-\mu(c,abc))}\Big/\frac{\mu(b,ab)}{\mu(b,ab)} = \frac{u(a)}{u(b)}\Big/\frac{u(a)}{u(b)} = 1.$$

$\square$

Step 2: If $a \succ b$, then $a \nsim^0 b$.

Proof of Step 2: By Richness, there is c with $c \succ a \succ b$ or $a \succ b \succ c$. In either case, by Lemma 1, $\frac{\rho(a,abc)}{\rho(b,abc)} < \frac{\rho(a,ab)}{\rho(b,ab)}$. Hence, $a \nsim^0 b$. $\square$

Step 3: If $a \succsim^0 b$, then $a \succsim b$.

Proof of Step 3: We show that if $a \not\succsim b$ then $a \not\succsim^0 b$. Let $a \not\succsim b$. Then by completeness, $b \succ a$. By Richness, there is c with $c \succ b \succ a$ or $b \succ a \succ c$. Suppose with out loss of generality that $c \succ b \succ a$. By Lemma 1, we have $\frac{\rho(b,abc)}{\rho(a,abc)} < \frac{\rho(b,ab)}{\rho(a,ab)}$. Moreover, since $c \succ b$ and $c \succ a$, Step 2 shows that $c \nsim^0 a$ and $c \nsim^0 b$. Hence, $b \succ^0 a$, so that $a \not\succsim^0 b$. $\square$

Step 4: If $a \sim^* b$ then $a \sim b$.

Proof of Step 4: Let $a \sim^* b$. By the definition of $\sim^*$, $a \succsim^* b$ and $b \succsim^* a$. Then $a \succsim^* b$ implies that there exist $c_1, \ldots, c_k$ such that $a = c_1 \succsim^0 c_2 \succsim^0 \ldots \succsim^0 c_k = b$. By Step 3 and the transitivity of $\succsim$, we have that $a \succsim b$. Similarly, $b \succsim^* a$ implies that $b \succsim a$. Thus $a \sim b$. $\square$

$\blacksquare$

In the following, we prove that $a \succ b$ if and only if $a \succ^* b$.

Lemma 3 *If $a \succ^* b$ then $a \succ b$.*

Proof: Let $a \succ^* b$. It suffices to consider the following two cases.

Case 1: $a \succ^0 b$. Suppose, towards a contradiction, $a \not\succ b$. By the completeness of $\succsim$, $b \succsim a$. Note that $a \succ^0 b$ implies $a \nsim^0 b$, so $a \nsim b$ by Lemma 2. Then $b \succ a$. By Richness there is c such that $c \succ b \succ a$ or $b \succ a \succ c$. In either case, $\frac{\rho(a,abc)}{\rho(b,abc)} / \frac{\rho(a,ab)}{\rho(b,ab)} > 1$ by Lemma 1, in contradiction with $a \succ^0 b$.

Case 2: There exist $c_1, \ldots, c_k \in X$ such that $a \succ^0 c_1 \succ^0 \cdots \succ^0 c_k \succ^0 b$. Then, by the proof in Case 1, $a \succ c_1 \succ \cdots \succ c_k \succ b$. Hence, by transitivity, $a \succ b$. ■

The next lemma show the converse.

Lemma 4 *If $a \succ b$ then $a \succ^* b$.*

Proof: To simplify the exposition, we use the following notation in this proof: $a \vdash b$ if $a \succ b$ and there is no $c \in X$ with $a \succ c \succ b$.

Let $a \succ b$.

Case 1: $a \vdash b$. It suffices to show that $a \succ^0 b$. By Richness, there exists c such that $c \succ a \succ b$ or $a \succ b \succ c$; so there is at least one c such that $a \not\sim c$ and $b \not\sim c$. By Lemma 2, $a \not\sim^* c$ and $b \not\sim^* c$.

Choose any $d \in X$ such that $a \not\sim^* d$ and $b \not\sim^* d$. By Lemma 2, $a \not\sim d$ and $b \not\sim d$. Since $a \vdash b$, it is not true that $a \succ d \succ b$. That is, either $d \succ a$ or $b \succ d$. Since $a \succ b$, then $d \succ a \succ b$ or $a \succ b \succ d$. In either case, by Lemma 1, $\frac{\rho(a,abc)}{\rho(b,abc)} / \frac{\rho(a,ab)}{\rho(b,ab)} < 1$. Thus $a \succ^0 b$. Hence, $a \succ^* b$.

Case 2: $a \not\vdash b$. There exist $c_1, \ldots, c_k \in X$ such that $a \vdash c_1 \vdash \cdots \vdash c_k \vdash b$. By the argument in Case 1, $a \succ^0 c_1 \succ^0 \cdots \succ^0 c_k \succ^0 b$. Therefore, $a \succ^* b$. ■

Finally, to complete the proof of the necessity, we prove that ρ satisfies Hazard Rate Regularity if and only if $\rho_{(u,\succsim)}$ satisfies the regularity. Since $\frac{1}{\mu(a,abc)} - \frac{1}{\mu(a,ab)} = \frac{u(a)+u(b)+u(c)+u(ab)}{u(a)} - \frac{u(a)+u(b)+u(ab)}{u(a)}$, $\mu(a,ab) \geq \mu(a,abc)$ if and only $u(c) \geq u(ab) - u(abc)$. Moreover, $b \nsim c$ if and only if $b \not\sim^0 c$.

6.2 Sufficiency

In this section, we prove the sufficiency. Choose a nondegenerate stochastic choice function ρ that satisfies axioms in the theorem. Let $\succ^*$ be the derived revealed perception priority.

For all $A \in \mathscr{A}$ and $a \in A$, define

$$\nu(a, A) = \frac{q(a, A)}{\sum_{a \in A} q(a, A)}.$$

Since ρ is nondegenerate, $1 > \rho(a, A) > 0$ for all $a \in A$. Remember that $A_a = \{b \in A | b \succ^0 a\}$. Since $a \notin A_a$, $1 - \rho(A_a, A) > 0$. Hence, q is well defined. Moreover, $q(a, A) > 0$ because ρ is non-degenerate. Thus, ν is also well defined.

Step 1: There exists $u : X \to \mathbb{R}_{++}$ such that $q(a, A) = \frac{u(a)}{\sum_{a' \in A} u(a')} \sum_{a \in A} q(a, A)$.

Proof of Step 1: First, we show that ν satisfies Luce's IIA. For all $a, b, c \in X$

$$\frac{\nu(a, \{a, b\})}{\nu(b, \{a, b\})} = \frac{q(a, \{a, b\})}{q(b, \{a, b\})} = \frac{q(a, \{a, b, c\})}{q(b, \{a, b, c\})} = \frac{\nu(a, \{a, b, c\})}{\nu(b, \{a, b, c\})}.$$

Moreover, $\sum_{a \in A} \nu(a, A) = 1$. Therefore, by Luce's theorem (Luce (1959)), there exists $u : X \to \mathbb{R}_{++}$ such that $\nu(a, A) = \frac{u(a)}{\sum_{a' \in A} u(a')}$. Hence, by definition, $q(a, A) = \nu(a, A) \sum_{a \in A} q(a, A) = \frac{u(a)}{\sum_{a' \in A} u(a')} \sum_{a \in A} q(a, A)$. □

For all $A \in \mathscr{A}$, define

$$u(A) = \sum_{a \in A} u(a) \Big(\frac{1}{\sum_{a \in A} q(a, A)} - 1 \Big).$$

Step 2: $q(a, A) = \frac{u(a)}{\sum_{a' \in A} u(a') + u(A)}$.

Proof of Step 2: By a direction calculation,

$$\begin{aligned}\frac{\sum_{a'\in A}u(a')+u(A)}{u(a)} &= \frac{\sum_{a'\in A}u(a')}{u(a)}+\frac{\sum_{a'\in A}u(a')}{u(a)}\left(\frac{1}{\sum_{a'\in A}q(a',A)}-1\right)\\ &= \frac{\sum_{a'\in A}u(a')}{u(a)}\frac{1}{\sum_{a'\in A}q(a',A)}\\ &= \frac{1}{q(a,A)},\end{aligned}$$

where the last equality holds by Step 1. □

Step 3: $\rho=\rho_{(u,\succsim^*)}$.

Proof of Step 3: Choose any $A\in\mathscr{A}$. Since $\succsim^*$ is a weak order, therefore the indifference relation $\sim^*$ is transitive. Then, the set of equivalence classes $A/_{\succsim^*}$ is well defined and finite. That is, there exist a partition $\{\alpha^1,\alpha^2,\dots\alpha^k\}$ of A such that $a_j\succ^* a_i$ for all $a_i\in\alpha^i$ and $a_j\in\alpha^j$ with $j>i$ and $a_i\sim^* a_{i'}$ for all $a_i,a_{i'}\in\alpha^i$.

Define $p_i\equiv\rho(\alpha^i,A)=\sum_{a'\in\alpha^i}\rho(a',A)$. Then for $a\in\alpha^i$, $q(a,A)=\frac{\rho(a,A)}{1-\sum_{j>i}p_j}$. Therefore,

$$\sum_{a\in\alpha^i}q(a,A)=\sum_{a\in\alpha^i}\frac{\rho(a,A)}{1-\sum_{j>i}p_j}=\frac{\sum_{a\in\alpha^i}\rho(a,A)}{1-\sum_{j>i}p_j}=\frac{p_i}{1-\sum_{j=i+1}^k p_j}.$$

Hence,

$$1-\sum_{a\in\alpha^i}q(a,A)=1-\frac{p_i}{1-\sum_{j=i+1}^k p_j}=\frac{1-\sum_{j=i+1}^k p_j-p_i}{1-\sum_{j=i+1}^k p_j}=\frac{1-\sum_{j=i}^k p_j}{1-\sum_{j=i+1}^k p_j}.$$

Therefore, for any $s\in\{1,\dots,k\}$,

$$\prod_{i=s+1}^k(1-\sum_{a\in\alpha^i}q(a,A))=\prod_{i=s+1}^k\frac{1-\sum_{j=i}^k p_j}{1-\sum_{j=i+1}^k p_j}=\frac{1-\sum_{j=s+1}^k p_j}{1}=1-\rho(A_a,A).$$

For all $a\in A$ and $A\in\mathscr{A}$, define $\mu(a,A)=q(a,A)$.

Choose $a\in A$. Without loss of generality assume that $a\in\alpha^s$. Then, $\rho(a,A)=q(a,A)(1-\rho(A_a,A))=\mu(a,A)(1-\rho(A_a,A))=\mu(a,A)\prod_{i=s+1}^k(1-\sum_{a'\in\alpha^i}\mu(a',A))\equiv$

$\rho_{(u,\succsim^*,u)}(a, A)$. □

■

Appendix A Appendix: Relation to Manzini and Mariotti

The model of Manzini and Mariotti (2013) is specified by a probability measure g on X, and a linear order $\succ_M$. Their representation is then

$$\rho(a, A) = g(a) \prod_{a' \succ_M a} (1 - g(a')).$$

Superficially, this representation looks similar to ours, but it is actually very different: It is incompatible with our model, in the sense that the set of stochastic choices that satisfy our model is disjoint from the set of stochastic choices in Manzini and Mariotti's model. We now proceed to prove this fact.

Let ρ have a Manzini and Mariotti (2013) representation as above and let X have at least three elements. Suppose, towards a contradiction that it also has a representation using our model.

We are going to prove that the two models differ in a strong sense, because we are going to show that there is no subset of X of three elements on which the two models can coincide.

Let $a, b, c \in X$. The preference relation $\succ_M$ is a linear order. Suppose, without loss of generality, that $a \succ_M b \succ_M c$. Given the Manzini-Mariotti representation, then

$$\rho(a, abc) = \rho(a, ab) = \rho(a, ac) = g(a),$$

and

$$\rho(b, abc) = \rho(b, ab) = g(b)(1 - g(a)).$$

We have assumed that ρ has a PALM representation given by some $(u, \succsim, u)$. Now consider how a, b, c are ordered by $\succsim$.

There are seven cases to consider; each one of these cases end in a contradiction.

1. $a \succsim b$, $a \succsim c$, and $b \not\sim c$: By Regularity, since $b \not\sim c$, $\rho(a, abc) = q(a, abc) < \rho(a, ab) = q(a, ab)$.

2. $b \succsim a$, $b \succsim c$ and $a \not\sim c$: By Regularity, since $a \not\sim c$, $\rho(b, abc) = q(b, abc) < \rho(b, ab) = q(b, ab)$.

3. $c \succ a \succsim b$: By Regularity, $\rho(a, abc) = q(a, abc)(1 - q(c, abc)) < q(a, abc) \leq q(a, ab) = \rho(a, ab)$.

4. $a \succ b \sim c$: By Regularity, since $\rho(a, abc) = q(a, abc) = \rho(a, ab) = q(a, ab)$ and $q(b, abc) < q(b, ab)$ because $a \not\sim c$, $\rho(b, abc) = q(b, abc)(1 - q(a, abc)) < \rho(b, ab) = q(b, ab)(1 - q(a, ab))$.

5. $b \succ a \sim c$: By Regularity, $\rho(a, abc) = q(a, abc)(1 - q(b, abc)) < q(a, abc) \leq q(a, ac) = \rho(a, ac)$.

6. $c \succ b \succ a$: By Regularity, $\rho(b, abc) = q(b, abc)(1 - q(c, abc)) < q(b, abc) \leq \rho(b, ab) = q(b, ab)$.

7. $a \sim b \sim c$: In this case, Luce's IIA is cannot be violated in PALM. However, in Manzini and Marriott's Model, there is always at least one violation of Luce's IIA.

Appendix B Appendix: Similarity Effect and Compromise Effect

B.1 Alternative Condition for Similarity Effect

In Proposition 1, we have shown that PALM can capture the similarity effect under that condition that $x \succ y \succ z$.

Remember our example of similarity effect in which x and z are the same Beethoven symphony. One may think that x and z are similar and hence $x \sim z$. Even in this case, PALM can capture the similarity effect with the following property of μ:

Definition: A hazard rate function μ satisfies *increasing impact at* $(x, y; z)$ if

$$\mu(x, \{x, y\}) - \mu(x, \{x, y, z\}) > \mu(z, \{x, y, z\}).$$

The increasing impact property has a natural implication: the effect on the hazard rate μ of adding z should not be smaller than the magnitude of z. Let us assume that $x \sim z \succ y$: the two Beethoven recordings are more salient from the viewpoint of perception than Debussy.

Proposition A1: *If $x \sim z \succ y$ and μ satisfies increasing impact at $(x, y; z)$, then $\rho_{(u,\succsim)}$ exhibits the similarity effect at $\{x, y\}$ with respect to z.*

Proof of Proposition A1: Since $x \succ y$, then $\rho(x, xy) = \mu(x, xy)$ and $\rho(y, xy) = \mu(y, xy)(1 - \mu(x, xy))$. Since $x \sim z \succ y$, then $\rho(x, xyz) = \mu(x, xyz)$ and $\rho(y, xyz) = \mu(y, xyz)(1 - \mu(x, xyz) - \mu(z, xyz))$. Note also that $\frac{\mu(x,xy)}{\mu(y,xy)} = \frac{u(x)}{u(y)} = \frac{\mu(x,xyz)}{\mu(y,xyz)}$. By increasing impact, $\mu(x, \{x, y\}) - \mu(x, \{x, y, z\}) > \mu(z, \{x, y, z\})$; so $1 - \mu(x, \{x, y\}) < 1 - \mu(x, \{x, y, z\}) - \mu(z, \{x, y, z\})$. Hence, $\frac{\rho(x,\{x,y\})}{\rho(y,\{x,y\})} = \frac{u(x)}{u(y)} \frac{1}{1-\mu(x,xy)} > \frac{u(x)}{u(y)} \frac{1}{1-\mu(x,xyz)-\mu(z,xyz)} = \frac{\rho(x,\{x,y,z\})}{\rho(y,\{x,y,z\})}$. ■

B.2 Compromise Effect by Simonson and Tversky (1992)

The definition of the compromise effect in Simonson and Tversky (1992) differs from the one we have discussed in the text. They say that ρ exhibits the *compromise effect* at (x, y, z), if

$$\frac{\rho(x, \{x, y, z\})}{\rho(y, \{x, y, z\})} < \frac{\rho(x, \{x, y\})}{\rho(y, \{x, y\})} \text{ and } \frac{\rho(z, \{x, y, z\})}{\rho(y, \{x, y, z\})} < \frac{\rho(z, \{y, z\})}{\rho(y, \{y, z\})}. \tag{7}$$

PALM can explain the compromise effect under this definition as well:[3]

Proposition A2: *If $x \sim z \succ y$, μ satisfies increasing impact at $(x, y; z)$ and $(z, y; x)$, then ρ exhibits the compromise effect at (x, y, z).*

Proof of Proposition A2: By Proposition A1, since μ satisfies increasing impact at $(x, y; z)$, $x \sim z \succ y$ implies $\frac{\rho(x,\{x,y,z\})}{\rho(y,\{x,y,z\})} < \frac{\rho(x,\{x,y\})}{\rho(y,\{x,y\})}$. Similarly, since μ satisfies increasing impact at $(z, y; x)$, $x \sim z \succ y$ implies also $\frac{\rho(z,\{x,y,z\})}{\rho(y,\{x,y,z\})} < \frac{\rho(z,\{y,z\})}{\rho(y,\{y,z\})}$. ■

B.3 Alternative Condition for Compromise Effect

In Proposition 2, we show that PALM can capture the compromise effect under that condition that $x \succ y \succ z$.

Remember our example of the compromise effect in which x and z are the extremes and y is the compromised option. One may think that the extremes are perceived more prominently than the compromised option, and hence, $x \sim z \succ y$. Even in this case, PALM can capture the compromise effect. The following proposition shows all configurations of a PALM for which it can generate the compromise effect under the definition of Rieskamp et al. (2006), beside the one in the proposition.

Proposition A3:

1) When $x \sim z \succ y$, ρ exhibits the compromise effect at $\{x, z\}$ with respect to y if and only if

$$u(y) > u(x) \text{ and } u(xyz) - \frac{u(x)u(z)}{u(y) - u(x)} > \frac{u^2(x) - u^2(y) + u(x)u(y)}{u(y) - u(x)} > u(xy).$$

[3] In particular, Simonson and Tversky (1992) called the effect *enhancement*. They define the effect by $\frac{\rho(y,xyz)}{\rho(y,xyz)+\rho(z,xyz)} > \rho(y, yz)$ and $\frac{\rho(y,xyz)}{\rho(x,xyz)+\rho(y,xyz)} > \rho(y, xy)$. Their definition is equivalent to (7) when the probability of choosing the outside option is zero, which is true in experiments. That is, if $\rho(x, xy)+\rho(y, xy) = 1$, then $\frac{\rho(x,\{x,y,z\})}{\rho(y,\{x,y,z\})} < \frac{\rho(x,\{x,y\})}{\rho(y,\{x,y\})} \Leftrightarrow \frac{\rho(y,xyz)}{\rho(x,xyz)+\rho(y,xyz)} > \rho(y, xy)$. If $\rho(y, yz)+\rho(z, yz) = 1$, then $\frac{\rho(z,\{x,y,z\})}{\rho(y,\{x,y,z\})} < \frac{\rho(z,\{y,z\})}{\rho(y,\{y,z\})} \Leftrightarrow \frac{\rho(y,xyz)}{\rho(y,xyz)+\rho(z,xyz)} > \rho(y, yz)$.

2) When $y \succ z \succ x$, ρ exhibits the compromise effect at $\{x, z\}$ with respect to y if and only if

$$u(x) > u(y) \text{ and } u(xyz) - \frac{u(y)u(z)}{u(x) - u(y)} < \frac{u^2(y) - u^2(x) + u(x)u(y)}{u(x) - u(y)} < u(xy).$$

The proofs are by straightforward calculations.

References

Barberá, S. and P. K. Pattanaik (1986): "Falmagne and the Rationalizability of Stochastic Choices in Terms of Random Orderings," *Econometrica*, 54, pp. 707–715.

De Clippel, G. and K. Eliaz (2012): "Reason-based choice: A bargaining rationale for the attraction and compromise effects," *Theoretical Economics*, 7, 125–162.

Debreu, G. (1960): "Review of R. Duncan Luce, Individual choice behavior: A theoretical analysis," *American Economic Review*, 50, 186–188.

Falmagne, J. (1978): "A representation theorem for finite random scale systems," *Journal of Mathematical Psychology*, 18, 52–72.

Gul, F., P. Natenzon, and W. Pesendorfer (2010): "Random Choice as Behavioral Optimization," Mimeo, Princeton University.

Krajbich, I. and A. Rangel (2011): "Multialternative drift-diffusion model predicts the relationship between visual fixations and choice in value-based decisions," *Proceedings of the National Academy of Sciences*, 108, 13852–13857.

Lleras, J. S., Y. Masatlioglu, D. Nakajima, and E. Y. Ozbay (2010): "When More is Less: Choice with Limited Consideration," Mimeo, U. of Michigan.

Luce, R. D. (1959): *Individual Choice Behavior a Theoretical Analysis*, John Wiley and sons.

Manzini, P. and M. Mariotti (2013): "Stochastic choice and consideration sets," Forthcoming in Econometrica.

Masatlioglu, Y., D. Nakajima, and E. Y. Ozbay (2012): "Revealed attention," *The American Economic Review*, 102, 2183–2205.

McFadden, D. and M. Richter (1990): "Stochastic rationality and revealed stochastic preference," in *Preferences, Uncertainty, and Optimality, Essays in Honor of Leo Hurwicz, Westview Press: Boulder, CO*, 161–186.

Mochon, D. and S. Frederick (2012): "Compromise or response order effects?" Mimeo, Yale University School of Management.

Reutskaja, E., R. Nagel, C. F. Camerer, and A. Rangel (2011): "Search Dynamics in Consumer Choice under Time Pressure: An Eye-Tracking Study," *The American Economic Review*, 101, 900–926.

Rieskamp, J., J. R. Busemeyer, and B. A. Mellers (2006): "Extending the bounds of rationality: evidence and theories of preferential choice," *Journal of Economic Literature*, 631–661.

Roe, R. M., J. R. Busemeyer, J. T. Townsend, et al. (2001): "Multialternative decision field theory: A dynamic connectionist model of decision making," *PSYCHOLOGICAL REVIEW-NEW YORK-*, 108, 370–392.

Simonson, I. and A. Tversky (1992): "Choice in context: tradeoff contrast and extremeness aversion." *Journal of marketing research.*

Tversky, A. (1972a): "Choice by elimination," *Journal of mathematical psychology*, 9, 341–367.

——— (1972b): "Elimination by aspects: A theory of choice." *Psychological review*, 79, 281.

Usher, M. and J. L. McClelland (2004): "Loss aversion and inhibition in dynamical models of multialternative choice." *Psychological review*, 111, 757.