

## Precise Determination of the Fine Structure Constant from X-Ray Spin Doublet Splitting<sup>1</sup>

R. F. CHRISTY<sup>2</sup> AND J. M. KELLER<sup>3</sup>

*Department of Physics, University of California, Berkeley, California*

(Received December 17, 1941)

Corrections to the Sommerfeld formula for the  $L_{II}$ - $L_{III}$  x-ray spin doublet splitting, to take into account departures from a pure Coulomb field for heavy elements caused by interactions between electrons, are calculated by using Dirac wave functions for a Coulomb field and calculating, with all relativistic corrections, the terms in the electron interaction of order  $e^2$ , i.e., of relative order  $1/Z$ , compared to the Sommerfeld splitting. The result is applied to experimental data for elements varying from  $Z=60$  to  $Z=92$ , to obtain a direct experimental value for the fine structure constant. By the method of least squares, we find  $1/\alpha = \hbar c/e^2 = 136.93$  with an estimated *limit* of error of 0.18.

### INTRODUCTION

IN view of the discrepancy that has existed between different determinations of the various fundamental atomic constants, as well as because of the interest attached to the value of the fine structure constant,  $\alpha = e^2/\hbar c$ , it has again seemed desirable to attempt a precise determination of this quantity from experimental data. One might hope to use for this purpose a measurement of the fine structure of the hydrogen atom spectrum. But as is well known, difficulties of resolving the small splitting, of the same order as the line breadths, as well as theoretical uncertainties in its interpretation, do not permit a precise evaluation. The splitting, however, becomes very much greater for the heavier atoms, varying roughly as the fourth power of the atomic number; and, as we shall see, the uncertainties of quantum electrodynamics do not here play a decisive role.

The well-known relativistic formula for the energy levels of an electron in a Coulomb field of charge  $Ze$ , where  $e$  is the charge on the proton, is

$$E = mc^2/[1 + (\alpha Z)^2/(n' + \gamma)^2]^{1/2}, \quad (1)$$

where  $E$  is the total energy (including rest energy),  $n'$  is the radial quantum number,  $\gamma = [k^2 - (\alpha Z)^2]^{1/2}$ ,  $|k| = j + \frac{1}{2}$ ,  $j$  is the total angular

momentum, and  $m$  and  $c$  have their usual meanings. Thus the difference in energy of the  $L$  shell levels ( $2p_{1/2}$  and  $2p_{3/2}$ ) is

$$\begin{aligned} E(2p_{3/2}) - E(2p_{1/2}) &= mc^2 \left\{ \frac{1}{2} [4 - (\alpha Z)^2]^{1/2} \right. \\ &\quad \left. - \frac{1}{2} [2 + 2[1 - (\alpha Z)^2]^{1/2}]^{1/2} \right\} \quad (2) \\ &= mc^2 S(\alpha Z), \end{aligned}$$

which we shall refer to as "the Sommerfeld formula."<sup>4</sup>

About two years ago, R. T. Birge<sup>5</sup> suggested that one could take advantage of this greatly magnified "fine structure" in the heavier elements to determine an accurate value for the fine structure constant. At that time he assembled data on the splitting of the x-ray spin doublets ( $L_{II}$  and  $L_{III}$  levels) as obtained from several different x-ray lines, and from elements varying in atomic number from  $Z=29$  to 92. He showed that these data formed a body of material of sufficient accuracy and consistency for such a determination.

However, in interpreting the experiments, it was clearly not adequate to consider the field in which one of the electrons of a heavy atom finds itself as just the Coulomb field of the nucleus. One could get around this difficulty in a rather unsatisfactory manner by introducing a screening constant  $s$ , defined in such a way that the level splitting is given by  $S[\alpha(Z-s)]$ . One

<sup>1</sup> A brief report of this work has been given previously—R. F. Christy and J. M. Keller, *Phys. Rev.* **58**, 658 (1940). Certain additions and corrections to the original calculations have been made in the present paper.

<sup>2</sup> Now at Illinois Institute of Technology, Chicago, Illinois.

<sup>3</sup> Now at Washington University, St. Louis, Missouri.

<sup>4</sup> First derived by Sommerfeld from the Bohr theory. See A. Sommerfeld, *Atomic Structure and Spectral Lines* (Methuen, 1934).

<sup>5</sup> R. T. Birge, *Phys. Rev.* **55**, 1119 (1939).

expects that the electrons in the  $K$  shell will be quite efficient at screening the nucleus from the  $L$  shell, that the electrons of the  $L$  shell will partially screen the nuclear field from each other, and that electrons of the outer shell will be successively less significant, until a point is reached beyond which the addition of further outer electrons has no appreciable effect. The value of the screening constant calculated from the experiments was about 3.5 for the heaviest atoms, and did not vary appreciably with atomic number above about  $Z=60$ . The value of  $s$ , and its dependence on  $Z$ , were altered when one used different values of  $\alpha$  in the determination of  $S[\alpha(Z-s)]$ . This was roughly what one would expect.

But this approach is unsatisfactory for several reasons. In the first place, the field that acts on electrons in a heavy atom is certainly not a pure Coulomb field, and use of an uncalculable screening constant to correct for this circumstance is unsatisfactory. Furthermore, an increase in the nuclear charge itself produces a distortion in the charge and current distribution of the electrons, and not just a change of scale. This is true even if the field is considered as that of a Coulomb potential, provided one takes into account relativistic effects. (The Sommerfeld splitting is a manifestation of this distortion in the  $2p$  electrons alone.) The wave functions of the several states are functions of  $\alpha Z$ , entirely apart from any screening.

The essential point is that we must distinguish between two distinct parameters which occur in the problem. One is the atomic number  $Z$ ; this parameter tells us the number of electrons in an atom, rough values of the mean radii, and energies of the several states, and in general it may appear in connection with those aspects of the problem that are essentially non-relativistic. In the present calculation,  $1/Z$  will be treated as a small number. The other parameter involved is  $\alpha Z = Ze^2/\hbar c$ . This quantity indicates the extent to which relativistic effects are significant; for the present problem, it cannot be considered small, for it is actually of order  $\frac{1}{2}$ .

To illustrate this point, we make an expansion

of  $S[\alpha(Z-s)]$  in terms of these two parameters  $S[\alpha(Z-s)]$

$$\begin{aligned}
 &= \left[ \frac{1}{32}(\alpha Z)^4 + \frac{5}{256}(\alpha Z)^6 + \frac{53}{4096}(\alpha Z)^8 + \dots \right] \\
 &\quad - \frac{s}{Z} \left[ \frac{4}{32}(\alpha Z)^4 + \frac{30}{256}(\alpha Z)^6 \right. \\
 &\quad \quad \quad \left. + \frac{424}{4096}(\alpha Z)^8 + \dots \right] \quad (3) \\
 &\quad + \left( \frac{s}{Z} \right)^2 \left[ \frac{6}{32}(\alpha Z)^4 + \frac{75}{256}(\alpha Z)^6 + \dots \right] \\
 &\quad - \left( \frac{s}{Z} \right)^3 \left[ \frac{4}{32}(\alpha Z)^4 + \dots \right] + \dots
 \end{aligned}$$

The first line of Eq. (3) is just the Sommerfeld formula  $S(\alpha Z)$ . The supposition that  $s$  approaches a constant value, as one approaches the larger values of  $Z$ , imposes an unnecessary restriction on the form of the higher order terms as functions of  $\alpha Z$ .

We wish to make a theoretical calculation to replace Eq. (3) for the spin doublet splitting in elements of large atomic number. The effect of the electrons in the atom on the splitting will be of order  $1/Z$  compared to that of the nuclear charge  $Ze$ . Hence we can calculate this effect by perturbation theory. The first-order term will correspond to the second line in Eq. (3). But we may obtain this term to an arbitrary accuracy, both in magnitude and in its functional dependence on  $\alpha Z$ . In the higher order terms in  $1/Z$ , only the lowest term in  $\alpha Z$  could in principle be determined by higher order perturbation calculation. Further terms in  $\alpha Z$  cannot be so determined because of fundamental limitations of present quantum-electrodynamic theory. But these terms are of order  $\alpha^2$  times the splitting and are negligible. Actually we shall not calculate the coefficient even of the leading terms in  $\alpha Z$  of the second and higher order perturbation terms, but consider it sufficient to introduce coefficients that will be determined by fitting the experimental data.

### CALCULATION OF THE FIRST-ORDER PERTURBATION

For unperturbed wave functions, we consider the individual electrons each in a Coulomb potential of charge  $Ze$ , independent of the presence of the other electrons. This leads to the well-known Dirac wave functions for a Coulomb field. The effect of the interactions of the electrons among themselves we consider as a perturbation and calculate to first order. For this purpose we use the Breit Hamiltonian interaction

$$\frac{e_i e_j}{|r_{ij}|} - \frac{1}{2} \left[ \frac{\boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j}{|r_{ij}|} + \frac{(\boldsymbol{\alpha}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\alpha}_j \cdot \mathbf{r}_{ij})}{|r_{ij}|^3} \right], \quad (4)$$

where  $r_{ij}$  is the displacement between points  $i$  and  $j$ , and the  $\boldsymbol{\alpha}$ 's are the Dirac current operators. The Breit Hamiltonian was originally derived from classical electrodynamics<sup>6</sup> as a quasi-static approximation, by keeping terms in the energy of order  $(v/c)^2$ . The two terms in Eq. (4) correspond, of course, to the electrostatic repulsion, and to magnetic and retardation effects, respectively. (When the divergence of the current vanishes, the second term reduces to the Biot-Savart law.)

The Breit Hamiltonian can also be obtained from quantum electrodynamics.<sup>7</sup> The magnetic contribution results from a second-order perturbation calculation involving the emission and absorption of (virtual) quanta. The energy denominators that occur contain the energy of the quanta and the difference in energy between the interacting states of the electrons involved. Again, to obtain a quasi-static approximation, the latter energy difference is neglected. This results in an expression that can be represented as an interaction in configuration space, as above. For any *particular* pair of states, the magnetic interaction can be expressed in configuration space without this approximation, but it contains explicitly the energy difference involved. One can make an expansion in powers of  $\epsilon r/\hbar c \sim v/c \sim \alpha Z$ , where  $\epsilon$  is this energy difference. The first term gives the magnetic interaction as in the Breit formula. Higher terms have been neglected in the main part of the calculations,

and their effect is subsequently introduced as a correction and in the estimate of error.

We have both direct and exchange interactions to consider. As we are interested not in the actual energy levels of the (excited) atoms, but only in the difference in energies for atoms in the cases of a  $2p_{1/2}$  electron or a  $2p_{3/2}$  electron missing, it is not necessary to calculate the interaction energies of all pairs of electrons, but only of a  $2p$  electron with other electrons. We take advantage of the spherical symmetry of subshells to calculate the interaction on a  $2p$  electron of complete subshells. (By a subshell we mean all those electrons of a particular orbital and total angular momentum in a particular shell—for example the two  $2s$  electrons.) We do this even in calculating the interaction of, for instance, three  $2p_{3/2}$  electrons on the fourth (missing) electron of that subshell. Including this “phantom” electron in the calculations actually makes no difference, as the addition to the direct integral will just be cancelled by the addition to the exchange integral.

The Dirac wave functions are products of spherical harmonics, exponentials in the radius, and polynomials in powers of the radius. Using the expansions of  $1/|r_{ij}|$  and  $1/|r_{ij}|^3$  in terms of  $r_s/r_o$  and spherical harmonics of the angular coordinates of the separate electrons (where  $r_s$  and  $r_o$  are, respectively, the smaller and greater of  $r_1, r_2$ ), we can reduce the integrals to products of angle and radial integrals. The former are just products of spherical harmonics. Taking the polynomials in the wave functions term by term, we can evaluate the radial integrals in terms of gamma-functions and hypergeometric functions. The resulting expressions are, however, very complicated and unilluminating, and in order to use them, we have calculated their value numerically for six values of  $\alpha Z$ , including  $\alpha Z=0$ . The values of  $\alpha Z$  were chosen rather arbitrarily, to simplify the numerical work as much as possible, and to cover fairly smoothly the region in which data exist.

The numerical results for the largest value of  $\alpha Z$  used are given in Table I. All numerical values given are energies of interaction of a  $2p_{1/2}$  or a  $2p_{3/2}$  electron with a complete subshell, and in units  $Ze^2/a = mZe^4/\hbar^2$  where  $a$  is the Bohr radius. Electrostatic and magnetic parts of the

<sup>6</sup> G. Breit, Phys. Rev. **34**, 553 (1929).

<sup>7</sup> J. R. Oppenheimer, Phys. Rev. **35**, 461 (1930).

TABLE I. Interaction energies (in units  $Ze^2/a = mZe^4/\hbar^2$ ) between various subshells and a  $2p_{1/2}$  or  $2p_{3/2}$  electron, for  $[1 - (\alpha Z)^2]^{\frac{1}{2}} = 0.76$ , ( $\alpha Z \sim 0.65$ ).

Subshell	$2p_{1/2}$	$2p_{3/2}$	Difference
Electrostatic direct interaction			
1s	0.64192	0.51552	0.12640
2s	0.39988	0.36876	0.03112
$2p_{1/2}$	0.46014	0.41311	0.04702
$2p_{3/2}$	0.82623	0.76061	0.05562
3s	0.20613	0.19861	0.00752
$3p_{1/2}$	0.22651	0.21640	0.01011
$3p_{3/2}$	0.41345	0.39792	0.01553
$3d_{3/2}$	0.46181	0.45312	0.00869
$3d_{5/2}$	0.66454	0.65408	0.01046
Electrostatic exchange interaction			
1s	-0.01796	-0.01498	-0.00298
2s	-0.03227	-0.03426	0.00199
$2p_{1/2}$	-0.23007	-0.01716	-0.03890
$2p_{3/2}$	-0.03432	-0.20833	
Magnetic exchange interaction			
1s	0.02773	0.00998	0.01776
2s	0.00192	-0.00095	0.00287
$2p_{1/2}$	0.00725	0.00170	0.00556
$2p_{3/2}$	0.00339	0.00960	-0.00621
Total			0.30256

interaction are treated separately. Magnetic direct integrals all vanish identically. This corresponds to the fact that there is no net current from a closed subshell. The values for  $\alpha Z = 0$  (non-relativistic limit) are just those calculable from Schroedinger wave functions. One sees, even though the electrostatic interactions are larger than the energies due to currents by order  $(c/v)^2$ , that the contributions to the differences for the two cases of a missing  $2p_{1/2}$  or  $2p_{3/2}$  electron are roughly of the same order of magnitude. This is of course due to the complete cancellation in the non-relativistic limit of the electrostatic terms.

Nevertheless, one sees that the dominant effects are the contributions of the electrostatic direct integrals. Furthermore, these contributions are always in a direction to decrease the splitting predicted by the Sommerfeld formula. Except in the case of the  $2p$  subshells, the electrostatic exchange energies are of the order of a twentieth of the direct terms. Because the exchange effects for the  $2p$  subshells involve the interaction of these subshells with electrons missing from one of the states of the same subshell, we expect their contribution to be large and of the same order as the direct integrals. Current effects are also small, being roughly a tenth of the corresponding electrostatic direct terms.

It is not practicable to carry to completion an

asymptotic calculation of this type, where effects of successive shells of electrons are taken into account one by one. One continues the calculation to the point beyond which the remainder seems negligible. Accordingly, we have included direct interactions of the  $K$ ,  $L$ , and  $M$  shells, and exchange interactions of the  $K$  and  $L$  shells. The numerical values of these interactions suggest that the still uncalculated contributions are perhaps one percent of the total first-order term. Actually, a fairly careful estimate of the contribution of the  $N$  shell direct integrals leads to a value about 0.2 percent of the previously calculated interaction. A comparison of the non-relativistic values of the  $M$  shell electrostatic exchange with the corresponding direct integrals indicates that the contribution of the former is  $\sim 0.5$  percent of the total first-order term.

In any case, the neglect of these outer shell electrons certainly will not alter significantly the shape of the first-order term as a function of  $\alpha Z$ : at most this value will slightly change its magnitude. Hence we will take for the total first-order perturbation term  $W$  the sum of the parts already calculated.  $W$  is tabulated in Table II. We later consider the error produced by multiplying  $W$  by a factor to allow for its estimated error.

There is also a correction to the calculations due to the approximate nature of the Breit Hamiltonian. As mentioned above, the Breit Hamiltonian actually represents just the first term of an expansion in powers of  $e\hbar/c \approx v/c$ . To determine the magnitude of the error introduced, we have calculated the effect of the next term in the interaction of the 1s electrons with a  $2p_{1/2}$  or  $2p_{3/2}$  electron. This should be by far the largest part of the neglected effect, because of the large kinetic energy of the  $K$  shell electrons. For  $\gamma = 0.76$ , ( $\alpha Z \sim 0.65$ ) we thus find a change in the original calculation of energy difference by 0.00317 in units  $mZe^4/\hbar^2$ .

TABLE II. Calculated correction to the Sommerfeld formula,  $W(\alpha Z)$ , in units  $Ze^2/a = mZe^4/\hbar^2$ .

$[1 - (\alpha Z)^2]^{\frac{1}{2}}$	$Z$ (approx.)	$W(\alpha Z)$
1.00	0	0.00000
0.96	38	0.03995
0.91	57	0.09508
0.86	70	0.15666
0.82	78	0.21110
0.76	89	0.30256

**DETERMINATION OF  $\alpha$  FROM  
EXPERIMENTAL DATA**

We now wish to compare our calculation with the observed x-ray spin doublet splittings ( $\Delta\nu$ ), and from this to obtain a value for  $\alpha$ . To do this, we write in analogy with Eq. (3)

$$h\Delta\nu = mc^2 S(\alpha Z) - W(\alpha Z) + O(1/Z^2). \quad (5a)$$

$S(\alpha Z)$  is the Sommerfeld formula, Eq. (2), for the splitting in a pure Coulomb field.  $W(\alpha Z)$  is the correction of order  $1/Z$  that we have calculated. The dependence of the  $1/Z^2$  and higher terms as functions of  $(\alpha Z)$  is as  $(\alpha Z)^4$  and higher powers. The present state of quantum-electrodynamic theory does not justify an attempt to calculate functional dependence of these higher order terms. But their effect must be small, and we shall include just the term  $(mc^2/Z^2)(\alpha Z)^4$ , multiplied by an arbitrary constant. We can write

$$\begin{aligned} W(\alpha Z) &= (mZe^4/h^2)(\alpha Z)^2 f(\alpha Z) \\ &= (mc^2/Z)(\alpha Z)^4 f(\alpha Z). \end{aligned} \quad (6)$$

$f(\alpha Z)$  is a quantity that approaches a constant value, as  $\alpha Z$  approaches zero. To eliminate the quantities  $h$ ,  $e$ , and  $m$ , where they occur explicitly in this relation, we introduce the Rydberg constant  $R$ . The value of  $R$  is known experimentally to a precision considerably higher than the uncertainties in  $h$ ,  $e$ , and  $m$ , and there seems to be no reason to doubt the correctness of the relation

$$R = 2\pi^2 me^4/h^3 = \alpha^2 mc^2/2h. \quad (7)$$

Hence we reformulate the connection between the doublet splitting  $\Delta\nu$  and the fine structure constant  $\alpha$  by the expression

$$\Delta\nu/R = (2/\alpha^2)S(\alpha Z) - 2\alpha^2 Z^3 f(\alpha Z) + B_1 \alpha^2 Z^2. \quad (5b)$$

$\alpha$  appears in this expression in several ways. In the third term of the right-hand side,  $\alpha^2$  appears multiplied by an undetermined constant. We can absorb our ignorance of the precise value for  $\alpha$  in that of the constant, writing as a single constant to be determined,  $B = B_1 \alpha^2$ .  $\alpha$  also appears in the argument of the first two terms on the right-hand side of Eq. (5b). These terms we treat in the familiar way, by expanding about an approximate value of  $\alpha$  and neglecting second

TABLE III. Experimental data. (These data were compiled, and kindly given to us, by R. T. Birge. They are based on the value,  $R = 109737$ .)

$Z$	$\Delta\nu/R$ (grating scale)	Weight	$Z$	$\Delta\nu/R$ (grating scale)	Weight
60	37.74799	148	74	98.34305	228
62	43.91743	92	75	104.68251	216
63	47.11417	90	76	111.06647	126
64	50.55852	90	77	118.30763	208
65	54.28796	90	78	125.67344	198
66	58.20409	110	79	133.39571	228
67	62.15851	110	80	141.73230	106
68	66.66333	110	81	150.07457	228
69	71.14493	110	82	159.21877	198
70	76.08290	110	83	168.48213	216
71	81.23862	142	90	249.60507	208
72	86.71362	110	91	263.14925	120
73	92.30472	228	92	277.73164	228

and higher derivatives. We use the value  $1/\alpha_0 = 137$ . Finally

$$\begin{aligned} \frac{\Delta\nu}{Z^4 R} - \frac{2}{\alpha_0^2 Z^4} S(\alpha_0 Z) + \frac{2\alpha_0^2}{Z} f(\alpha_0 Z) \\ = \left[ \frac{4}{\alpha_0 Z^4} S(\alpha_0 Z) - \frac{2}{\alpha_0 Z^3} \frac{d}{dZ} S(\alpha_0 Z) \right. \\ \left. + \frac{4\alpha_0^3}{Z} f(\alpha_0 Z) + 2\alpha_0^3 \frac{d}{dZ} f(\alpha_0 Z) \right] \Delta \left( \frac{1}{\alpha} \right) + \frac{B}{Z^2}. \end{aligned} \quad (8)$$

Equation (8) differs from Eq. (5b) by a factor of  $1/Z^4$ . This makes the two sides of Eq. (8) roughly independent of  $Z$ , and gives the observations of  $\Delta\nu/R$  for different elements and the same *a priori* weights roughly equal influence in determining the best value of  $\Delta(1/\alpha)$ .  $W(\alpha Z)$ , and hence  $f(\alpha Z)$ , was obtained by fitting a power series in  $(\alpha Z)$  to the six calculated values (Table II). A correction was then applied to  $W$  for terms of order  $(v/c)^4$  in the interaction energy between electrons. This correction to the Breit Hamiltonian contributes to Eq. (5b) a term of order  $(mc^2/Z)(\alpha Z)^6$ . Thus such a term, whose magnitude calculated for the  $K$  shell is 0.0178  $(mc^2/Z)(\alpha Z)^6$ , was added to the initially calculated value of  $W$ .<sup>8</sup>

Professor Birge kindly supplied us with his

<sup>8</sup> Another effect that alters the splitting of the  $2p$  levels is the modification of a Coulomb field at distances from the nucleus of order  $\hbar/mc$ , due to virtual creation of positrons (polarization of the vacuum). This correction to the energy difference is approximately  $(1/160\pi)(mc^2/Z)(\alpha Z)^7$ , and is too small to alter appreciably our calculated value of  $\alpha$ . The interaction between atomic electrons is not affected by negative energy states to this order of perturbation ( $1/Z$  or  $\alpha$ ).

values of  $\Delta\nu/R$  compiled from experiments on the doublets from many different x-ray lines, and weighted according both to the number of lines entering into the determination of each value of  $\Delta\nu/R$ , and to the *a posteriori* consistency of each line used. Values of  $\Delta\nu/R$ , and of the corresponding weights, are compiled in Table III for the range of atomic number used (26 elements, running from  $Z=60$  to  $Z=92$ ). These values have been converted from the conventional Siegbahn scale, on which all experimental results are published, to the presumably true values of the so-called grating scale of wave-lengths, by the use of the factor  $\lambda_g/\lambda_s=1.002034$ .

Finally, the values of  $\Delta(1/\alpha)$  and of  $B$  were found from these data by a least-square calculation, based directly on Eq. (8). The values obtained are  $\Delta(1/\alpha)=-0.072$ ,  $B=4.858\times 10^{-4}$ . The statistical probable error in the determina-

tion of  $\Delta(1/\alpha)$  is 0.034. These results give  $1/\alpha=136.928\pm 0.034$ .

The determination of the  $f(\alpha Z)$ , the correction of order  $1/Z$  to the spin splitting, is incomplete due to our neglect of outer shells. We have estimated that this incompleteness might falsify the magnitude of  $f$  by not more than one percent, which would produce a change in  $1/\alpha$  of 0.081. Adding three times the statistical probable error to this gives a figure of 0.18 for the limit of error on  $1/\alpha$ . Hence we conclude that the value of  $1/\alpha$  is 136.93, correct to within 0.18.

We gratefully acknowledge our indebtedness to Professor J. R. Oppenheimer for suggesting this problem, and for pointing out the method of calculation. We also wish to express our gratitude to the National Youth Administration for furnishing assistance in the numerical calculations.

## The Mean Life of Neutrons in Water and the Hydrogen Capture Cross Section

J. H. MANLEY, L. J. HAWORTH, AND E. A. LUEBKE  
*Department of Physics, University of Illinois, Urbana, Illinois*  
 (Received September 22, 1941)

Direct observations of the time variation of slow neutron density in a large volume of water during and after irradiation by neutrons from the D-D reaction have been made. An exponential growth and decay is found. From an analysis of these data and data on the spatial density distribution which determines the effect of diffusion, the mean life of neutrons in water was found to be  $205\pm 10$  microseconds. This leads to a cross section of  $0.33\times 10^{-24}$  cm<sup>2</sup> for the capture of slow neutrons by hydrogen.

**C**ONTROL of the ion beam responsible for neutron production from an artificial neutron source makes it possible to observe transient neutron phenomena. One such phenomenon is the time rate of change of neutron density in hydrogenous material. In water, for example, this depends on the probability of neutron escape from the surfaces, the loss by capture and the distribution of sources. For pure water, in which the capture is chiefly due to hydrogen nuclei, suitable measurements will lead to the capture probability and hence to the hydrogen capture cross section.

Baker and Bacher<sup>1</sup> have observed the mean

<sup>1</sup>C. P. Baker and R. F. Bacher, Phys. Rev. **59**, 332 (1941).

life of neutrons emerging from a paraffin block after the source has been removed. The effect of diffusion, however, complicates considerably the proper interpretation of such data. The analysis is more straightforward if the time variation of neutron density is obtained in a region where the effect of diffusion can be evaluated.

Previous results on the mean life of neutrons in paraffin and water vary from 170 to 270 microseconds<sup>2,3</sup> although the hydrogen density, which is the important factor, varies by only about 30 percent. Such a fundamental quantity as the cross section for the  $n+p\rightarrow d$  reaction

<sup>2</sup>E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936).

<sup>3</sup>O. R. Frisch, H. v. Halban, and J. Koch, Proc. Danske Vidensk. Selskab **15**, 10 (1938).