Higher-dimensional black-hole solution with dilaton field

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Using dimensional reduction by isometry from a higher-dimensional Einstein theory, the higher-dimensional black-hole solutions are considered. We obtain the spherically symmetric black-hole solutions in $(4+n)$-dimensional spacetime with the dilaton field. It is shown that the contribution of the dilaton field affects the gravitational coupling.

I. INTRODUCTION

As the higher-dimensional unification[1,2] of gravitation with other interactions became interesting, many authors began to study higher-dimensional theory in an effort to find the generality and the universality of the theories of physics.

In a local direct-product basis the $(4+n)$-dimensional unified metric g_{AB} may be written as

$$g_{AB}=\begin{bmatrix} g_{\mu\nu}+e^2\kappa^2\phi_{ab}B^a{}_\mu B^b{}_\nu & e\kappa\phi_{ab}B^b{}_\nu \\ e\kappa B^a{}_\mu\phi_{ab} & \phi_{ab}\end{bmatrix}, \qquad (1.1)$$

where e is the coupling constant, κ is a scale parameter of the isometry group, and $B^a{}_\mu$ is the gauge potential of the isometry. Here $g_{\mu\nu}$ $(\mu,\nu=0,1,2,3)$ is the four-dimensional metric and ϕ_{ab} $(a,b=1,2,\ldots,n)$ is the internal metric. Many solutions have been found which are static spherically symmetric in higher-dimensional spacetime. These solutions are classified by various viewpoints: (1) definitions of the radial coordinate, e.g., three-dimensional or general-dimensional radial coordinate; (2) dimensions of spacetime, e.g., five or higher than five; (3) topologies of the internal space, e.g., flat, sphere, torus, or other geometry; (4) parameters of the solutions; (5) the types of action, e.g., Einstein-Hilbert action, or additional actions; (6) conformal transformation of the base spacetime metric, etc.

There are simple generalizations of the usual Schwarzschild, Reissner-Nordström, and Kerr solutions with[3,4] or without the cosmological constant.[5] These higher-dimensional solutions are established on the higher-dimensional, general sphere. Thus the radial coordinate r in these solutions is treated as the general one: $r^2=\sum_{A=1}^{n+3}(x^A)^2$, where n denotes the extra dimension added to four-dimensional spacetime. As a special case of the solution of Myers and Perry,[4] Chakrabarti[6] found the eight-dimensional Kerr-type solutions with octonion algebra.

Static spherically symmetric or axisymmetric solutions of the five-dimensional vacuum Einstein equations, which have the form $\mathcal{M}^4\times S^1$ asymptotically, have been discussed as black-hole solutions[7-11] and monopole- or instanton-type solutions.[12-14] All static, spherically symmetric solutions of the five-dimensional vacuum Einstein equations have been found and classified by Chodos and Detweiler.[8]

In the aspects of internal space topology, some authors have considered the case of Ricci-flat internal space. In particular, Dereli[15] and Yoshimura[16] have classified all solutions for which internal space is Ricci flat while the base space is a two-sphere. Dobiasch and Maison[7] considered a more general case, in which the internal space is flat and the external base space represents a two-sphere, but the metric is supplemented by additional nondiagonal terms corresponding to Abelian gauge fields. Myers[17] found Majumdar-Papapetrou-type solutions of the $(4+n)$-dimensional Einstein-Maxwell solutions for the case of the torus internal space. Van Baal and coworkers[18] studied the case in which the internal space is a seven-sphere, in the context of $4+n=11$ supergravity.

In the viewpoint of a parameter for charge Chodos and Detweiler[8] derived a class of five-dimensional solutions, which contains three parameters but only two of these are independent. The solutions of Gibbons and Wiltshire[19] are more general than those of Chodos and Detweiler, in that the metric contains an extra nondiagonal term, which brings about the third independent parameter: magnetic charge. Dobiasch and Maison[7] considered an internal space with the Abelian isometry group in a space of more than five dimensions, which gives the corresponding parameters. There also had been the non-Abelian gauge field case in general[2] or special dimensions.[20] Sokolowski and Carr[21] had shown that there are no neutral black-hole solutions with a nontrivial scalar coupling between four-dimensional spacetime and the n-dimensional internal space, except in the charged black-hole cases.

There can be solutions for actions to which are added the terms containing higher powers of the Riemann curvature to the $(4+n)$-dimensional action. Since such higher-power terms allow the possibility of spontaneous compactification[22] they also give the regular dimensionally reduced black-hole solutions. Wiltshire[23] had the solutions on an $(n+2)$-sphere, which is the generalized form of Myers and Perry.[4] Some work has been done by Tomimatsu[24] and Myers and co-workers,[25] especially to see the effect in string theory. They obtained the Schwarzschild solution and then treated terms arising from the higher-order curvature corrections as perturbations.

In any higher-dimensional unification a central issue is how to make the dimensional reduction. So far a widespread method has been the use of the "zero-mode approximation"[26] of the harmonic expansion, after spontaneous compactification[27] of internal space. Unfortunately the method has serious defects in many aspects.[28,29] Thus we adopt the method of dimensional reduction by isometry.[2,30] Since internal space need not be compact in this reduction, the internal curvature can assume a negative as well as a positive value. The important point of our dimensional reduction is the way we identify the physical spacetime metric. Now the point is that $g_{\mu\nu}$ may not be viewed as the spacetime metric.[31] To remove the defect one has to make the conformal transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \sqrt{\phi}\, g_{\mu\nu} \tag{1.2}$$

and must identify the new metric $\tilde{g}_{\mu\nu}$ as the physical one.[31] Here the new physical metric is assumed to have the spherically symmetric form $\tilde{g}_{\mu\nu} = \mathrm{diag}(-B(r), A(r), r^2, r^2\sin^2\theta)$. There also have been solutions with a dilaton or conformally transformed metric. Pollard[9] used such a type of metric, showing the antigravity effect in five-dimensional spacetime. Mazur and Bombelli[11] had stationary rotating axisymmetric five-dimensional solutions with a dilaton field and showed that the rotating Kaluza-Klein black hole is described uniquely by the trivial embedding of the Kerr metric in five-dimensional spacetime. Gross and Perry[13] and Yoshimura[16] also obtained solutions in five or higher dimensions with a conformally transformed metric. But they did not treat or find the roles[32] of the dilaton field, such as renormalization at a classical level or the antigravity effect with a variable gravitational constant.

The purpose of this paper is to represent a higher-dimensional generalization of the black-hole solution, to understand the dynamical roles of the dilaton,[31] and to compare their characters with other black-hole solutions. Here we investigate the higher-dimensional solution by dimensional reduction onto the three-sphere, not the general sphere, to see the roles of the dilaton field clearly. For convenience $\phi_{ab} = \phi^{1/n}(r)\rho_{ab}$, $\rho_{ab} = \delta_{ab}$, and $B^a{}_\mu = 0$ are assumed in this paper. That is, the internal space is Ricci flat with a dilaton field not considering the gauge field. Black-hole solutions are given in Sec. II without the cosmological constant Λ and in Sec. III with Λ. Concluding remarks are in Sec. IV.

II. BLACK-HOLE SOLUTION

Instead of solving the field equation for $\tilde{g}_{\mu\nu}$, we start from the $(4+n)$-dimensional Einstein field equation by the old metric $g_{\mu\nu}$:

$$R_{AB} - \tfrac{1}{2}(R_{4+n} + \Lambda) g_{AB} = -8\pi G T_{AB} \;. \tag{2.1}$$

To find the metric components B and A of $\tilde{g}_{\mu\nu}$ we substitute $g_{\mu\nu} = \tilde{g}_{\mu\nu}/\sqrt{\phi}$ into (2.1) and obtain the differential equations for B, A, and ϕ. The nonvanishing Christoffel symbols are

$$\begin{aligned}
&\Gamma^0_{01} = \frac{B'}{2B} - \frac{k}{2}\sigma', \quad \Gamma^1_{11} = \frac{A'}{2A} - \frac{k}{2}\sigma' , \\
&\Gamma^1_{22} = -\frac{r}{A} + \frac{r^2}{A}\frac{k}{2}\sigma', \quad \Gamma^1_{00} = \frac{B'}{2A} - \frac{B}{A}\frac{k}{2}\sigma' , \\
&\Gamma^1_{33} = \Gamma^1_{22}\sin^2\theta, \quad \Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r} - \frac{k}{2}\sigma' , \\
&\Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{32} = \cot\theta , \\
&\Gamma^1_{ab} = \frac{k}{An}\sigma' e^{k(2/n+1)\sigma}\delta_{ab}, \quad \Gamma^a_{b1} = \frac{k}{n}\sigma'\delta^a_b ,
\end{aligned} \tag{2.2}$$

where σ is the dilaton field defined by[31]

$$\sigma = \frac{1}{2k}\ln\phi = \frac{1}{2}\left[\frac{n+2}{n}\right]^{1/2}\ln\phi \tag{2.3}$$

and a prime denotes differentiation with respect to r. The parameter $k = \sqrt{n/(n+2)}$ shows the extent that the effective gravitational coupling constant[32] G_N is varied, as we can see later. Both δ_{ab} and δ^a_b in (2.2) play the role of the Kronecker delta regardless of the location of the indices. When $k=0$ and/or $\sigma=$const, (2.2) is reduced to the four-dimensional case. In the case where the internal metric ϕ_{ab} is equal to the Kronecker delta δ_{ab} purely, i.e., the vanishing of the dilaton field, it also approaches the four-dimensional problem.

Using (2.2) the nonvanishing Ricci tensor components are

$$\begin{aligned}
R_{00} = &-\frac{B''}{2A} + \frac{B'}{4A}\left[\frac{A'}{A} + \frac{B'}{B}\right] - \frac{B'}{rA} + \frac{B}{A}\frac{k}{2}\sigma'' \\
&+ \frac{B}{A}\left[\frac{B'}{B} - \frac{A'}{A} + \frac{4}{r}\right]\frac{k}{4}\sigma' ,
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
R_{11} = &+\frac{B''}{2B} - \frac{B'}{4B}\left[\frac{A'}{A} + \frac{B'}{B}\right] - \frac{A'}{rA} - \frac{k}{2}\sigma'' \\
&+ \left[\frac{A'}{A} - \frac{B'}{B} - \frac{4}{r}\right]\frac{k}{4}\sigma' + \tfrac{3}{2}k^2\sigma'^2 ,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
R_{22} = &\frac{1}{A} + \frac{r}{2A}\left[-\frac{A'}{A} + \frac{B'}{B}\right] - 1 - \frac{r^2}{A}\frac{k}{2}\sigma'' \\
&+ \frac{r^2}{A}\left[\frac{A'}{A} - \frac{B'}{B} - \frac{4}{r}\right]\frac{k}{4}\sigma' ,
\end{aligned} \tag{2.6}$$

$$R_{33} = R_{22}\sin^2\theta , \tag{2.7}$$

$$R_{ab} = -e^{k(2/n+1)\sigma}\frac{1}{nA}\left[\frac{A'}{A} - \frac{B'}{B} - \frac{4}{r} - \frac{2\sigma''}{\sigma'}\right]\frac{k}{2}\sigma'\delta_{ab} \;. \tag{2.8}$$

When the matterless case without the cosmological constant Λ is considered, the Einstein equation (2.1) becomes

$$R_{AB}=0 \ . \tag{2.9}$$

Then (2.8) gives the relation

$$\frac{\sigma''}{\sigma'}=\frac{A'}{2A}-\frac{B'}{2B}-\frac{2}{r} \ , \tag{2.10}$$

which means the wave equation $\Box\sigma=0$, since its internal manifold is Ricci flat. By integration (2.10) we obtain

$$\sigma' \propto \frac{1}{r^2}\sqrt{A/B} \ . \tag{2.11}$$

Using (2.10), the Einstein equation (2.9) for the Ricci tensor (2.4), (2.5), and (2.6), respectively, corresponds to

$$-\frac{B''}{2A}+\frac{B'}{4A}\left[\frac{A'}{A}+\frac{B'}{B}\right]-\frac{1}{r}\frac{B'}{A}=0 \ , \tag{2.12}$$

$$\frac{B''}{2B}-\frac{B'}{4B}\left[\frac{A'}{A}+\frac{B'}{B}\right]-\frac{1}{r}\frac{A'}{A}=-\tfrac{3}{2}k^2\sigma'^2 \ , \tag{2.13}$$

$$-1+\frac{1}{A}+\frac{r}{2A}\left[-\frac{A'}{A}+\frac{B'}{B}\right]=0 \ . \tag{2.14}$$

Both (2.12) and (2.14) are the same as those of the Schwarzschild case. But the right-hand side of (2.13) represents the energy-momentum tensor component by σ-field contribution to the matter field[33] while the contribution does not appear in the Schwarzschild case.

From (2.12) and (2.13),

$$\frac{A'}{A}+\frac{B'}{B}=\tfrac{3}{2}k^2\sigma'^2 r \ . \tag{2.15}$$

In the Schwarzschild or other higher-dimensional cases[4-6] the right-hand side of (2.15) is zero, which yields the relation $A=1/B$. But in this higher-dimensional model (2.15) does not become such a simple form since the three-sphere instead of the general sphere is the base space for dimensional reduction.

The component R_{00} in (2.4) can be recombined as

$$-\left[\frac{B'}{2A}-\frac{B}{A}\frac{k}{2}\sigma'\right]' +\left[\frac{B'}{2A}-\frac{B}{A}\frac{k}{2}\sigma'\right]\left[\frac{B'}{2B}-\frac{A'}{2A}-\frac{2}{r}\right]=0 \ . \tag{2.16}$$

With (2.11), (2.16) gives the form as

$$B\sim e^{\rho\sigma} \ , \tag{2.17}$$

where ρ is the integration constant. It is the similar form to the solution of Brans-Dicke theory[34] and those of modified Brans-Dicke theory with torsion field.[35] We will follow the methods taken in those references to solve our problem.

With the help of (2.10), (2.14) becomes

$$\frac{\sigma''}{\sigma'}=-\frac{A+1}{r} \ . \tag{2.18}$$

From (2.10), (2.15), (2.17), and (2.18),

$$A=1+\rho\sigma' r-\tfrac{3}{4}k^2\sigma'^2 r^2 \ . \tag{2.19}$$

When $\rho=0$, since A does not have the second term in (2.19),

$$\rho\sim\frac{4}{3k^2}\ln\left\{r^{-1}\left[\frac{3k^2}{4\alpha}\right]^{1/2}+\left[1+\left[\frac{3k^2}{4\alpha}\right]r^{-2}\right]^{1/2}\right\} \ ,$$
$$B=1,\quad A=\left[1+\frac{3k^2}{4}\left[\frac{GM}{r}\right]^2\right]^{-1} \ , \tag{2.20}$$

where α is the proper constant for (2.11). This solution does not approach the Schwarzschild solution asymptotically. Equations (2.18) and (2.19) give the differential equation for

$$y'r+y+\rho y^2-\tfrac{3}{4}k^2y^3=0 \ , \tag{2.21}$$

where $y=r\sigma'$. If one gets the solution to y from (2.21), then the r dependence of σ, B, and A will be found. Though apparently it seems that y=const or $\rho\sim\ln r$ satisfies (2.21), this gives rise to $A=0$, from (2.18), which is unrealistic. In the case of a very small y, we can set (2.21) into $y'r+y\approx 0$ and solve it to have $y\sim 1/r$ or

$$\sigma\sim\frac{a}{r} \ , \tag{2.22}$$

where a is a constant to be determined by the requirement that the solutions approach the Schwarzschild metric in the asymptotically flat region. However, constants a and ρ will be found by comparing the form of B in (2.17) with the effective Newton coupling $G_N=Ge^{-k\sigma}$ which shows how the mass gets changed by the dilaton.[32] From the weak-field approximation

$$G_N\approx-\frac{r}{2M}(B-1)=-\frac{r}{2M}\left[\frac{\rho a}{r}+\frac{\rho^2a^2}{2r^2}+\cdots\right] . \tag{2.23}$$

It is taken in the large-r limit. The effective coupling with setting $G=1$ will be

$$G_N=e^{-k\sigma}=1-\frac{ka}{r}+\cdots \ . \tag{2.24}$$

If these two representations for the effective coupling, (2.23) and (2.24), are identified up to $O(r^{-2})$, then

$$\rho=-2k \quad\text{and}\quad a=\frac{M}{r} \ . \tag{2.25}$$

The explicit form of the factor of effectiveness of the coupling becomes

$$e^{-k\sigma}=e^{-M/r} \ . \tag{2.26}$$

Near the origin the coupling is zero and it approaches unity asymptotically, which shows that the coupling is constant for large r. Using (2.26) we get

$$B=1-\frac{2M}{r}e^{-k\sigma}\approx 1-\frac{2M}{r}+\frac{2M^2}{r^2} \ . \tag{2.27}$$

The topology of the metric (2.27) is the same as the case of Reissner-Nordström solutions of $e^2 > M^2$. Since the σ field, here, plays the role of the energy-momentum tensor in (2.10) and (2.13) analogous to the electromagnetic field, it is natural that this metric has the form of the Reissner-Nordström solution by the dilaton field (2.22) instead of the charge. In this metric there is no event horizon but a naked singularity which accords with the other results. Recently Mignemi and Wiltshire[36] proved that the spherically symmetric solutions for the asymptotically constant $\phi^{1/n}$ and asymptotically flat space without the gauge field have a naked singularity, which was well known.[22] Pollard[9] and Angus[12] also obtained solutions which had the naked singularity. Since there is an event horizon for a four-dimensional Schwarzschild metric without a dilaton field σ, one can say that the σ field removes the horizon to show a naked singularity.

If the y^3 term only is neglected in (2.10), $y'r + y + \rho y^2 \approx 0$, then the dilaton has the form of

$$\sigma = \frac{1}{\rho}\ln\left|1-\frac{\alpha}{r}\right| + \text{const} , \tag{2.28}$$

where α is the proper constant. For the exact solution of (2.21) without any approximation, the algebraic equation for y is given by proper integration

$$\frac{y}{\sqrt{1+\rho y - 3k^2y^2/4}}\left|\frac{\rho - 3k^2y/2+\sqrt{\Delta}}{\rho - 3k^2y/2-\sqrt{\Delta}}\right|^{\rho/2\sqrt{\Delta}} = \frac{\text{const}}{r} , \tag{2.29}$$

where $\Delta = 3k^2+\rho^2 > 0$. Therefore B and A will be obtained from the solution to y.

III. SOLUTIONS WITH Λ

When the cosmological constant Λ exists, the right-hand side of Einstein's field equation (2.9) is not zero, but it contains the Λ term:

$$R_{AB} = -\frac{g_{AB}}{2+n}\Lambda . \tag{3.1}$$

This equation can be rewritten by its components separately:

$$-\frac{B''}{2A}+\frac{B'}{4A}\left[\frac{A'}{A}+\frac{B'}{B}\right]-\frac{B'}{rA} = \tfrac{1}{2}\Lambda B e^{-k\sigma} , \tag{3.2}$$

$$\frac{B''}{2B}+\frac{B'}{4B}\left[\frac{A'}{A}+\frac{B'}{B}\right]-\frac{A'}{rA} = -\tfrac{1}{2}\Lambda A e^{-k\sigma} - \tfrac{3}{2}k^2\sigma'^2 , \tag{3.3}$$

$$\frac{1}{A}+\frac{r}{2A}\left[-\frac{A'}{A}+\frac{B'}{B}\right]-1 = -\tfrac{1}{2}\Lambda r^2 e^{-k\sigma} , \tag{3.4}$$

$$\frac{B'}{2B}-\frac{A'}{2A}+\frac{2}{r}+\frac{\sigma''}{\sigma'} = -k\Lambda\frac{A}{\sigma'}e^{-k\sigma} . \tag{3.5}$$

The Λ term can be a source term for the dilaton field σ, and it makes the inhomogeneous wave equation as (3.5) instead of (2.10). Thus the Λ term plays the role of energy-momentum added to the effect of the σ field. However, in Eqs. (3.2) and (3.3) the Λ terms cancel each other, and they give the same relationship (2.15) which is derived from (2.12) and (2.13) without the Λ terms.

Since it is very hard to solve the equation, the ansatz such as adopted in Ref. 33,

$$r = \beta e^{\gamma\sigma} \quad (\beta,\gamma = \text{const}) \tag{3.6}$$

will determine the r dependence of σ, B, and A (Ref. 37). With this ansatz (2.15) becomes $AB = r^{2\delta}(\delta = 3k^2/4\gamma^2)$. If we substitute this into (3.4),

$$\frac{d}{dr}(r^{1-\delta}B) = (1-\tfrac{1}{2}\Lambda r^2 e^{-k\sigma})r^{\delta} . \tag{3.7}$$

By integrating (3.7) with (3.6), σ, B, and A will become

$$\sigma = \frac{1}{\gamma}\ln\frac{r}{\beta} , \tag{3.8}$$

$$B = \frac{r^{2\delta}}{1+\delta}\left[1-\frac{2M}{r}r^{-\delta+k/\gamma}e^{-k\sigma} - \frac{1+\delta}{6-2k/\gamma}\Lambda r^2 e^{-k\sigma}\right] , \tag{3.9}$$

$$A = (1+\delta)\left[1-\frac{2M}{r}r^{-\delta+k/\gamma}e^{-k\sigma} - \frac{1+\delta}{6-2k/\gamma}\Lambda r^2 e^{-k\sigma}\right]^{-1} . \tag{3.10}$$

Because these metric components approach the Schwarzschild–de Sitter metric when $k\to 0$ or $\delta\to 0$, $2M$ is given as the constant of the second term in (3.9) and (3.10). The factor $e^{-k\sigma}$ in the metric plays the role of effective gravitational coupling except the second term which has another factor of $r^{-\delta+k/\gamma}$. Of course when $\delta = k/\gamma$, i.e., $\gamma = 3k/4$, the other factor of the second term becomes one not affecting the coupling constant, and then the effective coupling would be purely $e^{-k\sigma}$. One may leave k a parameter of the theory which has yet to be specified. Since σ increases logarithmically with r for positive γ from $-\infty$ to $+\infty$, the effective coupling grows weaker as the distance becomes farther. When the constant of the second term is chosen as zero, the metric will be a de Sitter solution with the effective gauge coupling $e^{-k\sigma}$ and the parameter can be determined from (3.2) and the equation for σ.

It is very hard to find the event horizon for the metric (3.9). Since (3.9) approaches the Schwarzschild–de Sitter metric as $k\to 0$, event horizons near $k/\gamma\to 0$ can be found. There are two horizons analogous to the Schwarzschild–de Sitter metric for small values of M and Λ: the event horizon $r_H \approx (2M)^{1/(1+\delta)}$ and the cosmological horizon $r_C \approx (1/\hat{\Lambda})^{1/(2-k/\gamma)}$, where $\hat{\Lambda} = \Lambda(1+\delta)/(6-2k/\gamma+2\delta)$. The horizons are calculated in the limits of $k, M, \Lambda \to 0$. But for the case of large k/γ there exist no horizons.

IV. CONCLUSIONS

We obtained the static spherically symmetric solution for a higher-dimensional metric through dimensional reduction into a three-sphere by isometry. The solution was determined from the Einstein equation derived by a new conformally transformed metric. Without the cosmological constant the solutions were similar to those of Brans-Dicke theory up to a constant since the dilaton field in our model played the role of the scalar field of Brans-Dicke theory. Thus the effective gravitational coupling realized Dirac's conjecture[38] naturally on the same footing of Brans-Dicke theory. With the ansatz (3.6) the solutions containing the cosmological constant were calculated. The effect of the dilaton field on the space-time topology was shown by approaching the Schwarzschild–de Sitter metric. Thus any other effects, such as the quantum radiation patterns for the outgoing and incoming waves near the horizons, would be investigated by changing the horizons on the analogy of Schwarzschild–de Sitter ones.[39]

As a further discussion our model and the corresponding problem can be extended into the case of $\rho_{ab} \neq \delta_{ab}$ and/or $B^a{}_\mu \neq 0$ in internal space. Then the dynamical roles of the gauge fields near the black hole and the effective gauge couplings which lead up to a classical renormalization[32] would be understood more clearly.

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