Testing the Inverse-Square Law of Gravity on a 465-m Tower

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We have performed a test of Newton's universal theory of gravitation. We measured gravity at 12 heights on a 465-m tower at the Nevada Test Site and, in addition, made measurements at 281 locations on the ground. The surface points fell within 91 optimally chosen sectors that extended out to 2.6 km from the tower. These data were combined with 60000 additional surface gravity measurements within 300 km of the tower. We used a surface integral derived from Laplace's equation to continue the surface gravity field upward and our observations are consistent with the Newtonian predictions to within $(-60\pm95)\times10^{-8}$ m sec^{-2} at the top of the tower.

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Modern experimental tests of Newtonian gravity are motivated by the observation that the theory has been tested at short and long ranges but not at intermediate distances.[1] Furthermore, quantum supergravity theories naturally include vector and scalar particles that may couple to matter with gravitational strength[2] and generate intermediate-range forces.

Several experiments have shown anomalous results.[3-9] Stacey and co-workers[1,3,4] have suggested that the anomalous gravity gradients observed in mines and boreholes are the result of a short-range Yukawa interaction. But recent borehole experiments do not agree on the sign or magnitude of the interaction,[5-8] suggesting that the effects of subsurface geology dominate non-Newtonian effects for these measurements.[5-7]

A more rigorous test that accounts for variations in subsurface geology is done by measuring gravity on the surface of the earth, and then using Laplace's equation to predict the gravitational field near the tower. Eckhardt *et al.*[9] have used this idea in their work on the WTVD television tower in North Carolina. They measured gravity at 6 heights up to 562 m on the tower and at 77 points within 5 km of the tower. At the top of the tower, they found a discrepancy of $(-500\pm35)\times10^{-8}$ m sec^{-2} between the measurements and the predictions, which they present as evidence for an attractive non-Newtonian force.

In this paper we report a test of Newtonian theory performed in Nevada on a stable tower with a carefully designed, evenly spaced, surface data set. To understand the principle of the test, assume that the tower rests on a spherical Earth on the axis of a geocentric spherical coordinate system, and define $g(\rho,\theta,\phi)$ to be the radial component of gravity. Newtonian theory implies that the product $g\rho$ is a solution of Laplace's equation outside the Earth, and can be determined from Poisson's integral,[10]

$$g(\rho,0,0)=\frac{a^2(\rho^2-a^2)}{4\pi\rho}\int g(a,\theta,\phi)\frac{d\Omega}{[\rho^2+a^2-2a\rho\cos(\theta)]^{3/2}}, \tag{1}$$

where $d\Omega$ is an element of solid angle and a is the radius of the Earth. If z is the elevation of the observation point on the tower (i.e., $z\equiv\rho-a$) and r is the distance along the surface to a measurement point (i.e., $r\equiv a\theta$), then we can expand Eq. (1) in powers of z/a, keeping leading terms, and obtain the following equation:

$$g(z,0,0)=\frac{z(1-z/a)^2}{2\pi}\int_0^\infty\int_0^{2\pi}g(0,r,\phi)\frac{r\,dr\,d\phi}{[r^2+z^2(1-z/a)]^{3/2}}. \tag{2}$$

Unfortunately, gravity measurements do not obey Eq. (2) because they are influenced by the Earth's (nonharmonic) rotation. So, we subtract an accurate model of the rotational acceleration plus a model of the Earth's gravity field from all the data. Thus we should use $\Delta g=g_{\text{observed}}-g_{\text{model}}$ in place of g on both sides of Eq. (2). A typical gravity model is the World Geodetic System model, 1984 (WGS84),[11] with free-air-gradient corrections.[12]

Our experiment took place at the 465-m Bare Reactor Experiment–Nevada (BREN) tower on Jackass Flats, Nevada. The tower was built in 1962 to support a reactor that has since been removed, leaving a stable platform for making gravity measurements. There is no electrical power on the tower. All measurements on the tower were made from July to October. At this time of year, the winds change direction twice a day due to convective heating. Therefore, we made our measurements at sunrise, when the wind velocities were below 5 km/h.

The BREN tower is located on Jackass Flats, a smooth desert plane which tilts to the west at an angle of 1.5°. There are no trees near the tower; so, there is a direct line of sight out to a distance of 5 km. This clear line of site allowed us to collect a statistically unbiased sample of the gravity field near the tower by laying out a radially symmetric pattern of surface points that is not correlated with the terrain or underlying geology. We collected surface gravity data along 9 rays at radii of 32.8, 67, and 131 m, and along 12 rays at 343, 564, 904, 1495, and 2566 m. 190 additional points were measured, on a similar but denser radial pattern, within 132 m of the tower, making a total of 281 points. Typically, we were able to place the gravimeter to within a meter of the predetermined position, and then measure its true position and elevation relative to the tower with a range-finding theodolite to ± 1 cm near the tower and to ± 10 cm at 2.5 km. In the vicinity of the tower, the pattern of surface gravity stations was designed to minimize the errors in the upward continuation of the gravity field.[13]

We collected surface gravity data in a series of 95 loops with a total of 632 measurements including repeats. The data were corrected for tides and drift in the conventional way. We used two LaCoste Romberg gravimeters, a Model D and a Model G. All measurements were referenced to the base of the tower and, typically, relative gravity is known at each station to $\pm 15\times 10^{-8}$ $\mathrm{m\,sec^{-2}}$. Two absolute gravity stations near the tower[14]—one only 6 km away—were used to establish a secondary gravity station on the tower base. Hence, absolute gravity, needed to combine our measurements with other data, is known at our surface stations to $\pm 30\times 10^{-8}$ $\mathrm{m\,sec^{-2}}$.

The absolute location of the tower was determined from ten Nevada Test Site (NTS) benchmarks that were located within 3 km of its base. Thus, we were able to combine our data with an extensive data set beyond the 2.6-km boundary of our survey. We used measurements from gravity data bases maintained by the U.S. Geological Survey[15,16] and the National Oceanic and Atmospheric Administration.[17] These data bases contain gravity measured typically every 0.5 $\mathrm{km^2}$ at NTS,[16] and approximately every 6 $\mathrm{km^2}$ in the western U.S. We used 60000 of these gravity measurements out to a truncation distance of 300 km from the tower.

The tower measurements were done with two Model-D LaCoste Romberg gravimeters. The calibration of one was checked by two independent methods. It was rebuilt and calibrated by the manufacturer before this experiment,[18] and we verified it on the Charleston loop, a gravimeter calibration loop near NTS.[19] Four stations on the loop cover the same 140×10^{-5} $\mathrm{m\,sec^{-2}}$ range measured on the tower. Our calibration and the manufacturer's agreed to within 15×10^{-8} $\mathrm{m\,sec^{-2}}$ over that range. Measurements with the second gravimeter, a standard instrument from the manufacturer's rental pool, agreed with those of the recently calibrated instrument.

The tower measurements were collected in a series of 11 loops for a total of 42 observations. A typical loop was 1.8 h long, occupied 4 elevations, and had a meter drift of about 15×10^{-8} $\mathrm{m\,sec^{-2}}$. The drift was removed from the data as if it occurred linearly in time. Repeated measurements from different loops were used to verify

TABLE I. Observations and predictions on the BREN tower. In all cases, gravity values shown are the differences between each station on the tower and the station at 9.76 m. The estimated uncertainties for the observations are all 20×10^{-8} $\mathrm{m\,sec^{-2}}$. The global model WGS84 and a free-air correction have been subtracted from the columns labeled Δg. The predicted values were derived by continuing upward free-air anomalies, using Eq. (2). The quoted error includes random and systematic errors.

Elevation above base (m)	Observed g (10^{-5} $\mathrm{m\,sec^{-2}}$)	Observed Δg (10^{-5} $\mathrm{m\,sec^{-2}}$)	Predicted Δg (10^{-5} $\mathrm{m\,sec^{-2}}$)	Difference (10^{-5} $\mathrm{m\,sec^{-2}}$)
9.76	0.000	0.000	0.000	0.000
42.42	−9.920	0.165	0.161	0.004 ± 0.040
85.09	−22.843	0.412	0.422	−0.010 ± 0.046
127.74	−35.750	0.671	0.699	−0.028 ± 0.051
170.40	−48.642	0.949	0.976	−0.027 ± 0.057
213.05	−61.543	1.217	1.252	−0.035 ± 0.063
255.71	−74.462	1.468	1.524	−0.056 ± 0.068
298.38	−87.367	1.739	1.793	−0.054 ± 0.073
341.04	−100.267	2.010	2.059	−0.049 ± 0.080
383.71	−113.186	2.265	2.323	−0.058 ± 0.085
426.38	−126.103	2.525	2.584	−0.059 ± 0.090
454.86	−134.727	2.695	2.755	−0.060 ± 0.095

the assumption of linear drift. To account for possible thermal expansion of the tower, we used an electronic distance meter to determine the elevation of the gravimeter to within 1 cm at the time of each measurement. We also measured the tilt of the tower, and found it to be vertical to within 20 sec of arc, eliminating the need to correct the elevation measurements. The gravimeters yielded consistent results summarized in Table I.

The predicted gravity values on the tower were calculated using a weighted sum of the surface gravity values where the weights were derived by discretizing Eq. (2).[13] The space around the tower was divided into several sectors of annuli to match the ground survey. We then averaged the gravity anomaly measurements, Δg, in each sector to get an estimate of the mean for the whole sector and applied the appropriate weights. We also estimated the variation of the mean in each sector by calculating the standard deviation of all measurements around the ring that includes the sector (after removing a linear regional trend in the data).

The fluctuations in the gravity field give an uncertainty in the prediction at the top of the tower of 80×10^{-8} m sec^{-2}. The effect of truncating the integral in Eq. (2) at a 300-km radius contributes 30×10^{-8} m sec^{-2}, and there is a 30×10^{-8} m sec^{-2} uncertainty caused by the arbitrary phase of the location of the sectors on the ground.

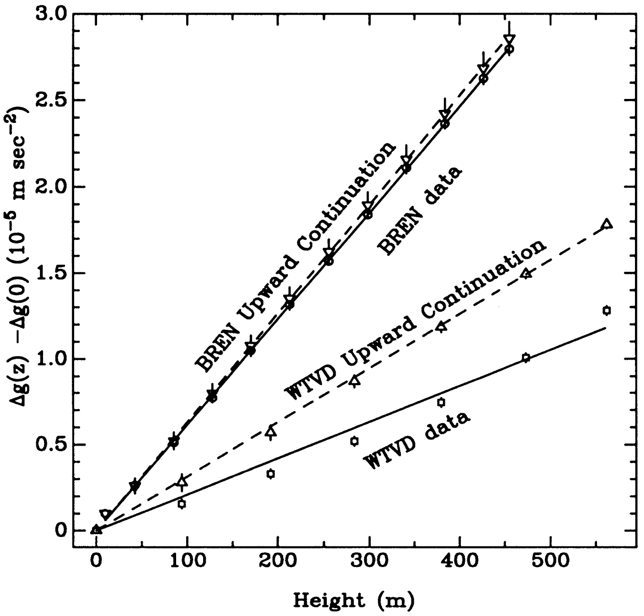

FIG. 1. The anomalous gravity observed on the BREN tower (solid line) is in very good agreement with the predictions based on the Newtonian theory of gravity (dashed line). The error bars for the BREN data include random and systematic errors added in quadrature. The data and predictions on the WTVD tower (Ref. 21) are shown for comparison. In all cases, the plotted lines are straight-line fits to the points.

A systematic bias in the upward continuation is introduced by assuming that the measured gravity anomalies represent the values on a smooth sphere. But there are nearby hills and valleys that represent a departure from this model. We estimated the bias by creating a model data set using digitized topographic maps of the area surrounding the tower.[20] We calculated gravity at the map elevations and on the tower assuming that the material under the surface had a constant density of 2.2 g/cm^3. We than continued the model data upward using Eq. (2), the spherical Earth approximation. The model tower data were higher than the upward continuation by $(150\pm30)\times10^{-8}$ m sec^{-2} at the top of the tower, and the discrepancy decreased linearly at lower heights. We applied these bias estimates as corrections to the upward continuation of the real data to produce the predictions shown in Table I.

The measured Δg values are compared to the upward continuation in Fig. 1. We observe that the tower data are linear (total $\chi^2=1.7$ for 10 degrees of freedom), with a gradient that is significantly lower (-0.3026×10^{-5} sec^{-2}) than the predictions of the globally symmetric model of the Earth (-0.3086×10^{-5} sec^{-2}). Interpreted by themselves, as are the data in borehole experiments,[3-8] these measurements would incorrectly suggest that a new, strongly repulsive force exists with a scale length greater than 450 m. However, the field predicted from upward continuation is in good agreement with the tower data and the largest discrepancy is only

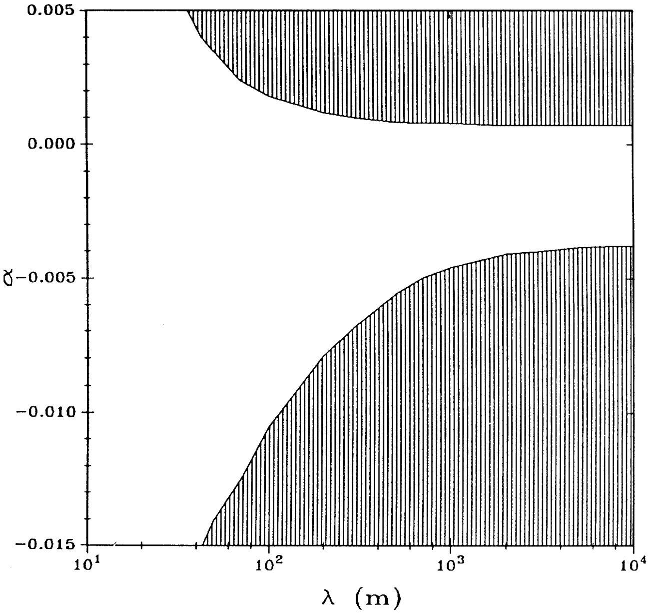

FIG. 2. The strength (α) and range (λ) of a hypothetical Yukawa interaction are highly constrained by the BREN tower data. The allowed, nonhatched region, is obtained by requiring the curve generated by Eq. (3) to lie within the envelope created by the error bars on $\Delta g(z,0,0)_{\text{observed}}-\Delta g(z,0,0)_{\text{predicted}}$.

$(-60\pm95)\times10^{-8}$ m sec^{-2} at 454 m above the ground.

A weak short-range Yukawa interaction is not excluded by our data but the possible strength of such an interaction is highly constrained. Figure 2 illustrates the allowed range of strengths and ranges for a fit by

$$\Delta g(z,0,0)_{\text{observed}}-\Delta g(z,0,0)_{\text{predicted}}=2\pi G\rho\alpha\lambda(1-e^{-z/\lambda}). \quad (3)$$

In conclusion, our observations of gravity on the BREN tower are in good agreement with the Newtonian theory of gravity at the one-standard-deviation level. We observe a residual discrepancy of 60 ± 95 parts per billion of the total field at the top of the tower. The residual is comparable in magnitude to the gravitational attraction of the atmosphere (which has been included in our analysis) and is much smaller than the effects previously reported.[9]

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