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Comments on $\mathcal{N} = 2$ *AdS* Orbifolds

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Abstract

We discuss twisted states of AdS orbifolds which couple to $\mathcal{N} = 2$ chiral primary operators *not* invariant under exchange of the gauge factors. Kaluza-Klein reduction on the fixed circle gives the correct conformal dimensions of operators in the superconformal theory and involves some aspects of monopole dynamics in the non-trivial background. As a byproduct we found evidence for decoupling of $U(1)$ factors in the four-dimensional gauge theory.

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1. Introduction

Since the remarkable conjecture of Maldacena [1] a lot of progress has been made towards understanding the dynamics of superconformal field theories (SCFT) in four dimensions. According to [1], $\mathcal{N} = 4$ SCFT is dual to type IIB string theory compactified on anti-de Sitter (AdS) space of the form $AdS_5 \times S^5$. To make this relation more precise one has to compare the states in both theories and their interactions [2,3]. Correlation functions of the fields in the four-dimensional theory on the boundary can be evaluated via the asymptotic dependence of the supergravity action.

The conjecture was extended to the theories with lower number of supersymmetries by means of the orbifold construction [4,5]. The idea is to place N three-branes at the orbifold point of $\mathbb{R}^4 \times \mathbb{R}^6/\Gamma$ where the discrete subgroup $\Gamma \subset SO(6)$ leaves the brane world-volume intact [6]. Then the near-horizon geometry looks like $AdS_5 \times S^5/\Gamma$, so that the isometry group $SO(4,2)$ of the AdS_5 space still corresponds to the conformal symmetry of the SCFT, while the isometry of the S^5/Γ becomes the R-symmetry group. The field content and the interactions are nicely encoded in the corresponding quiver diagram [6,5]. The gauge group is defined by irreducible representations of Γ ²:

$$G = \prod_i \otimes SU(n_i N) \tag{1.1}$$

where the product is over all the representations of dimension n_i . Each arrow in the quiver diagram from the node i to the node j gives rise to the bifundamental matter $(n_i N, \overline{n_j N})$.

In this paper we focus on $\mathcal{N} = 2$ superconformal theories that correspond to ADE-type subgroups $\Gamma \subset SU(2)$. Although the discussion is very general, it is convenient to think of the particular example of the $A_{n-1} = \mathbb{Z}_n$ orbifold. The low-energy theory on the world-volume of N three-branes is then a $SU(N)^n$ gauge theory where \mathbb{Z}_n acts as a permutation of the gauge factors. The conformal dimensions of the relevant and marginal operators in this $\mathcal{N} = 2$ superconformal field theory were studied in [7]. The authors of [7] considered only \mathbb{Z}_n -symmetric operators. From the string theory point of view these operators correspond to the Γ -invariant, i.e. untwisted, states [8]. However, because the orbifold action is not free, there are also twisted sectors not invariant under \mathbb{Z}_n permutation. Following the AdS/SCFT correspondence, these states couple to the

² The question of whether the diagonal $U(1)$ s decouple or not will be addressed later.

operators not invariant under exchange of the gauge factors. It is convenient to choose the following basis of such operators:

$$\mathcal{O}(i) - \mathcal{O}(i + 1) \tag{1.2}$$

where \mathcal{O} stands for a certain combination of chiral fields. Some marginal deformations of the form (1.2) have clear physical interpretation corresponding to differences between coupling constants of the i -th and $(i + 1)$ -th gauge factors [4].

In the next section we work out the chiral primary operators in $\mathcal{N} = 2$ SCFT that are not invariant under exchange of the gauge factors and calculate their conformal dimensions. Section 3 is devoted to the string theory analysis of twisted states and comparison to the gauge-theoretic results. A similar question was recently posed in the investigation of Brane Box Models [9]. When the present paper was completed, we received the preprint [10] where the arguments analogous to our section 3.2 were applied to fields localized on seven-branes in F-theory.

2. $\mathcal{N} = 2$ Superconformal Gauge Theories

We start this section by enumerating the chiral fields in the $\mathcal{N} = 2$ field theory with the gauge group (1.1) which can be used as building blocks for construction of the chiral primary operators. Each node of the corresponding Dynkin diagram contributes one $\mathcal{N} = 2$ vector multiplet, i.e. in terms of $\mathcal{N} = 1, 0$ chiral fields:

$$\Phi_i = \begin{pmatrix} \psi_i \\ a_i \end{pmatrix} \quad W_i = \begin{pmatrix} A_i \\ \lambda_i \end{pmatrix} \tag{2.1}$$

To simplify the notations we suppress space-time indices. In the specific case of \mathbb{Z}_n orbifold there are also n bifundamental matter hypermultiplets that we write as (Q_i, \tilde{Q}_i) in $\mathcal{N} = 1$ notations. The global symmetry of the $\mathcal{N} = 2$ gauge theory is $SU(2)_R \times U(1)_R$.

Now using these fields we construct all the possible chiral operators of the form (1.2) whose dimensions are protected by supersymmetry. Such operators come in short multiplets, so that their lowest components have scaling dimensions determined by the R-charges [11]:

$$\Delta = \left\lfloor \frac{R}{2} \right\rfloor + d - 1 \tag{2.2}$$

where d is the dimension of $SU(2)_R$ representation.

It is sufficient to consider only bosonic members of the multiplets, since the fermionic superpartners follow by supersymmetry³. Operators (1.2) which involve fields Q_i and \tilde{Q}_i , are not primary because they contain derivatives of the superpotential⁴:

$$\sum_i \int d^2\theta [\text{Tr}(\tilde{Q}_i \Phi_i Q_i) - \text{Tr}(Q_{i+1} \Phi_i \tilde{Q}_{i+1})] + c.c. \quad (2.3)$$

Using the building blocks of the form $\mathcal{O} = \text{Tr}WW\Phi^k$, $\mathcal{O} = \text{Tr}\Phi^k$ and $\mathcal{O} = \text{Tr}W\Phi^k$ we come to four families of the "twisted" bosonic operators:

1) The family of states whose lowest representative is difference of the gluino bilinears:

$$\text{Tr}(\lambda_i \lambda_i a_i^{k-1}) - \text{Tr}(\lambda_{i+1} \lambda_{i+1} a_{i+1}^{k-1}) \quad (2.4)$$

This chiral operator is a triplet with respect to the $SU(2)_R$ symmetry and its $U(1)_R$ charge equals $2k$ where k is a positive integer. Conformal dimension is given by the classical expression $\Delta = k + 2$.

2) There is another bosonic state in the same supermultiplet which corresponds to the Lagrangian density of the kinetic term for the superfield W . Integration only over the part of the superspace ensures that the states are in the short multiplet. The chiral operator⁵:

$$\text{Tr}(F_i^2 + iF_i \tilde{F}_i) a_i^{k-1} - \text{Tr}(F_{i+1}^2 + iF_{i+1} \tilde{F}_{i+1}) a_{i+1}^{k-1} \quad (2.5)$$

couple to the difference between the i -th and the $(i + 1)$ -th holomorphic couplings, $\tau_j = \frac{\theta_j}{2\pi} + \frac{4\pi j}{g_j^2}$, while the corresponding anti-chiral operator:

$$\text{Tr}(F_i^2 - iF_i \tilde{F}_i) \bar{a}_i^{k-1} - \text{Tr}(F_{i+1}^2 - iF_{i+1} \tilde{F}_{i+1}) \bar{a}_{i+1}^{k-1} \quad (2.6)$$

couple to the difference between the anti-holomorphic couplings $\bar{\tau}_i$. Altogether they form a complex $SU(2)_R$ singlet representation, and their linear combinations couple to differences between gauge couplings g_i and theta-angles θ_i . The R-charges of (2.5) and (2.6) are equal to $\pm(2k - 2)$ respectively. The conformal dimension $\Delta = k + 3$ is in agreement with the fact that (2.5) can be obtained from (2.4) by the action of two supercharges.

³ Actually due to the extended supersymmetry we might consider only one representative of each $\mathcal{N} = 2$ supermultiplet.

⁴ Note the difference from the notations of [7] in the relative sign.

⁵ From now on tilde refers to the Hodge dual. There will be no confusion with the fields \tilde{Q}_i since we do not encounter the latter any more.

3) In turn, the operator (2.4) with a fixed k can be obtained by action of two supercharges on the corresponding difference between Coulomb moduli:

$$\mathrm{Tr}(a_i)^{k+1} - \mathrm{Tr}(a_{i+1})^{k+1} \quad (2.7)$$

Obviously, the state (2.7) is a primary operator, the lowest component of the short multiplet. Therefore its conformal dimension $\Delta = k + 1$ and R-charges $R = 2k + 2$, $d = 1$ obey the formula (2.2).

4) The last family of chiral primary operators has the following bosonic representatives:

$$\mathrm{Tr}(F_i a_i^k) - \mathrm{Tr}(F_{i+1} a_{i+1}^k) \quad (2.8)$$

Note that the lowest ($k = 0$) state exists only if the gauge group is $(\prod_i \otimes U(n_i N))/U(1)$ and not (1.1). Therefore, the presence of the corresponding states in the spectrum of supergravity harmonics can tell us about decoupling of the $U(1)$ s. The state (2.8) is a $SU(2)_R$ singlet and carries $2k$ units of the $U(1)_R$ charge. The scaling dimension $\Delta = k + 2$ is again protected from quantum corrections.

While the operators described above manifestly comprise bosonic content of a short $\mathcal{N} = 2$ multiplet in the boundary SCFT, it is instructive to mention the supermultiplet structure of supergravity harmonics they couple to. Scalars $\mathbf{3}_{2k} + \mathbf{1}_{2k-2} + \mathbf{1}_{2k+2}$ and the tensor $\mathbf{1}_{2k}$ naturally fall into anti-self-dual tensor multiplet of $\mathcal{N} = 4$ AdS_5 superalgebra [12]. Because of the multiplet shortening their masses are also protected from quantum and stringy corrections. Next, following arguments of [10], it would be sufficient to check only R-symmetry representations of Kaluza-Klein excitations since their masses were completely determined by supersymmetry.

To conclude this section we make some predictions for the masses of Kaluza-Klein harmonics with $R = 2k$ coming from the twisted sectors. According to [2,3], the operator \mathcal{O} of spin zero on the boundary couples to the supergravity field ϕ with the mass:

$$m^2 = \Delta_{\mathcal{O}}(\Delta_{\mathcal{O}} - 4) \quad (2.9)$$

For the scalar states (2.4), (2.5) and (2.7) it means⁶:

$$m^2 = k^2 - 4, \quad k \geq 1 \quad (2.10)$$

⁶ The $k = 0$ state would correspond to the non-chiral operator $\mathrm{Tr}(W_i \overline{W}_i) - \mathrm{Tr}(W_{i+1} \overline{W}_{i+1})$ with zero R charge.

$$m^2 = k^2 + 4k \tag{2.11}$$

and

$$m^2 = k^2 - 4k \tag{2.12}$$

correspondingly.

The eigenvalues of the Maxwell-like operator widely used in the supergravity literature for the tensor operator (2.8) are given by [13,14]:

$$m^2 = (\Delta_{\mathcal{O}} - 2)^2 \tag{2.13}$$

which entails:

$$m^2 = k^2. \tag{2.14}$$

For the sake of convenience we outline the expected twisted states with a given R-charge in the following table:

State	$SU(2)_R \times U(1)_R$	Mass
Family 1: scalars	$\mathbf{3}_{2k}, k \geq 1$	$m^2 = k^2 - 4$
Family 2: scalars	$\mathbf{1}_{2k}, k \geq 0$	$m^2 = k^2 + 4k$
Family 3: scalars	$\mathbf{1}_{2k}, k \geq 2$	$m^2 = k^2 - 4k$
Family 4: 2-forms	$\mathbf{1}_{2k}, k \geq 1$	$m^2 = k^2$

3. Twisted Sectors of $\mathcal{N} = 2$ *AdS* Orbifold

The orbifold geometry is manifestly singular because of the continuous set of fixed points – a circle $S^1 \subset S^5$. The twisted states are localized on this circle [8], so that the corresponding fields propagate in the six-dimensional space-time: $AdS_5 \times S^1$. The Kaluza-Klein harmonics of these states are not invariant under \mathbb{Z}_n permutations. As explained in the introduction, they couple to operators charged under the corresponding discrete symmetry group.

As was shown in [6], type IIB string theory on the ADE orbifold leads to the $(2, 0)$ six-dimensional effective theory. Particularly, the twisted sectors include as many $(2, 0)$ tensor multiplets as the number of non-trivial conjugacy classes of Γ . The bosonic content of a tensor multiplet consists of the anti-self-dual antisymmetric tensor F_j , three scalars $\vec{\xi}_j$ in the triplet representation of six-dimensional $SU(2)_R$ global symmetry and two scalar singlets ϕ_j and φ_j . It is easy to see that the number of fields and their quantum numbers indeed match the results of the previous section if we identify $SU(2)_R$ symmetry in six and four dimensions, and associate the four-dimensional $U(1)_R$ symmetry with rotations over the fixed circle. By this definition, the k -th Fourier harmonic on the circle carries $2k$ units of the $U(1)_R$ charge, $k \in \mathbb{Z}$.

Naive Kaluza-Klein reduction on the S^1 gives the masses $m^2 = k^2$ for $\mathcal{N} = 4$ supergravity multiplets on AdS_5 . Except for the tensor (2.8), this result differs essentially from the dimensions of the spin zero operators (2.4) - (2.7). Below we match the Kaluza-Klein modes from twisted sectors to these operators and find several interesting subtleties which lead to the mass corrections. The key difference from the flat space orbifold \mathbb{C}^2/Γ is due to the curvature and the five-form flux $G^{(5)}$ through S^5 . These background fields induce effective interaction in the $(2, 0)$ six-dimensional theory. Even though the three-point amplitude involves only two twisted states, direct calculation of it for the S^5/Γ type IIB background does not seem promising. We choose another way and use the blow-up of the singularity: $X \rightarrow S^5/\Gamma$. At least locally we can represent X as $S^1 \times \mathcal{M}$, where \mathcal{M} is an Einstein manifold with the cosmological constant $\Lambda = 4$. If size of \mathcal{M} is large enough, we can rely on type IIB supergravity calculations. The only non-trivial cohomology group of \mathcal{M} is $H^2(\mathcal{M}, \mathbb{Z})$, generated by $(n - 1)$ anti-self-dual normalizable two-forms ω_i :

$$\int_{\mathcal{M}} \omega_i \wedge \omega_j = \delta_{ij}. \quad (3.1)$$

With this picture in mind, let us now work out the Kaluza-Klein spectrum for each family of fields step by step. The strategy is to find linearized equations of motion of the $(2, 0)$ theory in six dimensions taking into account the non-trivial background ($G^{(5)}$ and Λ).

3.1. Family 1: $SU(2)_R$ Triplets

The mass correction to the triplet of real scalars $\vec{\xi}_i$ comes from the interaction with the background curvature. Analogous to the flat space solution, the manifold \mathcal{M} corresponds to n Kaluza-Klein monopoles in a universe with repulsive cosmological constant Λ . Indeed, locally the metric on the Einstein manifold \mathcal{M} [15]:

$$ds^2(\mathcal{M}) = V^{-1}(d\tau + \vec{A}d\vec{r})^2 + Vd\vec{r}^2 \quad (3.2)$$

resembles the metric of the Euclidean multi-centered Taub-Nut solution. Here $V(\vec{r})$ and $\vec{A}(\vec{r})$ depend only on the coordinate \vec{r} on the three-dimensional base \mathcal{B} .

The metric on \mathcal{M} depends on n three-vectors \vec{r}_i corresponding to the positions of Kaluza-Klein monopoles. It is convenient to place one of the monopoles to the origin, and choose the basis of $(n-1)$ independent positions: $\vec{\xi}_i = \vec{r}_i$. The motion of the monopoles can be approximated by geodesic motion on the $3(n-1)$ -dimensional moduli space [16,17]. From the six-dimensional point of view, $\vec{\xi}_i$ correspond to the $(2,0)$ triplet of fields with the effective Lagrangian [18]:

$$\mathcal{L} = \int_{\mathcal{M}} [\sqrt{g}(R - 2\Lambda) + G^{\alpha\beta\gamma\delta}\partial_\mu g_{\alpha\beta}\partial^\mu g_{\gamma\delta}] \quad (3.3)$$

obtained via integration over \mathcal{M} . The greek letters from the beginning of the alphabet refer to directions along \mathcal{M} , and μ is one of the six-dimensional coordinates $X^{0\dots 5}$. The last term in the expression (3.3) gives the standard kinetic energy for $\vec{\xi}_i$, while the former refers to the potential energy. It turns out that the potential energy can be represented as an integral over the base \mathcal{B} where it reduces to the classical expression $\sum_i U(\vec{\xi}_i)$, i.e. the sum over classical potential energies of each monopole [15,16]. Because, to the second order in \vec{r}_i , equally charged monopoles exert no mutual force, we end up with external gravitational potential $U \approx 1 - \Lambda r^2$ [19]. In six dimensions it gives the tachyonic mass $m^2 = -4$ to the scalar triplet. And the reduction on the fixed circle gives the expected answer (2.10): $m^2 = k^2 - 4$.

3.2. Families 2-3: Periods of B Fields

The mass correction to scalar singlets comes from the interaction with the background flux $G^{(5)} = dA^{(4)}$. These scalars are periods of the $B^{(NS)}$ and $B^{(RR)}$ two-form fields over homology two-cycles:

$$B^{(NS)} = \sum_i \varphi^i \wedge \omega_i \quad B^{(RR)} = \sum_j \phi^j \wedge \omega_j. \quad (3.4)$$

In ten dimensions the linearized equations of motion for the B fields look like:

$$\nabla^\mu H_{\mu\nu\lambda}^{(NS)} = \frac{2}{3} G_{\nu\lambda\alpha\beta\gamma}^{(5)} H_{(RR)}^{\alpha\beta\gamma} \quad \nabla^\mu H_{\mu\nu\lambda}^{(RR)} = -\frac{2}{3} G_{\nu\lambda\alpha\beta\gamma}^{(5)} H_{(NS)}^{\alpha\beta\gamma} \quad (3.5)$$

where $H_{(NS/RR)}$ refer to the field strengths of $B^{(NS/RR)}$ respectively. Because the self-dual field $G^{(5)}$ is not dynamical, it is convenient to write an effective action for the fields $B^{(NS)}$ and $B^{(RR)}$ that leads to the equations (3.5)[20]:

$$S = \int d^{10}X \left[\frac{1}{12} (H_{(NS)})^2 + \frac{1}{12} (H_{(RR)})^2 + 4A^{(4)} \wedge H_{(NS)} \wedge H_{(RR)} \right] \quad (3.6)$$

Using (3.1) and (3.4), we perform the dimensional reduction of (3.6) to six dimensions:

$$S = \sum_i \int_{AdS_5 \times S^1} d^6X \left[\frac{1}{2} (d\phi_i)^2 + \frac{1}{2} (d\varphi_i)^2 + 4\phi_i \wedge d\varphi_i \wedge G_{(5)} \right] \quad (3.7)$$

The background flux $G_{(5)} = \epsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5} dX^{\mu_1} dX^{\mu_2} dX^{\mu_3} dX^{\mu_4} dX^{\mu_5}$ is proportional to the volume form on the AdS_5 , so that the derivative in the last term of (3.7) acts in the X^5 direction along the S^1 . Hence the Fourier harmonics of (ϕ_i, φ_i) propagating on the AdS_5 space become mixed by the following operator:

$$\begin{pmatrix} \Delta(AdS_5) - k^2 & 4k \\ 4k & \Delta(AdS_5) - k^2. \end{pmatrix} \quad (3.8)$$

It gives the eigenvalues of the Laplace operator $\Delta(AdS_5)$: $m^2 = k^2 \pm 4k$. The mode corresponding to the positive sign, $m^2 = k^2 + 4k$, has exactly the same mass as expected by SCFT analysis (2.11) to couple to the chiral operators (2.5) of family 2. The other harmonic has the right mass (2.12) to couple to dimension $\Delta = k$ operators (2.7) of family 3. The negative k harmonics couple to the corresponding anti-chiral operators (e.g. (2.6)) with R-charge $R = 2k < 0$ ⁷.

3.3. Family 4: Antisymmetric Tensors

The six-dimensional antisymmetric tensor F_i comes from the projection of type IIB self-dual four-form $A^{(4)}$ on the basis of two-cycles dual to ω_i [6]. In six dimensions the anti-self-duality equation for the field strength $G_i = dF_i$ has the usual form:

$$G_i = -\tilde{G}_i \quad (3.9)$$

⁷ In this case identification of the modes is inverse: the supergravity harmonics with $m^2 = k^2 + 4k$ couple to anti-chiral operators of family 3 while $m^2 = k^2 - 4k$ modes couple to the states (2.6).

It is easy to check that to linear order this equation remains unchanged unless we have a background flux G . Once the latter does not take place, we can make further reduction on the S^1 , and deduce the spectrum $m^2 = k^2$ in accordance with (2.14). However, there is no massless unitary representation of AdS_5 superalgebra $SU(2, 2|2)$ corresponding to such state [12]. Therefore, there is no $k = 0$ mode. This means that $U(1)$ gauge factors decouple, and the gauge group indeed has the proposed form (1.1).

4. Conclusions

We derived all the relevant, marginal and irrelevant (chiral) primary operators which couple to the twisted states of type IIB S^5/\mathbb{Z}_n orbifold. These operators are not invariant under exchange of the gauge factors in $\mathcal{N} = 2$ superconformal field theory. The mass spectrum of the twisted modes is obtained via reduction on the fixed circle. Blow-up of the singularity leads to the interesting features of monopole dynamics in presence of repulsive cosmological constant. The Kaluza-Klein reduction gives the correct conformal dimensions to the operators from section 2 only if the interaction with the background fields is properly taken into account.

Because the operators constructed in section 2 do not involve bifundamental matter, the discussion allows straightforward generalization to non-abelian discrete subgroups Γ along the lines of [21]. In that case, index i labels gauge factors and runs over all the conjugacy classes of Γ . Since \mathcal{M} supports as many anti-self-dual harmonic two-forms as the number of nodes in the corresponding Dynkin diagram, the analysis of sections 3.2 and 3.3 also remains unchanged. All the other fields come in the same supermultiplet with the periods of antisymmetric RR forms. Therefore, their masses also correctly reproduce the conformal dimensions of the corresponding SCFT operators just from supersymmetry.

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References

- [1] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity", hep-th/9711200.
- [2] E. Witten, "Anti De Sitter Space And Holography", hep-th/9802150.
- [3] S. S. Gubser, I. R. Klebanov, A. M. Polyakov, "Gauge Theory Correlators from Non-Critical String Theory", hep-th/9802109.
- [4] S. Kachru, E. Silverstein, "4d Conformal Field Theories and Strings on Orbifolds", hep-th/9802183.
- [5] A. Lawrence, N. Nekrasov, C. Vafa, "On Conformal Theories in Four Dimensions", hep-th/9803015.
- [6] M. R. Douglas, G. Moore, "D-branes, Quivers, and ALE Instantons", hep-th/9603167.
- [7] Y. Oz, J. Terning, "Orbifolds of $AdS_5 \times S^5$ and 4d Conformal Field Theories", hep-th/9803167.
- [8] L. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 261 (1985) 678; Nucl. Phys. B 274 (1986) 285.
- [9] A. Hanany, A. M. Uranga, "Brane Boxes and Branes on Singularities", hep-th/9805139.
- [10] O. Aharony, A. Fayyazuddin, J. Maldacena, "The Large N Limit of $\mathcal{N} = 2, 1$ Field Theories from Threebranes in F-theory", hep-th/9806159.
- [11] S. Minwalla, "Restrictions Imposed by Superconformal Invariance on Quantum Field Theories", hep-th/9712074.
- [12] M. Gunyadin, L. J. Romans, N. P. Warner, Nucl.Phys. **B272** (1986) 598; Phys. Lett. **B154** (1985) 268.
- [13] S. Ferrara, C. Fronsdal, A. Zaffaroni, "On N=8 Supergravity on AdS_5 and N=4 Superconformal Yang-Mills theory", hep-th/9802203.
- [14] O. Aharony, Y. Oz, Z. Yin, "M Theory on $AdS_p \times S^{11-p}$ and Superconformal Field Theories", hep-th/9803051.
- [15] G. W. Gibbons, S. W. Hawking, Commun. Math. Phys. **66** (1979) 291.
- [16] N. S. Manton, Phys.Lett. **B110** (1982) 54.
- [17] M. F. Atiyah, N. J. Hitchin, Phys.Lett. **A107** (1985) 21.
- [18] P. J. Ruback, Commun. Math. Phys. **107** (1986) 93.
- [19] G. W. Gibbons, S. W. Hawking, Phys. Rev. **D15** (1977) 2738.
- [20] G. Dall'Agata, K. Lechner, D. Sorokin, Class.Quant.Grav. **14** (1997) L195.
- [21] C. V. Johnson, R. C. Myers, Phys.Rev. **D55** (1997) 6382.