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Topological polaritons in a quantum spin Hall cavity

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We study the topological structure of matter-light excitations, so-called polaritons, in a quantum spin Hall insulator coupled to photonic cavity modes. We identify a topological invariant in the presence of time reversal (TR) symmetry, and demonstrate the existence of a TR-invariant topological phase. We find protected helical edge states with energies below the lower polariton branch and characteristic uncoupled excitonic states, both detectable by optical techniques. Applying a Zeeman field allows us to relate the topological index to the double coverage of the Bloch sphere by the polaritonic pseudospin.

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Recently, topologically nontrivial states of matter with protected edge or surface states have attracted much attention [1–3]. While the first realizations were found in electronic systems [4–7], topologically nontrivial phases of periodically driven systems [8–11] and photons [12–17] have been discovered within the last years. A time reversal (TR) invariant topological phase can exists when a band inversion occurs, as a function of momentum, between orbital states with different parity, and when these orbital states are coupled by spin-orbit interaction.

In electronic systems, the time reversal operator squares to minus one, $T^2 = -\mathbb{1}$, implying the existence of degenerate Kramers pairs, such that the crossing of topological edge states is protected. In contrast, for bosonic systems with $T^2 = +\mathbb{1}$, in general there is no TR-invariant topologically nontrivial phase [18–20]. The photonic topological insulators [12–17] either break TR or have a built-in degeneracy, which protects edge states in a way similar to the Kramer's degeneracy in fermionic systems.

We consider strongly coupled light-matter systems in two dimensions, so-called polaritons, in which the bosonic polariton can inherit its topological properties from the electronic part. Building on such an example, realized by quantum spin Hall (QSH) electrons [4,6] coupled to cavity photons, we develop a framework which allows us to characterize the topological states of polaritons (cf. Fig. 1). In contrast to recent proposals of topological polaritons [21–23] where TR broken systems were discussed, we focus on TR-invariant topological polaritons. Here, we go considerably beyond previous works and (i) define a topological invariant for TR-symmetric bosonic systems, (ii) explain that, contrary to Refs. [18–20], a topologically nontrivial phase is possible due to a vortexlike singularity in the exciton-photon coupling, and (iii) describe how TR-invariant topological polaritons can be detected experimentally by looking for dark excitonic states and edge states, and by studying the polarization in the presence of an external Zeeman field.

Main results. A polariton consists of an exciton coupled to a cavity photon, such that lower (LP) and upper polariton (UP) dispersion branches are formed [24,25]. The two polarization directions of the photonic component can be identified with a pseudospin [26], which can be described by an effective Hamiltonian, and whose direction can be observed by detecting photonic emission. For the LP branch it takes the form

$$H_{\rm LP}(\vec q) = \epsilon_{\rm LP}(\vec q) + \vec\sigma \cdot \vec h(\vec q), \qquad (1)$$

where $\vec h(\vec q)$ is a momentum dependent effective magnetic field. With $\epsilon_{\rm LP}(\vec q)$ we denote the lower polariton dispersion in the absence of pseudospin coupling, and $\vec\sigma$ is the vector of Pauli matrices. TR invariance $T H(\vec q) T^{-1} = H(-\vec q)$ with TR operator $T = -\sigma_x \mathcal{K}$ requires that the x and y components of the effective magnetic field $\vec h$ are even functions and the z component is an odd function of momentum. The pseudospin polarization of the eigenstate $|\chi_{1,2}\rangle$ is given by

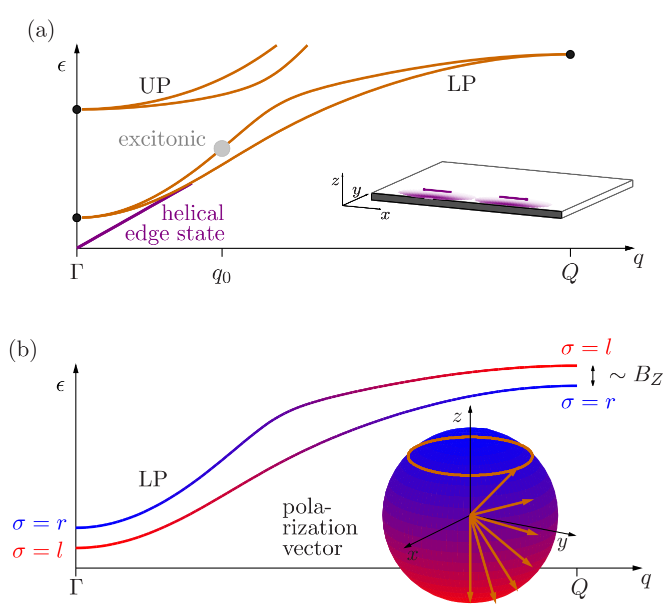

FIG. 1. Defining properties of a topologically nontrivial polariton: (a) LP and UP branches (orange lines) are split by spin-orbit coupling, except at TR-invariant momenta (Γ, Q) (thick black dots). For topological polaritons purely excitonic states (gray dot) emerge along a line $\vec q = \vec q_0$, and helical edge states (thick purple line) are present below the LP. (b) With a TR breaking Zeeman field $B_Z \neq 0$, the polarization vector $\vec n$ of each LP dispersion branch can be tracked over the whole Brillouin zone. The polariton is topologically nontrivial if $\vec n$ covers the entire Bloch sphere (inset). Here, the north (south) pole [blue (red) color] stands for right (left) circular polarization, while the x-y plane represents linearly polarized light.

$\vec{n}_{1,2} = \langle\chi_{1,2}|\vec{\sigma}|\chi_{1,2}\rangle = \mp\hat{h}$ with $\hat{h} \equiv \vec{h}/|\vec{h}|$. If $\vec{n}$ points north (south), the emitted light is right (left) polarized, while the x-y plane represents linearly polarized light [see the inset of Fig. 1(b)]. In the presence of an additional parity symmetry, $H_{\rm LP}(-\vec{q}) = H_{\rm LP}(\vec{q})$, one finds $h_z \equiv 0$, and the emitted light is always linearly polarized. Physical mechanisms which lead to a nonzero effective magnetic field are, for example, a longitudinal-transverse splitting of the electromagnetic field [26] or a spin-orbit coupling of the electronic building blocks, as discussed below. In both cases, it turns out that $\vec{n}_{1,2}$ winds twice around the z axis if $\vec{q}$ encircles the Γ point. Due to the continuity of $\vec{h}(\vec{q})$, we find that $\vec{h}(\vec{\Gamma}) = 0$, implying that the LP (UP) dispersion is degenerate at the Γ point [see Fig. 1(a)]; in our QSH-polariton model, $\vec{h}(\vec{\Gamma}) = 0$ at all other TR-invariant momenta as well.

The Chern number [27,28] for the eigenstate $|\chi_{1,2}\rangle$ counts how many times $\vec{n}_{1,2}$ wraps around the unit sphere if $\vec{q}$ covers the Brillouin zone, $C_{1,2} \sim \mp\int_{\vec{q}} \hat{h}\cdot(\partial_{q_x}\hat{h}\times\partial_{q_y}\hat{h})$. Clearly, in the presence of parity symmetry, $C_{1,2} = 0$ as $h_z \equiv 0$. However, even without parity, $C_{1,2} = 0$: The scalar triple product contains each component $\hat{h}_i$ with $i = x,y,z$ exactly once, and thus is an odd function of momentum, because $h_{x,y}$ is even and h_z is odd. Since the integral over the Brillouin zone is invariant under momentum inversion, the Chern number has to vanish, in agreement with the general classification [18–20].

For polaritons in a QSH cavity the LP eigenstates are superpositions of excitonic $|b_{1,2}\rangle$ and photonic wave functions $|a_{1,2}\rangle$: $|\Phi_{1,2}\rangle = \beta_{1,2}|b_{1,2}\rangle + \alpha_{1,2}|a_{1,2}\rangle$ with real coefficients $\beta_{1,2},\alpha_{1,2}$ (see below for details). An explicit calculation shows $T|\Phi_{1,2}(\vec{q})\rangle = \mp|\Phi_{1,2}(-\vec{q})\rangle$. By construction, the effective model is the projection onto the photonic sector, such that $|\chi_{1,2}\rangle = |a_{1,2}\rangle$. Both models have the same polarization vector $\vec{n}_{1,2}$, which implies that $C_{1,2} = 0$ for the microscopic model, too.

In order to reveal the nontrivial topology of the microscopic model, we define TR partners $|\Phi_\pm\rangle = (|\Phi_2\rangle \pm |\Phi_1\rangle)/\sqrt{2}$ which transform according to $T|\Phi_\pm(\vec{q})\rangle = |\Phi_\mp(-\vec{q})\rangle$. Remarkably, the corresponding Chern numbers are nonzero, $C_\pm = \mp 2$, an interesting result given that the general classification [18–20] rules out nontrivial Chern numbers for eigenstates of the Hamiltonian. The robustness of $C_\pm$ is due to a vortexlike nonanalyticity of the exciton-photon Bloch Hamiltonian Eqs. (6) and (7), which can only be removed when the splitting of the LP branch vanishes. As signatures of the topological phase we find an odd number of lines of momenta $\{\vec{q}_0\}$ encircling the Γ point for which $\alpha_2(\{\vec{q}_0\}) = 0$, experimentally accessible by looking for a dark excitonic state at $\{\vec{q}_0\}$. In addition, we predict polaritonic edge states. Our procedure of defining pairs of TR partners as suggested in Refs. [29,30] is analogous to defining spin eigenstates with a nontrivial spin-Chern number in a QSH insulator with broken spin symmetry [31,32].

If TR symmetry is broken by a small Zeeman field, $B_Z \neq 0$, the pseudospin degeneracy at the TR-invariant momenta is lifted, and two completely nondegenerate dispersion relations exist [see Fig. 1(b)]. For topological polaritons with $C_\pm \neq 0$, we then find that for each branch of the dispersion the experimentally measurable pseudospin polarization $\vec{n}(\vec{q})$ covers the Bloch sphere. Thus, in a TR broken setup, the experimentally measurable pseudospin polarization $\vec{n}(\vec{q})$ and the pseudospin model Eq. (1) allow us to distinguish between topologically trivial and nontrivial polaritons.

Microscopic model. The Hamiltonian of the two-dimensional QSH insulator in the basis of orbital states $\{|+1/2\rangle,|+3/2\rangle,|-1/2\rangle,|-3/2\rangle\}$ is [6]

$$H_{\rm e}(\vec{k}) = \begin{pmatrix} H_{\rm e}^+(\vec{k}) & 0 \\ 0 & H_{\rm e}^-(\vec{k}) \end{pmatrix},\quad H_{\rm e}^+(\vec{k}) = \vec{d}(\vec{k})\cdot\vec{\sigma}, \tag{2}$$

with wave vector $\vec{k}$ and spin-orbit field $\vec{d}$. Because of TR symmetry: $H_{\rm e}^-(\vec{k}) = H_{\rm e}^+(-\vec{k})^*$, where $\alpha = \{+,-\}$ labels a pseudospin. In the following we will use the parametrization $d_{x/y} = A\sin(k_{x/y})$, $d_z = M + B[2-\cos(k_x)-\cos(k_y)]$ with $A = \hbar v_F/a$, $B > 0$, and $M \in \mathbb{R}$, where v_F is the Fermi velocity, a the lattice spacing, and $\vec{k}$ is measured in a^{-1}. For $M < 0$ and $B > |M|/2$ the normalized spin-orbit field $\hat{d} \equiv \vec{d}/|\vec{d}|$ covers the Bloch sphere and the QSH insulator is topologically nontrivial [6].

An electromagnetic field $\vec{A}$ is coupled minimally to the semiconductor via $\vec{p} \to \vec{p} + e\vec{A}$ with elementary charge $e > 0$. We work in the Coulomb gauge, linearize the Hamiltonian in $\vec{A}$, and expand the photon field in plane waves with amplitudes $\vec{A}_{\vec{q}\sigma}$, where $\vec{q}$ is the photon momentum and σ labels the polarization [33]. We find for the optical transition matrix elements,

$$g_{\mu\nu}^{\sigma}(\vec{k}',\vec{k},\vec{q}) = \delta_{\vec{k}'\vec{k}+\vec{q}}\, e\vec{A}_{q\sigma}\cdot\langle\psi_{\mu k'}|\frac{\partial H_{\rm e}(\vec{k})}{\partial\vec{k}}|\psi_{\nu k}\rangle, \tag{3}$$

where μ,ν label the pseudospin and band index of the eigenstates of Eq. (2). In order to evaluate Eq. (3) we use the Wigner-Eckart theorem [34] and the specific form of the basis states of Eq. (2) [6]. We find that optical transitions do not change the pseudospin α.

A particle-hole transformation of Eq. (2) yields the hole Hamiltonian $H_{\rm h}^{\alpha}(\vec{k}) = -H_{\rm e}^{\alpha}(-\vec{k})^*$, with wave functions $\psi_{\alpha\vec{k}}^{\rm h} = (\psi_{\alpha-\vec{k}}^{\rm e})^*$, and energies $\epsilon_{\rm h}(\vec{k}) = -\epsilon_{\rm e}(-\vec{k}) > 0$ (the chemical potential is zero). Accounting for the attractive Coulomb interaction of electrons and holes, the wave function of optically active excitons is

$$|\psi_{\alpha\vec{q}}^{\rm x}\rangle = \sum_{\vec{k}}\phi_{\rm C}(\vec{k})|\psi_{\alpha\vec{q}/2-\vec{k}}^{\rm h}\rangle\otimes|\psi_{\alpha\vec{q}/2+\vec{k}}^{\rm e}\rangle, \tag{4}$$

where $\phi_{\rm C}(\vec{k})$ denotes the Fourier transform of the electron-hole wave function with respect to the relative coordinate. Semiconductors typically have a large dielectric constant, which screens the Coulomb interaction and results in an exciton Bohr radius much larger than the lattice constant. Thus, the binding function $\phi_{\rm C}(\vec{k})$ is strongly peaked around $k = 0$ and $|\psi_{\alpha\vec{q}}^{\rm x}\rangle \approx |\psi_{\alpha\vec{q}/2}^{\rm h}\rangle\otimes|\psi_{\alpha\vec{q}/2}^{\rm e}\rangle$ with energy $\epsilon_{\rm x}(\vec{q}) \approx \epsilon_{\rm h}(\vec{q}/2) + \epsilon_{\rm e}(\vec{q}/2)$. Such excitons are described by the Hamiltonian

$$H_{\rm x}^{\alpha}(\vec{q}) = H_{\rm h}^{\alpha}(\vec{q}/2)\otimes\mathbb{1}_{\rm e}^{\alpha} + \mathbb{1}_{\rm h}^{\alpha}\otimes H_{\rm e}^{\alpha}(\vec{q}/2), \tag{5}$$

acting on a four-dimensional Hilbert space of electron-hole pairs.

Diagonalizing Eq. (5) and projecting onto excitons gives the eigenstates $|b_\alpha\rangle$ with energy $\epsilon_{\rm x} = 2|\vec{d}|$. The cavity photon dispersion is $\omega_{\vec{q}} = \omega_0\sqrt{1+(D/\omega_0)\vec{q}^{\,2}}$ with ω_0 set by the cavity thickness, $D \equiv \hbar^2 c_{\rm ph}^2/\omega_0 a^2$, photon velocity $c_{\rm ph}$, and momentum $\vec{q}$ measured in a^{-1}. Coupling excitons and right (r) and left (l) circularly polarized photons yields polaritons. In the basis $\{|b_+\rangle,|b_-\rangle,|a_r\rangle,|a_l\rangle\}$ the Hamiltonian takes the form

$$H_{\rm P} = \begin{pmatrix} \epsilon_{\rm x}\mathbb{1} & G \\ G^\dagger & \omega\mathbb{1} \end{pmatrix}, \tag{6}$$

and the exciton-photon coupling is obtained via Eq. (3),

$$G = \frac{g_0}{4}\begin{pmatrix} (1-\hat{d}_z)e^{-2i\varphi} & -(1+\hat{d}_z) \\ (1+\hat{d}_z) & -(1-\hat{d}_z)e^{2i\varphi} \end{pmatrix}, \tag{7}$$

with $\hat{d}_z = d_z/|\vec{d}|$ and $e^{\pm i\varphi} \equiv (d_x \pm i d_y)/|d_x \pm i d_y|$. The coupling $g_0 \propto \epsilon_{\rm x}(\vec{q})$, and is proportional to the photon amplitude times a numerical constant from the transition matrix element Eq. (3). However, for the study of topological properties, we can safely neglect the continuous $\vec{q}$ dependency and treat g_0 as constant.

Topological invariant. The Hamiltonian Eq. (6) has two eigenstates for both LP and UP branches: $|\Phi_{1,2}^{\gamma}\rangle = b_{1,2}^{\gamma}|b_{1,2}\rangle + a_{1,2}^{\gamma}|a_{1,2}\rangle$ with $\gamma = \{\mathrm{LP},\mathrm{UP}\}$, exciton $|b_{1,2}^{\gamma}\rangle$, and photon pseudospinors $|a_{1,2}^{\gamma}\rangle$. We construct new basis states, $|\Phi_{\pm}^{\gamma}\rangle = (|\Phi_2^{\gamma}\rangle \pm |\Phi_1^{\gamma}\rangle)/\sqrt{2}$, which are not eigenstates, but obey $T|\Phi_{\pm}^{\gamma}(\vec{q})\rangle = |\Phi_{\mp}^{\gamma}(-\vec{q})\rangle$. We define a Chern number as

$$C_{\pm}^{\gamma} = -\frac{i}{2\pi}\int_{\vec{k}\in \mathrm{BZ}} \varepsilon_{ij}\langle \partial_{k_i}\Phi_{\pm}^{\gamma}|\partial_{k_j}\Phi_{\pm}^{\gamma}\rangle, \tag{8}$$

where ε_{ij} is the Levi-Civita symbol. An explicit evaluation of Eq. (8) yields $C_{\pm}^{\rm LP} = \mp 2$ and $C_{\pm}^{\rm UP} = 0$ for positive detuning, and vice versa for negative detuning. The existence of $C_{\pm}^{\rm LP} = \mp 2$ is due to the vortex structure on the diagonal of the coupling Eq. (7). For a transition to a phase with different Chern numbers to occur, this vortex structure has to be removed, implying vanishing diagonal elements in Eq. (7). At the transition point the LP splitting vanishes and the dispersion is twofold degenerate. Hence, the Chern numbers $C_{\pm}^{\gamma}$ are protected by the splitting of LP and UP branches.

It is enlightening to analyze the photonic component

$$|a_{\pm}^{\rm LP}\rangle = \frac{a_2^{\rm LP} \mp a_1^{\rm LP}}{2}e^{i\varphi}|a_r\rangle + \frac{a_2^{\rm LP} \pm a_1^{\rm LP}}{2}e^{-i\varphi}|a_l\rangle, \tag{9}$$

of $|\Phi_{\pm}^{\rm LP}\rangle$. The coefficients $a_1^{\rm LP}, a_2^{\rm LP} \in \mathbb{R}$ are continuous in the entire Brillouin zone, with $a_1^{\rm LP} \neq 0\ \forall\, \vec{q}$, $a_2^{\rm LP}(\Gamma) = -a_1^{\rm LP}(\Gamma)$, and $a_2^{\rm LP}(Q) = a_1^{\rm LP}(Q)$, such that $a_2^{\rm LP}$ has to have an odd number of lines of zeros when going from the Γ point to the Q points. Then, the polarization vector $\vec{n}_+ \sim \langle a_+^{\rm LP}|\vec{\sigma}|a_+^{\rm LP}\rangle$ points north (south) at the Γ point (Q points) and winds twice around the z axis in between. Thus, $\vec{n}_\pm$ covers the Bloch sphere twice. We find that the polarization vector of the excitonic component $|b_{\pm}^{\rm LP}\rangle$ does not cover the Bloch sphere and cannot contribute to Eq. (8). For all wave vectors $\vec{q}_0$ at which $a_2^{\rm LP} = 0$, a dark exciton eigenstate $|\Phi_2^{\rm LP}\rangle = |b_2^{\rm LP}\rangle$ exists. This constitutes a clear signature for topological polaritons.

Breaking TR symmetry with a Zeeman field B_Z, we find that the LP eigenstates $|\Phi_{1,2}^{\rm LP}\rangle$ for $B_Z \neq 0$ have the same topological structure as the states $|\Phi_{\pm}^{\rm LP}\rangle$ for $B_Z = 0$, i.e., the $|\Phi_{\pm}^{\rm LP}\rangle$ obtained for $B_Z = 0$ can be continuously deformed into the eigenstates $|\Phi_{1,2}^{\rm LP}\rangle$ for $B_Z \neq 0$. Thus, if $\vec{n}_\pm$ covers the Bloch sphere in the case $B_Z = 0$, then in the case $B_Z \neq 0$ the polarization vector $\vec{n}_{1,2}$ also covers the Bloch sphere.

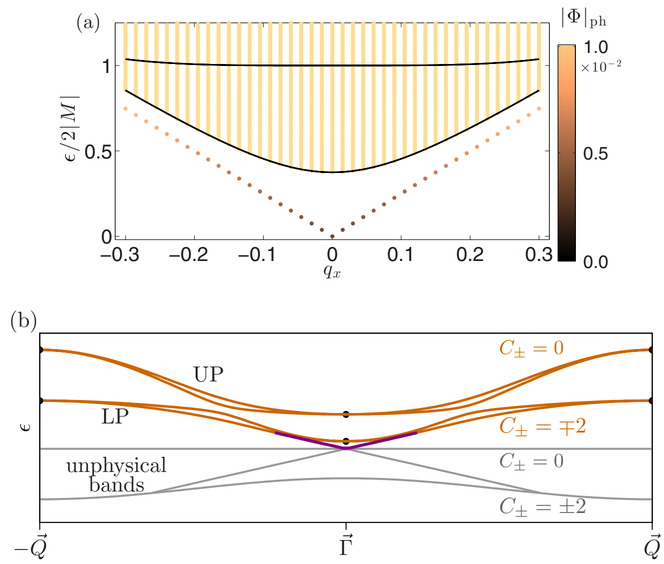

FIG. 2. Spectrum of topological polaritons for a system with cylindrical geometry: (a) The colored dots depict polaritons with photonic fractions $|\Phi|_{\rm ph}$. The solid black lines show the LP and UP branches (LP and UP splitting not visible on this scale). As parameters, $A = 5|M|$, $B = 25|M|$, $\omega_0 = 3|M|/4$, $D = 35|M|$, $g_0 = |M|/24$, and $N = 2000$ lattice points are used. (b) The polariton branches (orange) and the unphysical electron-hole bands (light gray) are sketched schematically. Because LP branch and negative energy bands carry opposite Chern numbers $C_\pm$, a pair of helical edge states (the physical part is marked purple) connects both.

Edge states. We first evaluate the electron Hamiltonian Eq. (2) on a cylindrical geometry with boundaries along the x direction and then couple the corresponding excitons to photon eigenmodes [35]. The numerically obtained spectrum is shown in Fig. 2(a). At each boundary we find one pair of edge states with energies below the LP branch. How can these edge states be related to the polaritonic Chern numbers discussed above, and why extend the edge states all the way down to zero energy? Since the excitons are a direct product of topologically nontrivial electrons and holes, it is expected to find nonvanishing Chern numbers $C_{\pm}^{\rm x} = \mp 2$. The pseudospin degeneracy of the exciton dispersion in addition to TR allows one to construct an operator which commutes with the exciton Hamiltonian and behaves exactly as a fermionic TR operator [for details, consult the Supplemental Material [35]). Due to this symmetry, the excitonic system is also in symmetry class AII and a $\mathbb{Z}_2$ index for excitons is given by $\nu = \frac{1}{4}|C_+^{\rm x} - C_-^{\rm x}|$ [29]. Here, the prefactor $1/4$ was introduced because of the Chern number doubling due to the tensor product of hole and electron space [35]. We note that the coupled system of excitons and photons does not have any (pseudo)fermionic TR symmetry and is solely in symmetry class AI. The nonanalyticity of the polariton Hamiltonian (6) does not allow one to obtain a lattice representation and to study edge states directly. A lattice Hamiltonian is obtained by embedding the polariton space into a larger Hilbert space which contains all

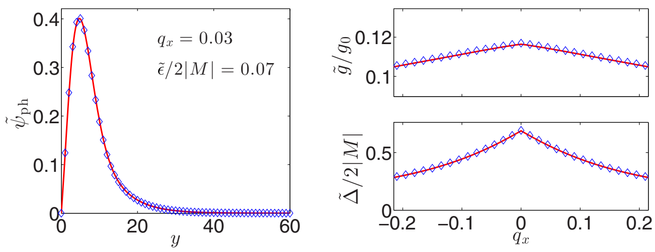

FIG. 3. Comparison of analytical (lines) and numerical (symbols) results for the polaritonic edge state. Left panel: The photon wave function [cf. Eq. (11)] is shown. Right panel: The coupling strength $\tilde{g}$ (upper plot) and detuning $\tilde{\Delta}$ (lower plot) of the effective model are depicted. Same parameters as in Fig. 2(a) are used.

possible electron-hole (*e-h*) states [35], including unphysical *e-h* pairs which have either zero or negative energies [see Fig. 2(b)]. Since the *e-h* bands at negative energy have Chern numbers ± 2, and since the sum of all Chern numbers in the Hilbert space of polaritons and artificial *e-h* pairs has to be zero, the polariton Chern numbers are compensated by the negative energy *e-h* Chern numbers, and edge states below the LP branch emerge (cf. Fig. 2).

Effective edge model. The polaritonic edge states are well described by coupling excitonic edge states to photonic eigenmodes [35]. To leading order perturbation theory we find

$$\tilde{\epsilon}(q_x) \approx \tilde{\epsilon}_{\rm x}(q_x) - \left(\frac{\tilde{g}(q_x)}{\tilde{\Delta}(q_x)}\right)^2 \tilde{\Delta}(q_x), \tag{10}$$

$$|\tilde{\Phi}_{\rho\, q_x}\rangle \approx |\tilde{\psi}^{\rm x}_{\rho\, q_x}\rangle - \frac{\tilde{g}(q_x)}{\tilde{\Delta}(q_x)}|\tilde{\psi}^{\rm ph}_{\rho\, q_x}\rangle, \tag{11}$$

where $\rho = \{{\rm R,L}\}$ labels the right and left mover, $\tilde{\Delta} \equiv \tilde{\omega} - \tilde{\epsilon}_{\rm x}$ is the detuning between the photon $\tilde{\omega}$ and exciton energy $\tilde{\epsilon}_{\rm x} = \hbar v_F |q_x|$, and $\tilde{g}$ the coupling strength. The excitonic and photonic edge-state wave functions are denoted by $|\tilde{\psi}^{\rm x}_{\rho\, q_x}\rangle$ and $|\tilde{\psi}^{\rm ph}_{\rho\, q_x}\rangle$, respectively. The right (left) moving exciton carries pseudospin $+\,(-)$, whereas both right and left moving photons are linearly polarized with electric field parallel to the plane of incident. We find that a high photonic fraction of the edge state relies on $v_F \sim c_{\rm ph}$ and $g_0 \sim \omega_0$. For $\tilde{\epsilon}_{\rm x} \ll 2|M|$ our perturbative results are in very good agreement with numerics, as shown in Fig. 3. Finally, the one-dimensional effective Hamiltonian (for one edge) takes the form

$$H_{\rm E} = \sum_{q\rho}\{\tilde{\epsilon}_q b^\dagger_{\rho q} b_{\rho q} + \tilde{\omega}_q a^\dagger_{\rho q} a_{\rho q} + (\tilde{g}_q b^\dagger_{\rho q} a_{\rho q} + {\rm H.c.})\}. \tag{12}$$

The operator $a^\dagger_{\rho q}$ creates a right, $\rho = {\rm R}$ (left, $\rho = {\rm L}$) moving photon with momentum q and dispersion $\tilde{\omega}_q$. In one dimension the elementary excitations are collective modes (plasmons) instead of excitons. Then, $b^\dagger_{\rho q}$ creates right and left moving plasmons with dispersion $\tilde{\epsilon}_q$. For a Luttinger liquid interactions renormalize the bare Fermi velocity [36], and the plasmon dispersion remains linear.

Experimental signatures. The signatures of topological polaritons are dark excitonic states along a line $\{\vec{q}_0\}$ in momentum space, and edge states below the LP branch [cf. Fig. 1(a)]. Both are detectable via optical techniques [24]. Applying a Zeeman field allows one to determine the Chern number from analyzing the polarization of the bulk polaritons [see Fig. 1(b)]. Realizing topological polaritons in a nontrivial QSH insulator is challenging. However, recent developments of engineering spin-orbit coupling in polaritonic systems [37] and accomplishing slow photons in photonic crystals [38] may pave the way for it.

Conclusion. We have considered a TR-invariant model of polaritons in a QSH cavity, and introduced a topological invariant for TR partners which is stabilized by the pseudospin splitting of polaritons. In the topological phase, polaritonic edge states below the LP branch exist, as well as lines in momentum space with uncoupled excitons, both detectable via optical techniques.

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