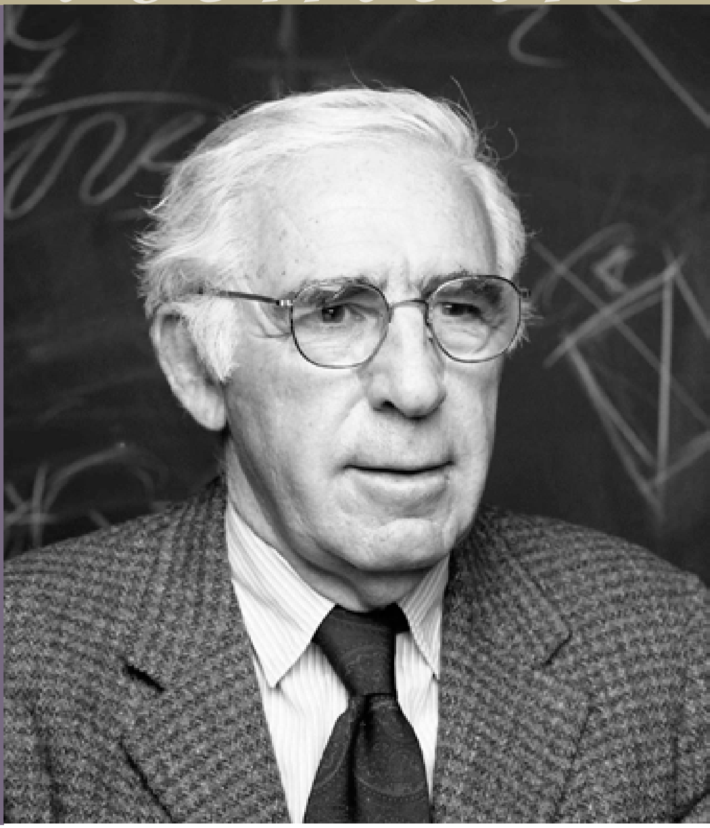
Daniel Gorenstein

1923-1992

BIOGRAPHICAL *Memoirs*



A Biographical Memoir by
Michael Aschbacher

NATIONAL ACADEMY OF SCIENCES

DANIEL **GORENSTEIN**

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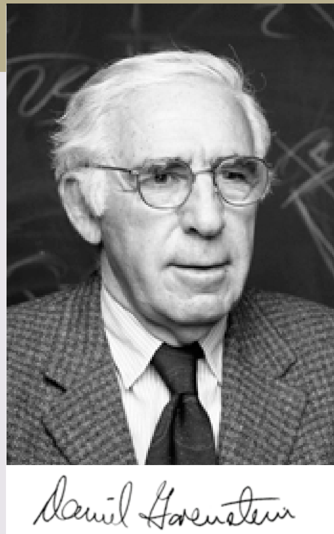

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By Michael Aschbacher

Daniel Gorenstein was one of the most influential figures in mathematics during the last few decades of the 20th century. In particular, he was a primary architect of the classification of the finite simple groups.

During his career Gorenstein received many of the honors that the mathematical community reserves for its highest achievers. He was awarded the Steele Prize for mathematical exposition by the American Mathematical Society in 1989; he delivered the plenary address at the International Congress of Mathematicians in Helsinki, Finland, in 1978; and he was the Colloquium Lecturer for the American Mathematical Society in 1984. He was also a member of the National Academy of Sciences and of the American Academy of Arts and Sciences.

Gorenstein was the Jacqueline B. Lewis Professor of Mathematics at Rutgers University and the founding director of its Center for Discrete Mathematics and Theoretical Computer Science. He served as chairman of the university's mathematics department from 1975 to 1982, and together with his predecessor, Ken Wolfson, he oversaw a dramatic improvement in the quality of mathematics at Rutgers.

Born and raised in Boston, Gorenstein attended the Boston Latin School and went on to receive an A.B. degree from Harvard University in 1943, married his wife Helen in 1947, and received his Ph.D. from Harvard in 1950. He served as a faculty member at Clark University and Northeastern University before moving to Rutgers in 1969.

Gorenstein's thesis, overseen by faculty advisor Oscar Zariski, was in algebraic geometry, but he published only one paper in the area afterward, as he soon changed his interests to finite group theory. However, geometers still know his name through the "Gorenstein ring," a concept growing out of his dissertation.

In his article in the book *A Century of Mathematics in America*, Gorenstein described how he made the transition from algebraic geometry to finite group theory (Gorenstein 1988). Like many American mathematicians of his era, he consulted during the summer with the U.S. government on problems in cryptography and coding theory. For example, he and N. Zierler produced the first algorithm for decoding general BCH-codes. One encryption scheme involved ABA-*groups*: finite groups G with proper subgroups A and B such that $G = ABA$. The study of such groups became Gorenstein's entry into group theory.

Gorenstein spent the academic year 1957–58 at Cornell University. He had intended to work there with Sheeram Abhyankar in algebraic geometry, but during the summer of 1957 Abhyankar was in a serious automobile accident and did not arrive at Cornell until late in the year. As a result, Gorenstein continued to work on ABA-groups with his good friend Yitz Herstein, who was then also at Cornell. By the time Abhyankar was in residence, Gorenstein had come to think of himself as a finite group theorist rather than an algebraic geometer.

In 1960 Gorenstein spent a sabbatical year at the University of Chicago, as part of a group theory year organized by A. A. Albert. It was there that he met John Walter and began what was to become a long and fruitful collaboration. It was also at this point that Gorenstein started to become a serious finite group theorist and to confront problems in this field that in time would lead to his renowned program to classify the finite simple groups.

Gorenstein contributed to the classification of the finite simple groups in many ways. First, he proved some of the deepest and most important theorems underlying the classification, and he did so throughout the roughly 30-year period of maximal activity in simple group theory, beginning about 1960. He was also an innovator, introducing some of the critical concepts and techniques that formed the foundation of the proof. His 1968 text, *Finite Groups*, became the basic reference in finite group theory; many young mathematicians, including myself, were introduced to the new theory of finite simple groups through his book. Gorenstein was the chief strategist in the effort to classify the simple groups; indeed, in his series of lectures at the University of Chicago in 1972, he was the first to dare to put forward a detailed program for classifying such groups. Finally, he was the organizer of the large and complex effort to classify the simple groups, which took shape in the 1970s.

In his 1972 lectures in Chicago, Gorenstein speculated in some detail as to how the finite simple groups might be classified. At the time, some of the senior people in finite group theory viewed his program as science fiction, but it was his vision rather than theirs that proved to be accurate. While the program that Gorenstein sketched in 1972 was not implemented in all its particulars, it turned out to be remarkably close in many respects to the eventual proof of the classification theorem. The text of his Chicago lectures appears as an appendix to his article on the classification in his *Bulletin of the AMS* article (Gorenstein 1979).

In addition to serving as chief strategist for the classification effort, Gorenstein was also its chief organizer and cheerleader. He suggested problems, and means to attack the problems, to numerous people, and he inspired everyone in the field with his enthusiasm and energy. In this way he had a great influence on young finite group theorists. A list of his students includes Jui-Chi Chang, Andrew Chermak, Martin Guterman, Thomas Hearne, Gerard Kiernan, Kenneth Klinger, Robert Miller, Michael O'Nan, and Bernadette Tutinas. (O'Nan received his degree from Princeton, but Gorenstein was his de facto advisor.) Moreover, almost all finite group theorists of my age group were at least indirectly his students.

To understand Gorenstein's work and its significance, one needs to know a little about the state of the art in finite group theory in 1960. The classification says that each finite simple group is a group of prime order, an alternating group, a finite simple group of Lie type, or one of 26 sporadic simple groups. Most mathematicians are familiar with the groups of prime order and the alternating groups. The groups of Lie type are linear groups, and they can be viewed as forms of algebraic groups. Finally, the sporadic groups, being pathologies, are members of no known naturally defined family of simple groups.

Before 1955, only certain of the finite simple groups of Lie type were known to exist, and of the 26 sporadic groups, only the five Mathieu groups had been discovered. Chevalley's *Tohuku* paper in 1955 showed that Chevalley groups could be defined over any field (Chevalley 1955). This work was extended by R. Steinberg (Steinberg 1959) and R. Ree (Ree 1961a,b) to show that other twisted versions of the Chevalley groups could be defined over suitable fields. Further work, related to the classification of simple algebraic groups, by T. Springer, Steinberg, J. Tits, and others, gave a good picture of the structure of these groups. Thus by the early '60s, all simple groups of Lie type were known to exist and were fairly well understood.

In his 1954 address to the International Congress of Mathematicians in Amsterdam (Brauer 1954), Richard Brauer proposed that finite simple groups might be characterized in terms of the centralizers of their involutions. The conceptual base for this proposal was the Brauer-Fowler Theorem, which states that there are only a finite number of finite simple groups possessing any given involution centralizer (Brauer and Fowler 1955). Moreover, Brauer and his students and collaborators went on to prove that various simple groups were indeed characterized by involution centralizers. Thus by 1960 the idea of classifying finite simple groups by using centralizers of involutions was well accepted and in time became the basis for the classification.

Next, in the late 1950s and early ’60s, local group theory began to assume its position as the preeminent tool in the study of simple groups—in papers by Michio Suzuki, in John G. Thompson’s thesis establishing the Frobenius conjecture, in the Feit-Thompson Theorem proving groups of odd order are solvable, and in Thompson’s work on N-groups. Given a prime p, a *p-local subgroup* of a finite group G is the normalizer of a nontrivial p-subgroup of G. In other words, the local theory of finite groups studied them from the point of view of their local subgroups. This attention to local group theory was particularly important to Gorenstein because he was soon to become primarily a local group theorist, and also because his early work on simple groups was motivated by his study of the local group-theoretic arguments in the Odd Order Paper (Feit and Thompson 1963) and the N-group papers (Thompson 1968, 1970, 1971, 1973, 1974a, 1974b).

In order to classify the simple groups in terms of their local structure, one needs to be able to describe that structure and to understand how the local subgroups of a simple group differ from those of a random finite group. In particular, it is necessary to understand the local structure of the groups of Lie type and, to a lesser extent, of the alternating and sporadic groups.

The proof of the classification that eventually evolved depends upon two partitions of the simple groups: a subdivision into groups of odd and even characteristic, and a subdivision into large and small groups. The finite groups of Lie type are linear groups over finite fields; the characteristic of such a group G is the characteristic p of the associated finite field. The p-elements of G are the unipotent elements of G and the elements of order prime to p are the semisimple elements. The centralizers of unipotent elements have a much different structure than that of the centralizers of semisimple elements.

These differences can be captured using definitions phrased in the language of abstract groups rather than in that of linear groups. As involution centralizers are the basis of the Brauer approach, we are led to a corresponding partition according to odd or even characteristic, in terms of the structure of the centralizers of involutions.

The size of a group G of Lie type is measured by its Lie rank, which is in turn a linear function of the dimension of the defining vector space for G. Again, this rank can be defined in abstract group-theoretic terms, rather than in linear group-theoretic terms.

Gorenstein's first major papers on simple groups, written with John Walter, concerned small groups of odd characteristic. Here a simple group G is of odd characteristic and small rank if its 2-rank $m_2(G)$ is small. Recall that if p is prime then the p-rank $m_p(G)$ of a finite group G is the maximum n such that G has a subgroup isomorphic to the direct product of n copies of the group of order p. The simple groups of Lie type over fields of odd order and Lie rank 1 correspond roughly to the simple groups of 2-rank 2. Such groups have dihedral, semidihedral, or wreathed Sylow 2-subgroups. In the early '60s, Gorenstein and Walter classified the groups with dihedral Sylow 2-subgroups (Gorenstein and Walter 1964) and some years later J. Alperin, Brauer, and Gorenstein classified the groups with semidihedral or wreathed Sylow 2-subgroups (Alperin, Brauer, and Gorenstein 1970).

After his classification with Walter of groups with dihedral Sylow 2-subgroups, Gorenstein wrote his text *Finite Groups*, which became the basic reference in finite group theory (Gorenstein 1968). The book had a tremendous influence on young mathematicians of my generation interested in finite groups. For example, during my last year in graduate school (1968–69), I took a course taught from a preprint version of *Finite Groups*. In part under the influence of this book, I switched my research interests from combinatorics to finite group theory, and my first job was as a postdoc at the University of Illinois, working with the prominent group theorists Suzuki and John Walter.

It is also worth noting that in the last chapter of *Finite Groups* we find the germ of one of the important innovations in finite group theory introduced by Gorenstein (in some cases, in collaboration with John Walter) during the late '60s and early '70s. Specifically, in Chapter 17 Gorenstein defines the maximal normal semisimple subgroup of a finite group G, which he came to call $L(G)$, but which is now commonly denoted by $E(G)$ and is used to define the generalized Fitting subgroup.

The generalized Fitting subgroup $F^*(G)$ of a finite group G is the characteristic subgroup $F(G)E(G)$, where the Fitting subgroup $F(G)$ is the product of the subgroups $O_p(G)$ over all primes p and $E(G)$ is the product of the components of G. Here $O_p(G)$ is the largest normal p-subgroup of G and the components of G are its subnormal quasisimple subgroups. Subnormality is the transitive extension of the normality relation. A group L is quasisimple if $L=[L,L]$ is perfect and $L/Z(L)$ is simple.

The generalized Fitting subgroup has become the principal tool for restricting the structure of local subgroups of simple groups, which a priori is rather arbitrary. Roots of the notion appear in work of Hans Fitting and Helmut Wielandt, and the concept was developed more fully by Gorenstein and Walter, particularly in their emphasis on components. In Bender (1970), Helmut Bender was the first to write down the formal definition of $F^*(G)$, establish some of its basic properties, and make use of the full strength of the concept.

Brauer denoted by $O(G)$ the largest normal subgroup of G of odd order, and Gorenstein called $O(G)$ the core of G. From his reading of the N-group papers and his work with Walter, Gorenstein was well aware that cores of 2-locals were often troublesome obstacles in local group theory. Gorenstein and Walter were led to define the *2-layer* $L_{2'}(G)$ of a finite group G to be the normal subgroup of G minimal subject to $L_{2'}(G)O(G)/O(G)=E(G/O(G))$ (Gorenstein and Walter 1971). Moreover, they proved the following crucial property of the embedding of a 2-local subgroup H of G, which they called *L-balance*: $L_{2'}(H)\le L_{2'}(G)$. In addition, they defined a 2-component of G to be a perfect subnormal subgroup K of G with $K/O(K)$ quasisimple.

Gorenstein and Walter began to focus on 2-components in involution centralizers. This eventually led them: to define a simple group G to be of odd characteristic if for some involution t in G, $C_G(t)$ has a 2-component; and to define G to be of even characteristic if for each involution t in G, $F^*(C_G(t))=O_2(C_G(t))$. (Actually, they used somewhat different terminology.) We will see that in time they were able to show that each large simple group is of either odd or even characteristic. This then is the partition of the simple groups via characteristic, promised earlier. Moreover, most simple groups of Lie type over fields of odd order are of odd characteristic with respect to this definition, while all simple groups of Lie type over fields of even order are of even characteristic.

Another innovation of Gorenstein and Walter was the notion of 2-connectivity of *G* at a prime *p*. Let $\mathrm{E}_k^p(G)$ be the set of subgroups of *G* isomorphic to the direct product of *k* copies of the group of order *p*; and regard $\mathrm{E}_k^p(G)$ as a graph, where *A* and *B* are adjacent if they commute elementwise. Motivated in part by work of Suzuki, Thompson, and Bender, Gorenstein and Walter pointed out the importance of determining the finite groups *G* such that $\mathrm{E}_2^2(G)$ is disconnected. One of my early contributions was to essentially determine all such groups, building on earlier work of Bender and Suzuki, who determined the groups *G* for which $\mathrm{E}_1^2(G)$ is disconnected.

Perhaps Gorenstein's most important innovation (this time accomplished without Walter) was his notion of a signalizer functor. This concept was to become the basis for the most effective tool for dealing with the difficulties caused by cores.

For $A \in \mathrm{E}_k^p(G)$, a signalizer functor on *A* is a map θ from the set $A^\#$ of nontrivial elements of *A* to the subgroups of *G*, such that $\theta(a)$ is an *A*-invariant subgroup of $C_G(a)$ of order prime to *p* satisfying $\theta(a) \cap C_G(b) = \theta(b) \cap C_G(a)$ for each $a, b \in A^\#$. The Signalizer Functor Theorem says that if $k \geq 3$ and θ is a signalizer functor on *A*, then the subgroups $\theta(a)$ for $a \in A^\#$ generate a subgroup of *G* of order prime to *p*. Gorenstein introduced the notion of a signalizer functor and proved the first signalizer functor theorem (Gorenstein 1969a,b). Later, stronger theorems were proved by D. Goldschmidt (Goldschmidt 1972a,b), G. Glauberman (Glauberman 1976), Gorenstein and Richard Lyons (Gorenstein and Lyons 1977), and P. McBride (McBride 1982a,b). Goldschmidt simplified Gorenstein's original definition of a signalizer functor to the one appearing above, and Bender gave a slick alternate proof of the signalizer functor theorem for $p = 2$ (Bender 1975).

Using signalizer functors, Gorenstein and Walter proved: If *G* is simple and $\mathrm{E}_2^2(G)$ is connected, then *G* is of odd or even characteristic (Gorenstein and Walter 1972). This result was later improved to the:

Gorenstein-Walter Dichotomy Theorem. If *G* is a finite simple group of 2-rank at least 3, then *G* is either of odd characteristic or even characteristic.

In the late '60s and early '70s, Gorenstein continued to spend much of his time on small groups of odd characteristic. Recall that a simple group *G* of odd characteristic is "small" if its 2-rank $m_2(G)$ is small. From the Dichotomy Theorem, *G* is certainly small when $m_2(G) \leq 2$, and we've seen that, in collaboration with Walter, Alperin, and Brauer, Gorenstein was able to determine all such groups. But when $m_2(G)$ is 3, or in some cases even 4, certain generic arguments sometimes do not go smoothly. Thus in Gorenstein and Harada (1974), he and Koichiro Harada were led to classify the simple groups of sectional 2-rank at most 4. Here a section of G is a group H/K, where $K \trianglelefteq H \leq G$. This work completed the classification of groups in one of the four blocks—the small-rank groups of odd characteristic—of the partition of the simple groups.

Gorenstein and his collaborators had successfully dealt with the small-rank groups of odd characteristic. At about the same time, in the mid-'70s, an approach for dealing with the remaining groups of odd characteristic was taking shape; indeed, that approach was not too different from the one sketched by Gorenstein in his Chicago lectures: First, establish what came to be called the B-conjecture, or as he phrased the problem, show that in a simple group G, $L_{2'}(C_G(t)) = E(C_G(t))$ for each involution t in G. Then show for some involution t, $C_G(t)$ is in standard form; that is, $C_G(t)$ closely resembles an involution centralizer in some known simple group. Finally, solve the corresponding standard-form problems: Prove that if $C_G(t)$ is in standard form for some involution t in G, then G is a known simple group.

With the groups of odd characteristic seemingly well in hand, Gorenstein turned to the groups of characteristic 2. This was also the time when he began his collaboration with Lyons, a relationship that was to continue until his death. Gorenstein and Lyons began to study groups of even characteristic by exploring various special subclasses as test cases. Initially they drew on Thompson's work in the N-group papers (Thompson 1968, 1970, 1971, 1973, 1974a, 1974b), in which he introduced an important invariant $e(G)$ of a group G of even characteristic. For p an odd prime, define the 2-local p-rank $m_{2,p}(G)$ to be the maximum of the p-ranks $m_p(M)$ over all 2-local subgroups M of G. Then define $e(G)$ to be the maximum over all odd primes p of the 2-local p-ranks of G. If G is of Lie type over a field of even order, then $e(G)$ is a good approximation of the Lie rank of G. Thus the small groups G of even characteristic are those with $e(G)$ small. Eventually Gorenstein and Lyons decided to concentrate on the groups G with $e(G) \geq 3$, with emphasis on those with $e(G) \geq 4$.

As usual, we need some definitions. Define a group G to be of $GF(2)$-type if G is of even characteristic and for some involution t in G, $O_2(C_G(t))$ is of symplectic type. A 2-group Q is of symplectic type if Q has no noncyclic characteristic abelian subgroups. The groups of $GF(2)$-type include most groups of Lie type over the field of order 2, plus some sporadic groups and a few groups of Lie type over the field of order 3.

Define a simple group G to be of uniqueness type if G is of even characteristic and, for some maximal 2-local subgroup M of G and for all odd primes p with $m_{2,p}(G) \geq 4$, M is almost strongly embedded in G. I won't burden the reader with the precise definition of "almost strong embedding," but I will say only that it includes the condition that M contains a connected component of $\mathrm{E}_2^p(G)$ for each odd prime p such that $m_{2,p}(G) \geq 4$.

A finite group G is said to be a κ-group if all its simple sections are in the class κ of "known" simple groups appearing in the statement of the classification theorem. Observe that if G is a minimal counterexample to the classification, then all proper subgroups of G are κ-groups.

In their 730-page article, Gorenstein and Lyons (1983) proved the:

Gorenstein-Lyons Trichotomy Theorem. Assume G is a finite simple group of even characteristic with $e(G) \geq 4$ and all proper subgroups of G are κ-groups. Then one of the following holds:

(1) G is of $GF(2)$-type.

(2) G is of uniqueness type.

(3) G is of standard type.

Here G is of standard type if for some odd prime p with $m_{2,p}(G) \geq 4$, the p-local structure of G resembles that of some group of Lie type over a field of even order. We must then use this p-local information to show that G is in κ. In so doing, we are switching our attention from the prime 2 to the odd prime p. In Gorenstein's 1972 program he also foresaw a change in primes, but there he suggested that p should be 3. In the end, it developed that a more judicious choice of p was necessary, but the fact remains that the ultimate approach did involve a change in primes; one reason was that the centralizers of semisimple elements seemed to be easier to work with than the centralizers of unipotent elements. But we can't totally leave the prime 2 behind, because we have a classification of groups G in which $\mathrm{E}_1^2(G)$ is disconnected, but not a classification of those G with $\mathrm{E}_1^p(G)$ disconnected for some odd prime p. This is the reason why the groups of uniqueness type must be treated.

Actually, Gorenstein and Lyons prove more, because they also obtain somewhat weaker results when $e(G) = 3$.

The effect of the Trichotomy Theorem is to reduce the classification of the generic simple groups of even characteristic to three special, albeit difficult, problems: determine all simple groups of each of the three types. Indeed, there are no groups of uniqueness type, so the challenge there is to obtain a contradiction. Of course, it also remains to determine the small groups G of even characteristic. They consist of the quasithin groups G with $e(G) \leq 2$ and the groups G of even characteristic with $e(G) = 3$.

The Gorenstein-Lyons memoir is comparable in depth, scope, and complexity to those of the best papers in finite simple group theory, such as the Odd Order Paper (Feit and Thompson 1963) and the N-group papers (Thompson 1968, 1970, 1971, 1973, 1974a, 1974b). The result was one of the last steps completed in the proof of the classification theorem, and one of the most difficult.

Sometime in 1981, as if by magic, there emerged a consensus among finite group theorists that the finite simple groups were classified. Unfortunately, this assessment was premature. Normally, a major theorem in mathematics is deemed to be proven when a manuscript containing the purported proof is circulated, read, and assessed by the community, refereed, and published. But the proof of the classification consists of numerous papers—too many in number and too intricately related to keep track of without a detailed outline, and such an outline probably did not exist in 1981, unless possibly it occupied a drawer in Gorenstein's desk. Moreover, some of those papers were not as yet published, and at least one turned out to be incomplete, resulting in a serious gap in the proof.

The gap was the absence of a full treatment of the quasithin groups of even characteristic. As time went by and no paper was published treating this problem, some mathematicians became justifiably skeptical that a complete proof of the classification existed. But by sometime in the '90s, experts had concluded that a gap did indeed exist. Eventually, work began so as to close the gap, and roughly 20 years after the classification was originally thought to exist, a classification of the quasithin groups appeared in Aschbacher and Smith (2004), finally completing the proof of the classification of the finite simple groups.

In any event, around 1981, when the finite simple groups were thought to be classified, Gorenstein began two projects. First, he wrote two books that supplied a detailed exposition of parts of the proof of the classification. The first of these books (Gorenstein 1982) contained a discussion of the known simple groups and an introduction to the basic notation, terminology, and concepts needed in the classification. The second book (Gorenstein 1983) provided a detailed outline of the proof of the theorem classifying the simple groups of odd characteristic. Gorenstein planned to write a third volume devoted to the simple groups of even characteristic, but his death prevented its realization.

While writing his two books, Gorenstein came to realize that the proof of the classification could not be left in its then-current state. So he decided to begin a program (the second of his two projects) to write out a simplified, self-contained proof of the classification that would be more accessible, thereby placing the classification on more solid ground. To aid him in this monumental effort, he recruited Lyons and Ronald Solomon as his major collaborators, and he subcontracted certain parts of the job to others. The revised proof was to appear in a series of volumes published by the American Mathematical Society. Unfortunately, Gorenstein died before he could see any of the volumes in print. However, Lyons and Solomon continued the effort, and to date six volumes have appeared in the series, with more in the works (Gorenstein, Lyons, and Solomon 1994, 1996, 1998, 1999, 2002, 2005).

Gorenstein had a talent for interacting with people of all kinds. I was always surprised at the sheer number and variety of his friends. In his later years at Rutgers, the administration relieved him of his teaching duties so that he could serve as a troubleshooter—for example, by chairing various difficult committees—where his skill at working with people made him particularly effective. Still, he devoted lots of time to mathematics. Typically, mornings were reserved for mathematics and afternoons for more mundane tasks.

My friend Danny Gorenstein enjoyed good food, drink, and art. During an extended visit to Caltech when Danny was a Sherman Fairchild Distinguished Scholar, my wife and I had the pleasure of accompanying him to various famous Los Angeles restaurants of the day, such as Scandia and Rex (although he thought the portions at Rex were too small). Danny was a good conversationalist too, and from time to time he liked to argue politics with some of his more conservative colleagues; but these arguments were always good-natured.

I'm not much of a storyteller, so I'm including two anecdotes from Ron Solomon about Gorenstein:

Danny described going to a restaurant in northern Italy with some other mathematicians. After ordering his meal, he noticed several other diners getting served plates of mushrooms that looked tasty. So he asked the waiter to bring him some too. When the bill came it included one very expensive item; when Danny challenged it, he was told it was for the mushrooms he ordered. He had been so busy talking math that he hadn't even noticed eating them.

Danny and Ron entered a dining hall at Rutgers, deep in conversation about some group theory problem. Then Danny saw some administrators he needed to talk to, and excused himself. A while later he returned and resumed his and Ron's discussion of group theory at the exact point he had left it, as if there had been no interruption.

Danny died from cancer at his summer home in Martha's Vineyard on August 26, 1992. He is survived by his wife Helen of 45 years, his son Mark, his three daughters Diana Dalsass, Phyllis Erickson, and Julia Nourok, and his three grandchildren.

Danny's untimely death was a great loss to finite group theory. The Gorenstein-Lyons-Solomon revision project may be the only chance to produce a more readable proof of the classification, written down in one place, as the expertise that produced the proof is fast disappearing. But without Danny's boundless energy and enthusiasm to drive it, the project is not yet complete; if he were still around to crack the whip, things might well be different.

For Danny's friends, the loss is greater. Speaking for myself, he was one of the handful of people that I deeply enjoyed spending time with. I always looked forward to his visits or to meetings we would both attend, because I enjoyed his company so much. Mathematics conferences have not been the same without Danny, and life has not been as interesting.

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