

Luminosity dependence of the cyclotron line and the accretion regime transition in 2015 giant outburst of V 0332+53

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ABSTRACT

We report on the analysis of *NuSTAR* observations of the Be-transient X-ray pulsar V 0332+53 during the giant outburst in 2015. We confirm the cyclotron-line energy – luminosity correlation previously reported in the source, and for the first time observe a significant departure from this correlation below $L \sim 10^{37}$ erg s⁻¹. We interpret this finding as first observational evidence for the transition from super- to sub- critical accretion associated with the disappearance of the accretion column. We also discuss the discrepancy in cyclotron line energies corresponding to the same luminosities in rising and declining parts of the outburst and based on the spin evolution of the source conclude that it is likely caused by a change of the emission region geometry rather than the accretion-induced decay of the magnetic field as previously suggested.

Key words: X-rays: binaries – X-rays: individual: V 0332+53.

1 INTRODUCTION

In binary systems accretion of matter supplied by non-degenerate companion onto a strongly magnetised ($B \sim 10^{12}$ G) rotating neutron star results into pulsed X-ray emission from the vicinity of neutron stars (NSs) magnetic poles. The plasma is channeled to the polar caps by the magnetic field of the neutron star which has also profound effect on the observed X-ray spectra. In particular, the motion of the electrons in strong magnetic field is quantised which gives rise to the so-called cyclotron resonance scattering features (CRSFs, see [Mushtukov et al. \(2016\)](#) for a recent review). A single (fundamental) or multiple harmonics ([Santangelo et al. 1999](#)) absorption-like features can be observed in X-ray band depending on conditions in the line forming region. The energy of the fundamental is related to the magnetic field strength in the line forming region $E_{\text{cycl}} \sim 12\text{keV } B/10^{12} \text{ G}$, thus providing a unique possibility to measure it.

The structure of the emission region in the vicinity of the NS and thus the origin of the CRSF are, however, uncertain. At low accretion rates most of the observed emission

likely comes directly from the accretion mounds on the polar caps of the NS where the gravitational energy of the flow is released. However, the observed luminosities of bright pulsars by far exceed the local Eddington limit for kilometre-sized polar caps. At high accretion rates, the plasma must be, therefore, stopped above the NS surface by the radiative pressure and the observed emission has to emerge from the extended “accretion column” ([Basko & Sunyaev 1976](#); [Becker et al. 2012](#); [Mushtukov et al. 2015b](#)). Conditions for the transition between the two regimes are determined by the largely unknown geometry of the column, and by the angular- and energy-dependent plasma opacities which makes it extremely hard to make robust theoretical predictions on the transition luminosity ([Mushtukov et al. 2015a,b](#)).

On the other hand, analysis of luminosity dependence of the observed properties of X-ray pulsars reported for several sources ([Tsygankov et al. 2006](#); [Staubert et al. 2007](#); [Klochkov et al. 2012](#)) might help to constrain the critical luminosity observationally. Indeed, in low luminous sources the CRSF energy typically increases with the flux, whereas at higher

accretion rates an anti-correlation is observed. As discussed by [Staubert et al. \(2007\)](#), [Becker et al. \(2012\)](#), [Mushtukov et al. \(2015a\)](#), and [Mushtukov et al. \(2015c\)](#), this behaviour could hint on the two accretion regimes corresponding to sub- and super-critical accretion. Observing the transition between the two regimes in a single source would strongly support this interpretation. In this letter we report on the analysis of the CRSF luminosity dependence in the *Be*-transient X-ray pulsar V 0332+53 during the giant outburst in 2015 which provides the first evidence for such transition.

2 OBSERVATIONS AND DATA ANALYSIS

We focus on the analysis of five dedicated *NuSTAR* observations aimed to detect the transition from super- to sub-critical accretion regime. The observation log is presented in Table 1. In addition, we used *Swift*/XRT data contemporary to the *NuSTAR* observations to extend the low-energy coverage. To verify our results we analysed also *INTEGRAL*/SPI observations of the source obtained during the outburst (*INTEGRAL* revolutions 1565, 1570, and 1596). The *NuSTAR* data reduction was carried out using the HEASOFT 6.18 package and current calibration files (CALDB version 20160325) and standard data reduction procedures as summarised in the instruments documentation. Source spectra were extracted from a region of 120'' radius around the V 0332+53. The background spectra were extracted from a circular region of 80'' radius as far away from the source as possible for each observation. The spectra for two *NuSTAR* units were extracted and fitted simultaneously between 5 and 79 keV for each observation ([Fürst et al. 2013](#)). To extract the *Swift*/XRT spectra we used the Swift data products service provided by the UK Swift Science Data Centre¹ as described in [Evans et al. \(2009\)](#). All spectra were grouped to at least 25 counts per bin.

INTEGRAL observed V 0332+53 four times during spacecraft revolutions 1565, 1570, 1586 and 1596. Problems with the energy calibration of IBIS/JEM-X telescopes did not permit us to reconstruct the source spectrum properly. However, we were able to do it for three observations where the SPI spectrometer was operating (detectors annealing was performed during revolution 1586). The *INTEGRAL*/SPI data were screened and reduced in accordance with the procedures described by [Churazov et al. \(2011, 2014\)](#). The XSPEC version 12.9 was used to fit the spectra.

The broadband spectrum of the source has been previously described ([Tsygankov et al. 2006](#); [Lutovinov et al. 2015](#)) using a power law with cutoff at high energies (CUTOFFPL model in XSPEC) modified by interstellar absorption and one to three broad absorption features accounting for the CRSF at ~ 26 keV and its harmonics at ~ 50 keV and ~ 72 keV ([Tsygankov et al. 2006](#)). To model the CRSF and the first harmonic we use the multiplicative lorentzian profile in form of $L(E) = 1/(1 + ((E - E_{\text{cycl}})/(\sigma/2))^2)$ rather than the pseudo lorentzian model (CYCLABS in Xspec) used by [Tsygankov et al. \(2006\)](#). Indeed, this model was designed to mimic the high energy cutoff ([Mihara et al. 1990](#)) which is already included in the continuum model. As a consequence,

the measured line centroid E_0 becomes coupled to the cutoff energy and shifted by σ^2/E_0 with respect to true centroid ([Nakajima et al. 2010](#)) which complicates interpretation of the results.

On the other hand, we found that the gaussian profile yields a slightly worse fit with systematic $\sim 1-2\%$ residuals around the line, especially for CUTOFFPL continuum. This behaviour has been reported by [Pottschmidt et al. \(2005\)](#) and [Nakajima et al. \(2010\)](#) for 2005 outburst and was interpreted as evidence for a complex CRSF profile. We find, however, that the magnitude of the residuals depends on the continuum model used (for instance, they essentially disappear for HIGHECUT model). Furthermore, restricting the energy range to 20–80 keV or using a lorentzian line profile results in no significant residuals for any continuum model. We conclude, therefore, that given the existing uncertainties in modeling of the broadband continuum of X-ray pulsars, there is no evidence for a more complex line profile. This conclusion is consistent with Swift/BAT results ([Cusumano et al. 2016](#)) where gaussian line provided adequate description of the data.

We verified that measured CRSF centroid does not depend on the continuum or line model used and is well constrained for *NuSTAR* observations. In particular, we measured consistent CRSF energies (within the uncertainties) using the broadband fits of *NuSTAR*+*Swift*/XRT data and HIGHECUT or a comptonisation model CompTT by [Titarchuk \(1994\)](#), as well as for *NuSTAR* data in 20–80 keV range and the CUTOFFPL model for either lorentzian and gaussian line profiles. In all cases inclusion of the additional soft blackbody component with temperature of ~ 0.4 keV improves the fit for the XRT data, although, taking into account large systematic uncertainties in window-timing mode it is unclear whether this component is real. For all models we also accounted for interstellar absorption. It was sufficient to assume the absorption column fixed to the average value of $2 \times 10^{22} \text{ cm}^{-2}$ for all observations. We note that the absorption column is similar to one derived from XRT observations in later phases of the outburst ([Tsygankov et al. 2016](#)). Neither component significantly affects the derived CRSF parameters. The CompTT continuum model provides, however, the most stable and consistent fit for all observations, therefore, we use this model combined with the lorentzian profile for the CRSF for the rest of analysis. On the other hand, *INTEGRAL* data does not allow to reliably constrain the continuum parameters, so we fix these to the values derived from the *NuSTAR* data at closest luminosity and only fit for the CRSF parameters. Again, we have verified that the derived line parameters are not significantly affected by choice of the continuum model also in this case. The best-fit results are presented in Fig. 1-3 and Table. 1.

We also carried out the pulse-phase resolved analysis of all *NuSTAR* observations. The complex spin frequency evolution ([Doroshenko et al. 2016](#)) and long intervals between the individual observations prevented us from obtaining a single phase-coherent timing solution, so we phased the individual pulse profiles using the main minimum in 3–17 keV energy band which is present in all observations as a reference. In all cases the phase dependence of the CRSF energy seems to be similar and exhibits a single peak correlated with flux as shown in Fig. 4.

¹ http://www.swift.ac.uk/user_objects/

Table 1. Observation log and best-fit results for the phase averaged spectrum using the CompTT model. For INTEGRAL/SPI some of the parameters (shown in *italic*) were fixed to values derived from *NuSTAR* data at closest luminosity level.

Obs. ID.	Date MJD	Exposure ks	E_{cycl} keV	σ_{cycl} keV	τ_{cycl}	T_0 keV	kT_e keV	τ	$L_{37, 3-80 \text{ keV}}$ $10^{37} \text{ erg s}^{-1}$	χ^2/dof
80102002002	57223	10.5	28.04(2)	7.0(2)	0.863(5)	1.35(2)	5.5(1)	16.9(2)	15.7	1.16/2882
80102002004	57276	14.9	28.24(3)	6.4(1)	0.862(2)	1.22(2)	5.9(1)	17.7(2)	7.8	1.19/2625
80102002006	57282	17.3	28.44(3)	6.9(1)	0.882(3)	1.25(4)	8.0(8)	18.4(4)	5.4	1.05/2601
80102002008	57296	18.2	28.83(4)	5.4(1)	0.885(3)	0.19(5)	6.9(2)	18.5(2)	1.7	1.08/1723
80102002010	57300	20.1	28.71(5)	5.1(1)	0.881(4)	0.78(5)	6.8(2)	17.3(1)	0.9	1.04/1374
rev. 1565	57220	67.8	28.3(1)	8(1)	0.87(2)	<i>1.35</i>	<i>5.5</i>	≤ 16	~ 18	0.95/109
rev. 1570	57233	141.	27.3(2)	9(1)	0.86(2)	<i>1.35</i>	<i>5.5</i>	11(2)	~ 27	1.25/109
rev. 1596	57302	120.	28.2(6)	5	0.7(8)	<i>0.78</i>	<i>6.8</i>	<i>17.3</i>	~ 0.8	0.97/112

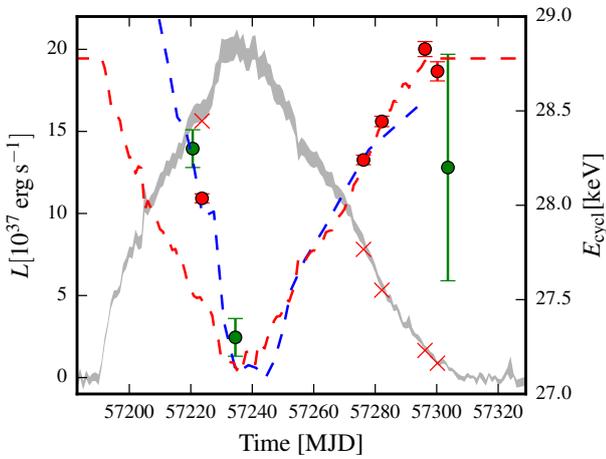


Figure 1. *Swift*/BAT light curve in 15–50 keV band (grey) of V 0332+53 during the 2015 giant outburst scaled to match the source luminosity measured using *NuSTAR* pointed observations (red crosses). The red and green error bars indicate the CRSF fundamental energy as measured by *NuSTAR* and *INTEGRAL*/SPI respectively. The red dashed line shows the model prediction for the fundamental energy based on the CRSF energy versus luminosity correlation measured by *NuSTAR* in the declining part of the outburst. The blue dashed line shows the fundamental energy reported by Cusumano et al. (2016) shifted by 1.2 keV. The shift is likely due to the difference in absolute energy scale of the *Swift*/BAT with respect to *NuSTAR* and SPI.

3 DISCUSSION

3.1 CRSF in brightening and declining phases of the outburst

The source is known to exhibit an anti-correlation of the CRSF centroid energy with luminosity which is believed to be caused by change of the accretion column height and was studied extensively during the 2004–2005 outburst (Tsygankov et al. 2006, 2010; Poutanen et al. 2013; Lutovinov et al. 2015). The *NuSTAR* and *INTEGRAL* observations reveal similar behaviour, although with two important differences. First, line energies measured during the brightening and declining phases of the 2015 outburst seem to be systematically different for comparable luminosity levels which was not the case during the previous outburst (Tsygankov et al. 2010). Both the *NuSTAR* and *INTEGRAL* data yield higher

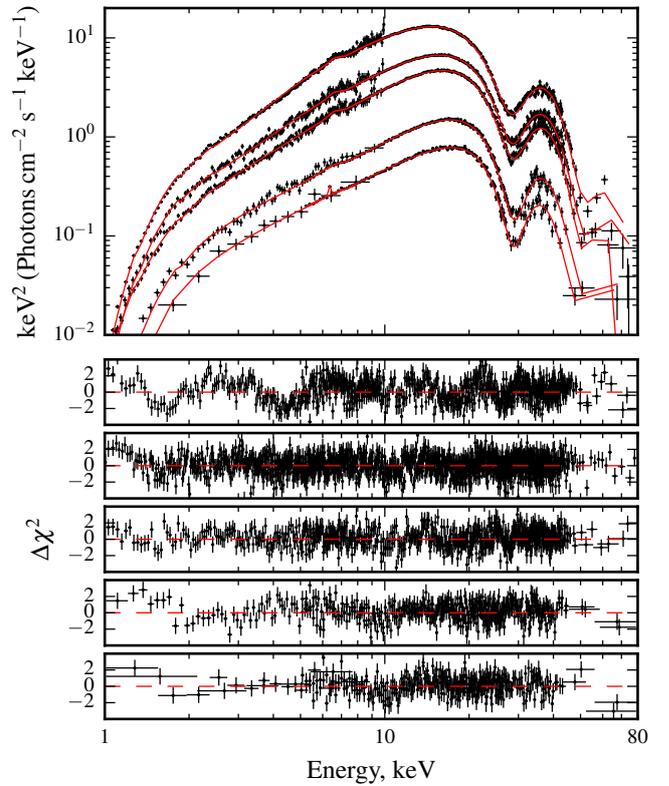


Figure 2. Unfolded spectra as observed by *NuSTAR* (5–80 keV) and *Swift*/XRT (1–10 keV), and the corresponding best-fit residuals for the brightest to the dimmest observations (top to bottom, top panel).

CRSF centroid energies at the same luminosity during the rising part in comparison to the decline part as it shown in Fig. 1 and Fig. 3. This result is consistent with recent report by Cusumano et al. (2016) who measured up to ~ 1.5 keV higher CRSF energies during the rising part of the outburst based on the *Swift*/BAT data (the blue dashed line in Fig. 1).

Cusumano et al. (2016) suggested that the observed decrease of the CRSF energy is caused by the accretion-induced decay of the intrinsic magnetic field of NS. We note that in this case the spin evolution of the pulsar is also expected to be affected as the net torque exerted onto the NS by the accretion flow depends on the magnetosphere size (Rappaport & Joss 1977). In particular, magnetic field

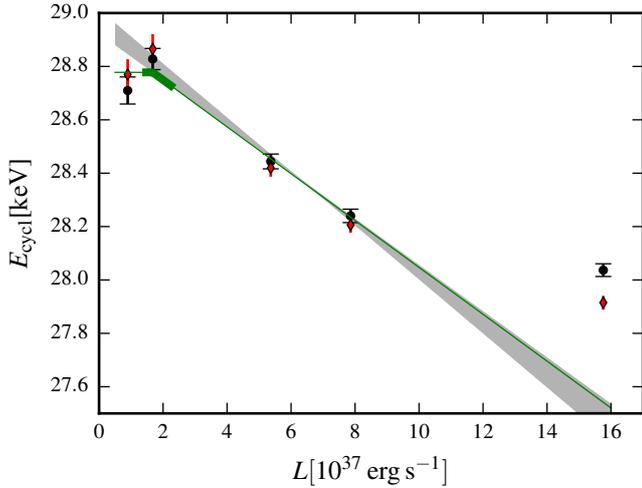


Figure 3. Dependence of fundamental centroid energy on luminosity from *NuSTAR* and *Swift*/XRT observations in 2015 using the *CompTT* model with lorentzian line profile for the CRSF (red points), and *NuSTAR* data in 20–80 keV energy range using the *CUTOFFPL* model with gaussian line profile for the CRSF (black points). The shaded area represents 1σ confidence region for CRSF energy – luminosity correlation fit obtained by fitting three intermediate luminosity points during the decline of the outburst. The green line shows the best-fit broken linear relation.

decay implies decrease of the magnetosphere size and the spin-down torque. Therefore higher spin-up rate for a given luminosity could be expected during the declining phase of the outburst (Ghosh & Lamb 1979; Lipunov et al. 1981; Wang 1987). However, the opposite was observed (Doroshenko et al. 2016) which is a strong argument against the magnetic field decay interpretation suggested by Cusumano et al. (2016). The reason for the observed decrease of the spin-up rate is unclear and we can only speculate that it is connected with the viscous evolution of the accretion disc during the outburst. Indeed, the accretion disc is expected to have higher surface density and thus might push further into the magnetosphere during the rising phase of the outburst resulting in higher spin-up rate. Difference of the outburst-long light curves of the current and 2005 outbursts suggests that while the total accreted mass was similar (Cusumano et al. 2016) the disc structure was different in two cases which could explain absence of hysteresis effects in 2005.

So what could be the reason for the observed CRSF energy change then? The CRSF energy in V 0332+53 is known to change by $\sim 1 - 2$ keV within the pulse phase as indicated by our phase resolved analysis and reported previously by Lutovinov et al. (2015). The magnetic field variation within the line forming region is thus comparable with that implied by the observed change of the CRSF energy from brightening to the decaying parts of the outburst, so it can not be excluded that the observed line energy decreases in the declining phase is related to a change of the geometry of line forming region rather than intrinsic field drop. For instance, in context of model by Poutanen et al. (2013), the CRSF is formed during reflection off the NSs atmosphere, so the observed change in CRSF energy would correspond to a change of its illumination pattern. Indeed, increase of the magnetospheric radius during the declining part of the

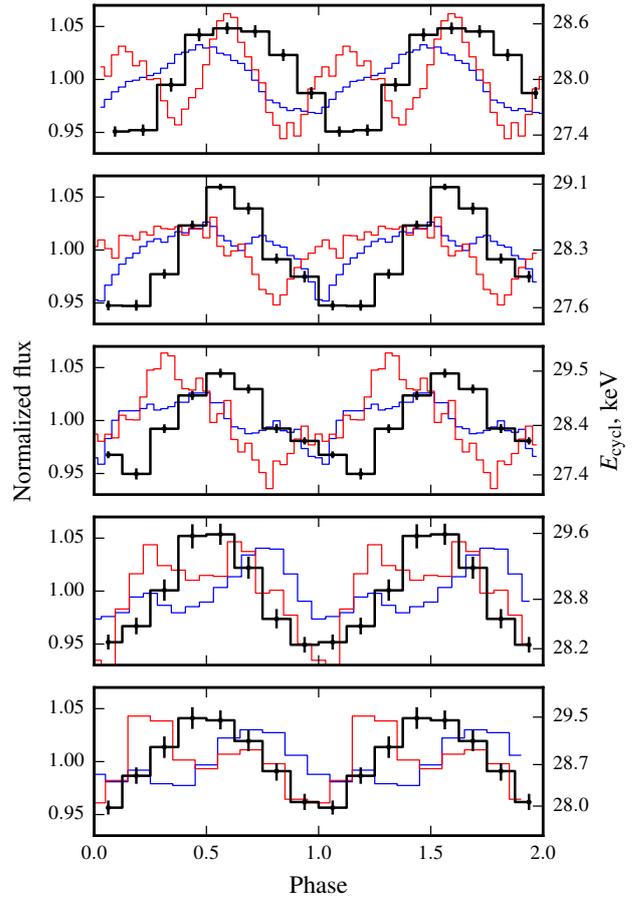


Figure 4. Pulse phase dependence of the CRSF energy as observed by *NuSTAR* (black histogram) at different luminosities ($L/10^{37} \text{ erg s}^{-1} = 15.6, 7.8, 5.3, 1.7, 0.9$ top to bottom). The pulse profiles in 3–17 and 17–40 keV range (blue and red steps) are also shown. The pulse phase of individual observations was aligned using the main pulse minimum in 3–17 keV band.

outburst is expected to reduce the footprint of the accretion column as plasma is expected to follow the field lines which are closer to the magnetic pole in this case. Decrease of the footprint implies that the accretion column becomes taller for a given luminosity and thus more effectively illuminates equatorial regions of the NS resulting in lower observed CRSF energy as illustrated schematically in Fig. 6.

Changes in the emission region geometry must be reflected in pulse profile shape. While no direct comparison of the pulse profiles in brightening and declining parts of the outburst is possible due to the lack of observations at comparable luminosities, the pulse profiles observed during the rising phase of the outburst (top panel in Fig. 4) do appear significantly different compared to other observations where the more gradual evolution is observed. On the other hand, in context of the reflection model almost entire NS surface is illuminated in both cases, so the phase dependence of CRSF parameters is not expected to change significantly, which is consistent with the results of phase resolved analysis (see Fig. 4.)

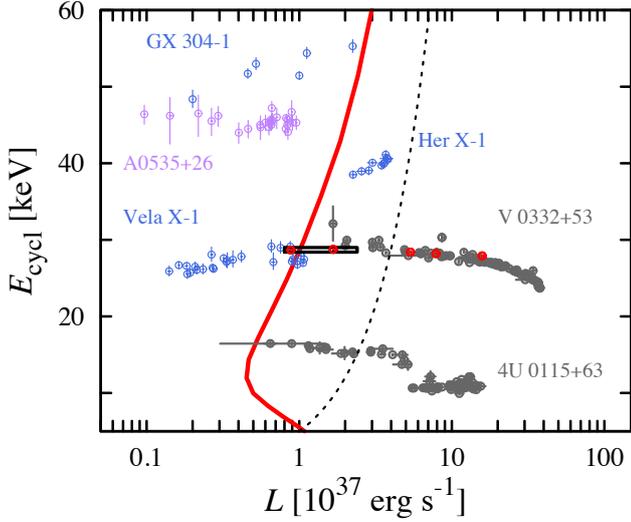


Figure 5. CRSF energy as function of luminosity for sources where either a correlation or anti-correlation was reported in the literature. The red circles correspond to *NuSTAR* data reported in this work. The black box corresponds to the estimated value of the transitional luminosity. The red solid curve corresponds to the critical luminosity value (Mushtukov et al. 2015a) for the case of $\Lambda = 0.5$, where Λ is the ratio of magnetospheric radius to Alfvén radius, and radiation dominated by *X*-mode polarisation.

3.2 The critical luminosity

Taking into the account the observed difference in CRSF energies measured during the rising and declining part of the outburst we focus on the declining part further on. The CRSF energies measured by *NuSTAR* show there similar trend as reported by Tsygankov et al. (2006) with $E_{\text{cycl}} = 28.97(5) - 0.094(7)L_{37}[\text{keV}]$. However, this correlation seems to break at the lowest flux (see Fig. 3). The centroid energy is very well constrained in the last two *NuSTAR* observations, and the line energy is actually slightly lower during the dimmer observation. To test whether the last observation follows the same correlation as the others during the declining phase one can compare the observed CRSF energy 28.71(5) keV with 28.88(4) keV expected from the extrapolating the correlation. These turn out to be different at $\sim 99.7\%$ significance. Alternatively, one can estimate probability that the break of the linear trend occurs at luminosity exceeding the lowest observed luminosity by fitting a broken linear model explicitly, which yields $\sim 97\%$ significance.

The break of the correlation is accompanied by a change of the pulse profile shape (see Fig. 4). The peak at phases 0.25–0.5 visible in soft band (3–17 keV) at higher luminosities disappears, and the second peak at phases 0.75–1 becomes dominant instead. The hard band (17–40 keV) shows opposite behaviour. This indicates a significant change of the intrinsic pulsars beam at lower luminosities, most likely from from a “fan”-like to a “pencil”-like beam which are expected to be shifted by half phase from one another. Such a transition can be associated with the recession of the accretion column which emits predominantly sideways (i.e. produces “fan”-like beam) while emission from accretion mounds is expected to produce “pencil”-like beam. The pulse-resolved behaviour

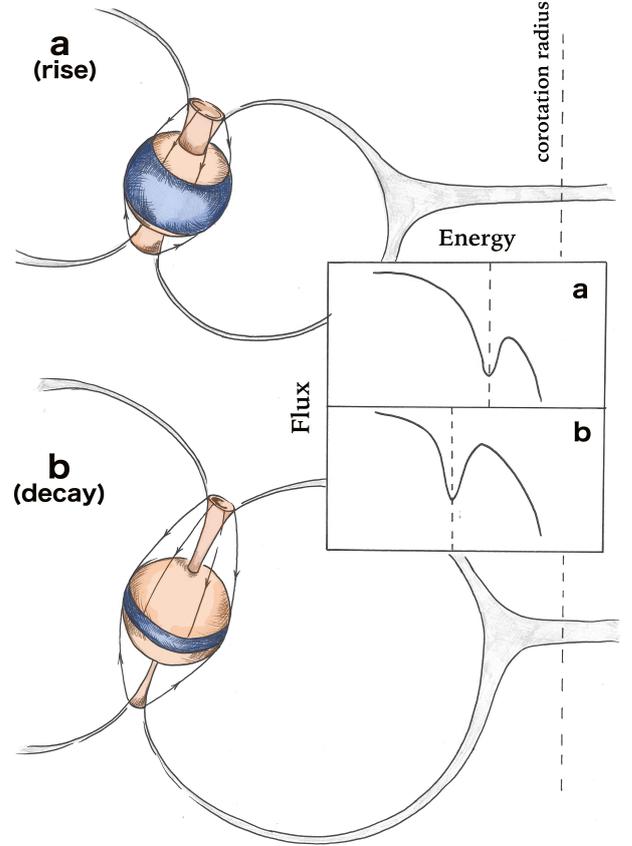


Figure 6. Change of the accretion disc structure and of the magnetospheric radius (with respect to the co-rotation radius indicated by vertical line) change both the net torque exerted onto the NS, and the accretion column height. During the declining phase the matter falls closer to the polar areas which increases the height of the accretion column so that illumination pattern shifts to the equatorial areas implying lower observed CRSF energy.

of the cyclotron line at the lowest observed luminosity is also in agreement with predictions highlighted in Mushtukov et al. (2015c) for sub-critical XRPs: the phase-resolved cyclotron line centroid energy and the width are negatively and positively correlated with the pulse intensity, respectively. Spectral analysis of the *INTEGRAL*/SPI data at even lower flux is also consistent with presence of a break.

A formal broken linear fit to the correlation implies the transitional luminosity of $L_{\text{crit}} = 1.7(5) \times 10^{37} \text{ erg s}^{-1}$, i.e. in agreement with theoretical predictions (see Fig. 5 and discussion in Mushtukov et al. 2015a) which further supports our interpretation. We conclude, therefore, that for the first time the critical luminosity has been measured.

4 CONCLUSIONS

Based on the analysis of *NuSTAR* observations of V 0332+53 during the declining part of the 2015 giant outburst we have confirmed the previously reported anti-correlation of CRSF energy with luminosity. We also confirm lower CRSF energies measured in the declining part recently attributed

by Cusumano et al. (2016) to the accretion-induced decay of the magnetic field of the NS. We find, however, that the later conclusion is inconsistent with the observed spin evolution of the source which would imply higher magnetic field during the declining phase. Alternatively, a change of the magnetosphere size could explain observed spin evolution and CRSF behaviour. Indeed, decrease of the magnetosphere size implies expansion of the emission and line forming regions and thus lower CRSF energies in context of the reflection model.

Finally we find that at luminosities below $\sim 1.6 \times 10^{37} \text{ erg s}^{-1}$ the correlation breaks, which we interpret as the first observational evidence for the transition from super- to sub-critical accretion. The transitional luminosity is in agreement with the theoretical predictions and cyclotron line luminosity dependence observed in other sources as shown in Fig. 5.

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